

**Homework 2: Asymptotics**

Handed out Thursday, September 22. Due at the start of class Tuesday, October 4, 2011 .

**Problem 1.** Use the formal definitions (not the Limit Rule) to establish the following. In each case state specific values of the constants (e.g.,  $c_1$ ,  $c_2$ ,  $n_0$ ) you used to satisfy the conditions, and show how you arrived at these values. (There are many potentially correct choices. Explaining your work is thus essential for full credit.)

(a)  $4n^2 + 4n + 12 \in O(n^2 - 4n + 8)$

(b)  $2n^3 - 3n^2 + 17n \in \Theta(n^3)$ .

(c)  $n^2 + 10n \lg^2 n \in O(n^2)$ . (Hint: Find  $n_0$  such that  $\lg^2 n \leq n$ , for all  $n \geq n_0$ .)

**Problem 2.** Repeat Problem 1, but this time use the Limit Rule to show that each function is in the set given in the problem.

**Problem 3.** For each pair of expressions ( $A, B$ ) below, indicate whether  $A$  is  $O$ ,  $o$ ,  $\Omega$ ,  $\omega$ , or  $\Theta$  of  $B$ . Note that zero, one or more of these relations may hold for a given pair; list all correct ones, and explain your work for partial credit.

	$A$	$B$
(a)	$n^{200}$	$4^n$
(b)	$(\log n)^{16}$	$\sqrt{n}$
(c)	$\sqrt{n}$	$n^{\sin(\pi n/4)}$
(d)	$16^n$	$65536^n$
(e)	$n^{\log n}$	$(\log n)^n$

**Problem 4.** Consider the following algorithm to sort the array `Keys`.

```

for i= 1 to n-1 do
  base_j = i;
  base_x = Keys[i]
  for j = i + 1 to n do
    If Keys[j] < base_x then
      base_j = j;
      base_x = Keys[j];
  end forj;
  If base_j < > i then
    Keys[base_j] = Keys[i];
    Keys[i] =base_x;
  end fori;

```

(a) Give a summation for the number of *key swaps* in the worst case and simplify your summation.

- (b) Give a summation for the number of key swaps in the **best** case and simplify your summation.
- (c) Give a double summation for the number of *comparisons* in the worst case and simplify your summation.
- (d) Give a double summation for the number of *comparisons* in the best case and simplify your summation.
- (e) Give a double summation for the number of *comparisons* in the **average** case, but do not solve.