Blind Deconvolution
Fall 2005
Friday afternoon lecture
Dianne P. O'Leary

Blind Image Deblurring through Structured Total Least Norm

Dianne P. O'Leary

Computer Science Dept. and
Institute for Advanced Computer Studies
University of Maryland
oleary@cs.umd.edu

http://www.cs.umd.edu/users/oleary

Coauthors:
Armin Pruessner
Anoop Kalsi
Nicola Mastronardi
Phillip Lemmerling
Sabine Van Huffel

Support: NSF

The Plan

- The deconvolution model
- How we solve it: 1, 2, and ∞ norms
- Savings when the 2-norm is used
- Exploiting sparsity structure
- Results
- Conclusions

The Model

$$(\mathbf{A} + \mathbf{E})\mathbf{x} = \mathbf{b} - \mathbf{r}$$

where

- $\mathbf{A}$ is the estimate of the point spread function (psf)
- $\mathbf{b}$ is the observed image

- $\mathbf{x}$ is the true image
- $\mathbf{E}$ is the error in the psf estimate
- $\mathbf{r}$ is the error in the observed image

In deconvolution, we try to recover $\mathbf{x}$ given $\mathbf{A}$ and $\mathbf{b}$, assuming that $\mathbf{E}$ is zero.

In blind deconvolution, we account for the fact that $\mathbf{E}$ is nonzero.

Ways to estimate $\mathbf{x}$: Least Squares

$$\mathbf{A}\mathbf{x} = \mathbf{b} - \mathbf{r} \qquad (*)$$

- Least squares
$$\min_{\mathbf{x}} \|\mathbf{r}\|_2^2$$
- Regularized Least Squares
$$\min_{\mathbf{x}} \|\mathbf{r}\|_2^2 + \|\mathbf{L}\mathbf{x}\|_2^2$$
where $\|\mathbf{L}\mathbf{x}\|$ measures the size of $\mathbf{x}$ in an appropriate weighted norm.

More ways to estimate $\mathbf{x}$: Total Least Squares

$$(\mathbf{A}+\mathbf{E})\mathbf{x} = \mathbf{b} - \mathbf{r} \qquad (*)$$

- Total Least Squares
$$\min_{\mathbf{x},\mathbf{E}} \|\mathbf{E}\|_F^2 + \|\mathbf{r}\|_2^2$$
subject to (*).
- Structured Total Least Squares
$$\min_{\mathbf{x},\mathbf{E}} \|\mathbf{E}\|_F^2 + \|\mathbf{r}\|_2^2$$
subject to $(*)$, and constraining $\mathbf{E}$ to have the same structure as $\mathbf{A}$: perhaps the same sparsity pattern or other properties.

- Regularized, Structured Total Least Squares

$$\min_{\mathbf{x},\mathbf{E}} \|\mathbf{E}\|_F^2 + \|\mathbf{r}\|_2^2 + \|\mathbf{L}\mathbf{x}\|_2^2$$

  subject to $(*)$, and constraining $\mathbf{E}$ to have the same structure as $\mathbf{A}$

Even more ways to estimate $\mathbf{x}$: Total Least Norm

Instead of the 2-norm, we can use alternatives:

Regularized, Structured Total Least Norm

$$\min_{\mathbf{x},\mathbf{E}} \|\mathbf{E}\|^{\ell} + \|\mathbf{r}\|^{\ell} + \|\mathbf{L}\mathbf{x}\|^{\ell}$$

subject to $(*)$, and constraining $\mathbf{E}$ to have the same structure as $\mathbf{A}$, where $\|.\|$ is some convenient norm.

The Context

- Rosen, Park, Glick: proposed an algorithm for STLN in 1-, 2-, and ∞-norms.
- Kamm and Nagy: developed algorithms for TLS when the matrix is Toeplitz. Proposed direct and iterative methods. Proposed a preconditioner.
- Mastronardi, Lemmerling, Van Huffel: developed a fast approach for RSTLS when the matrix is Toeplitz
- More recently: Fu and Barlow: developed an algorithm for RSTLS for BTTB matrices based on iterative solution of the linearized problems. Proposed a preconditioner.

Our contribution:

- Pruessner and O'Leary: extended Rosen et al STLN algorithm to include RSTLN in each norm and demonstrated its use on image deblurring.
- Kalsi and O'Leary: developed a fast approach for RSTLS for matrices with certain displacement structures including Toeplitz and BTTB.

- Mastronardi, Lemmerling, Kalsi, O'Leary, and Van Huffel: exploited sparsity structure in image deblurring.

The Plan

- The deconvolution model
- How we solve it: 1, 2, and ∞ norms
- Savings when the 2-norm is used
- Exploiting sparsity structure
- Results
- Conclusions

The Setup

$\mathbf{E} \in \mathbf{R}^{m \times n}$ is parameterized by elements of the vector $\boldsymbol{\alpha} \in \mathbf{R}^q$, $q < mn$.

Assume there exists a matrix $\mathbf{X}$ parameterized by $\mathbf{x}$ such that

$$\mathbf{X}\boldsymbol{\alpha} = \mathbf{E}\mathbf{x}.$$

The RSTLN formulation is to find vectors $\boldsymbol{\alpha}$ and $\mathbf{x}$ to minimize

$$\min_{\mathbf{x},\mathbf{E}} \|\mathbf{E}\|^{\ell} + \|\mathbf{r}\|^{\ell} + \|\mathbf{L}\mathbf{x}\|^{\ell}.$$

We will use the vector 1-, 2-, and ∞ norms.

