CMSC 311 Computer Organization

Boolean Algebra Worksheet Answers

Spring 2008 Dr. Hugue

Simplify as much as possible.

1. $(AB + C)(AB + C')$
   $= AB(C + C')$ (distributive law (16))
   $= AB(1)$ (identity 7)
   $= AB$ (identity 2)
   `A \& B`

2. $X'W + XW' + XW + X'W'$
   $= X'W' + X'W + XW' + XW$ (commutative law (10))
   $= X'(W' + W) + X(W' + W)$ (distributive law)
   $= (X' + X)(W' + W)$ (distributive law)
   $= 1 \cdot 1$ (identity 7)
   $= 1$ (identity 3)

3. $pqr + rp$
   $= p(qr + r)$ (distributive)
   $= p(q + 1)r$ (distributive)
   $= pr$ (identity 3)
   `p \& r`

4. $(y' + z')xyz$
   $= xyy'z + xyzz'$ (distributive)
   $= 0 + 0 = 0$ (identity 8)

5. $(C + DX)(C + EX)$
   $= C + (DX)(EX)$ (distributive law (15))
   $= C + DEX$ (identity 6)
   `C | (D \& E \& X)`

6. $T(L' + V) + TV'$
   $= T(L' + V + V')$ (distributive law)
   $= T(L' + 1) = T$ (identities 7 and 3)

7. $fg \oplus 1$
   $= 0fg + 1(fg)'$ (definition)
   $= f' + g'$ (DeMorgan's law)
   `~f | ~g`

8. $(m \oplus n) \oplus (m' \oplus n)$
   $= (m \oplus m') \oplus (n \oplus n)$ (XOR is commutative & associative)
   $= 1 \oplus 0 = 1$ (identities)

9. $(uw \oplus (t + u))'$
   $= ((uw)'(t + u) + uw(t + u)')'$ (definition)
   $= ((u' + w')(t + u) + uu'wt)'$ (DeMorgan's)
   $= (u' + w')' + (t + u)'$ (identities & DeMorgan's)
   $= uw + t'u'$ (DeMorgan's once again)
   `u \& w | ~t \& ~u`

10. $(xy+z)'(z(x'+y'))$
$= z'(xy)'z(x'+y') = 0$ (DeMorgan's and identity)

11. $[(x+a)(x+b)(x+c)(x+d)]'$
$= (x+abcd)' = x'(abcd)'$ (Distributive & DeMorgan's)
`~x \& ~(a \& b \& c \& d)`

Prove or Disprove the following expressions

1. $(X \oplus Y)' = (X' \oplus Y)$
$X' \oplus Y = X'Y' + XY = ((X+Y)(X'+Y'))' = (X \oplus Y)'$
True

2. $(A' \oplus T') = (A \oplus T)$
$A' \oplus T' = A''T' + A'T'' = AT' + A'T = A \oplus T$
True

3. $[a'(b+c')]' + b'c = a$
$[a'(b+c')]' + b'c = a + b'c + b'c = a + b'c$
False: let $a = 0$, $b = 0$, $c = 1$

4. $wvx' + wv'x + w'vx' + w'v'x = w \oplus v \oplus x$
$wvx' + wv'x + w'vx' + w'v'x = w(vx' + v'x) + w'(vx' + v'x) = v \oplus x$
False: let $w = v = x = 1$

5. $(A+D)(A+B+D) = (A+B)$
$(A+D)(A+B+D) = (A+D)((A+D)+B) = A+D$
False: let $A = 0$, $B = 1$, $D = 0$

6. $(M'N) \oplus (MN') = M \oplus N$
$M'N \oplus MN' = M'N(M'+N) + MN'(M+N') = M'N + MN'$
True

7. $(x+a)(y+b)(x+c)(y+a)(x+b)(y+c) = (xy+abc)$
$(x+a)(x+b)(x+c)(y+a)(y+b)(y+c) = (x+abc)(y+abc) = xy+abc$
True

8. $(K \oplus L \oplus M \oplus N \oplus P)' = (K' \oplus L' \oplus M' \oplus N' \oplus P')$
$((K \oplus L) \oplus (M \oplus N) \oplus P)' = (K' \oplus L' \oplus M' \oplus N' \oplus P)' = K' \oplus L' \oplus M' \oplus N' \oplus P'$
True