Supplemental Exercises: Unit 2
Scientific Computing with Case Studies
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1. Your research advisor wants to solve a least squares problem

$$\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|.$$

Given $\boldsymbol{b}$ and the results below explain how to solve the problem. (Make clear exactly what you would compute and why.)

```
>> [Q,R] = qr(A)

Q =

   -0.5765   -0.4958    0.2491    0.2479   -0.5127   -0.1884
   -0.1796   -0.1225   -0.8396   -0.3736   -0.3094    0.1119
   -0.4759    0.0858   -0.2201    0.5221    0.4019    0.5325
   -0.4737    0.3507   -0.1337   -0.0877    0.3476   -0.7115
   -0.2391    0.7489    0.2168   -0.1234   -0.5040    0.2566
   -0.3569   -0.2192    0.3460   -0.7095    0.3242    0.3104

R =

   -1.5911   -0.9448   -0.9956   -0.8336   -0.8998
         0    0.9456    0.5002    0.5695   -0.2729
         0         0   -0.0006   -0.0004    0.0099
         0         0         0   -0.0011   -0.6413
         0         0         0         0   -0.1391
         0         0         0         0         0
>> [Qp,Rp,Pp] = qr(A)

Qp =

   -0.5765   -0.4958   -0.1298    0.2511   -0.5536   -0.1884
   -0.1796   -0.1225    0.4180   -0.8460   -0.2231    0.1119
   -0.4759    0.0858   -0.5987   -0.2111    0.2821    0.5325
   -0.4737    0.3507    0.0100   -0.1339    0.3583   -0.7115
   -0.2391    0.7489    0.2307    0.2133   -0.4663    0.2566
   -0.3569   -0.2192    0.6298    0.3365    0.4672    0.3104
```

```
Rp =

   -1.5911   -0.9448   -0.8998   -0.9956   -0.8336
         0    0.9456   -0.2729    0.5002    0.5695
         0         0    0.6563   -0.0000    0.0010
         0         0         0   -0.0006   -0.0004
         0         0         0         0    0.0002
         0         0         0         0         0

Pp =

     1     0     0     0     0
     0     1     0     0     0
     0     0     0     1     0
     0     0     0     0     1
     0     0     1     0     0
```

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2. Suppose we have factored $\boldsymbol{A} = \boldsymbol{QR}$, and that we now want to change the 4th and 5th rows of $\boldsymbol{A}$. Recall the formula

$$(\boldsymbol{A} - \boldsymbol{Z}\boldsymbol{V}^T)^{-1} = \boldsymbol{A}^{-1} + \boldsymbol{A}^{-1}\boldsymbol{Z}(\boldsymbol{I} - \boldsymbol{V}^T\boldsymbol{A}^{-1}\boldsymbol{Z})^{-1}\boldsymbol{V}^T\boldsymbol{A}^{-1}.$$

(a) Define $\boldsymbol{Z}$ and $\boldsymbol{V}$ so that the new matrix is $\boldsymbol{A} - \boldsymbol{Z}\boldsymbol{V}^T$.

(b) Given a vector $\boldsymbol{b}$, describe how to use the formula to solve $(\boldsymbol{A} - \boldsymbol{Z}\boldsymbol{V}^T)\boldsymbol{x} = \boldsymbol{b}$.

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3. Your research advisor wants to solve a least squares problem

$$\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|.$$

The entries in the matrix $\boldsymbol{A}$ were measured with a precision of $\pm 10^{-3}$. Given $\boldsymbol{b}$ and the results on the attached sheet, explain how to solve the problem. (Make clear exactly what you would compute and why.)

```
> [U,S,V] = svd(A)

U =
```

```
   -0.5468    0.5684   -0.1184   -0.2047    0.5656   -0.0464
   -0.3451   -0.3358    0.5244    0.5478    0.2844   -0.3352
   -0.5090   -0.0719   -0.1584   -0.2461   -0.5874   -0.5523
   -0.4232   -0.4894    0.2331   -0.4489    0.0157    0.5703
   -0.3490    0.3332   -0.0486    0.5659   -0.4363    0.5042
   -0.1478   -0.4567   -0.7933    0.2752    0.2522    0.0323
```

S =

```
    3.2143         0         0         0         0
         0    1.1092         0         0         0
         0         0    0.3060         0         0
         0         0         0    0.0006         0
         0         0         0         0    0.0002
         0         0         0         0         0
```

V =

```
   -0.3278    0.5021   -0.4255   -0.6029   -0.3098
   -0.5640   -0.3190    0.6510   -0.2702   -0.2885
   -0.2664    0.2047   -0.1087    0.7069   -0.6129
   -0.6410    0.2826   -0.0163    0.2526    0.6672
   -0.3043   -0.7241   -0.6189   -0.0006   -0.0005
```