For notation ease, we will take $\mathbf{L} = \lambda \mathbf{I}$ and measure $\|\mathbf{E}\| = \|\mathbf{D}\boldsymbol{\alpha}\|$. This reduces the problem to minimizing

$$\left\| \begin{array}{c} \mathbf{r}(\boldsymbol{\alpha}, \mathbf{x}) \\ \mathbf{D}\boldsymbol{\alpha} \\ \lambda \mathbf{x} \end{array} \right\|_p^{\ell}$$

Now let $\Delta\mathbf{x}$ and $\Delta\mathbf{E}$ denote small changes in $\mathbf{x}$ and $\mathbf{E}$, respectively. Then in order to make $(\mathbf{X}+\Delta\mathbf{X})(\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha}) = (\mathbf{E}+\Delta\mathbf{E})(\mathbf{x}+\Delta\mathbf{x})$, we require

$$\mathbf{X}\Delta\boldsymbol{\alpha} = (\Delta\mathbf{E})\mathbf{x}.$$

If we expand $\mathbf{r}(\boldsymbol{\alpha},\mathbf{x})$ in a Taylor series about $[\boldsymbol{\alpha}^T\ \mathbf{x}^T]^T$ and ignore second order and higher terms, we have

$$\begin{aligned}\mathbf{r}(\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha},\mathbf{x}+\Delta\mathbf{x}) &\approx \mathbf{b}-(\mathbf{A}+\mathbf{E})\mathbf{x}-\mathbf{X}\Delta\boldsymbol{\alpha}-(\mathbf{A}+\mathbf{E})\Delta\mathbf{x}\\ &= \mathbf{r}(\boldsymbol{\alpha},\mathbf{x})-\mathbf{X}\Delta\boldsymbol{\alpha}-(\mathbf{A}+\mathbf{E})\Delta\mathbf{x}.\end{aligned}$$

This linearization results in:

$$\min_{\Delta\boldsymbol{\alpha},\Delta\mathbf{x}}\left\|\begin{bmatrix}\mathbf{X} & \mathbf{A}+\mathbf{E}\\ \mathbf{D} & \mathbf{0}\\ \mathbf{0} & \lambda\mathbf{I}\end{bmatrix}\begin{pmatrix}\Delta\boldsymbol{\alpha}\\ \Delta\mathbf{x}\end{pmatrix}+\begin{pmatrix}-\mathbf{r}\\ \mathbf{D}\boldsymbol{\alpha}\\ \lambda\mathbf{x}\end{pmatrix}\right\|_p.$$

$\mathbf{x}=\mathbf{x}+\Delta\mathbf{x}$ and $\boldsymbol{\alpha}=\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha}$

RSTLN Algorithm

1. Set $\mathbf{E}=\mathbf{0}_{m\times n}$ and $\boldsymbol{\alpha}=\mathbf{0}_{q\times 1}$.
2. Compute $\mathbf{x}$ by $\min_{\mathbf{x}}\|\mathbf{A}\mathbf{x}-\mathbf{b}\|_p$ (for $p=2$ this is just least squares).
3. Compute $\mathbf{X}$ from $\mathbf{x}$ and the residual $\mathbf{r}=\mathbf{b}-\mathbf{A}\mathbf{x}$.
4. For $k=1,2,...$ until $\|\Delta\mathbf{x}\|,\|\Delta\boldsymbol{\alpha}\|\le\epsilon$ repeat Steps 4.1 – 4.3
   4.1. Solve
   $$\min_{\Delta\boldsymbol{\alpha},\Delta\mathbf{x}}\left\|\begin{bmatrix}\mathbf{X} & \mathbf{A}+\mathbf{E}\\ \mathbf{D} & \mathbf{0}\\ \mathbf{0} & \lambda\mathbf{I}\end{bmatrix}\begin{pmatrix}\Delta\boldsymbol{\alpha}\\ \Delta\mathbf{x}\end{pmatrix}+\begin{pmatrix}-\mathbf{r}(\boldsymbol{\alpha},\mathbf{x})\\ \mathbf{D}\boldsymbol{\alpha}\\ \lambda\mathbf{x}\end{pmatrix}\right\|_p.$$
   4.2. Set $\mathbf{x}=\mathbf{x}+\Delta\mathbf{x}$ and $\boldsymbol{\alpha}=\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha}$.
   4.3. Construct $\mathbf{E}$ from $\boldsymbol{\alpha}$, and $\mathbf{X}$ from $\mathbf{x}$ and compute $\mathbf{r}=\mathbf{b}-(\mathbf{A}+\mathbf{E})\mathbf{x}$.
5. The recovered image is $\mathbf{x}$ and the recovered blurring matrix $(\mathbf{A}+\mathbf{E})$.

RSTLN for $p=2$

The minimization problem in the RSTLN formulation is equivalent to minimizing the function:

$$\phi(\boldsymbol{\alpha},\mathbf{x})=\frac{1}{2}\|\mathbf{r}(\boldsymbol{\alpha},\mathbf{x})\|_2^2+\frac{1}{2}\|\mathbf{D}\boldsymbol{\alpha}\|_2^2+\frac{1}{2}\|\lambda\mathbf{x}\|_2^2. \qquad (1)$$

As Rosen et al noted for the STLN method, Step 4.1 is a Gauss-Newton method which approximates the Hessian of $\phi(\boldsymbol{\alpha},\mathbf{x})$ by the positive definite matrix $\mathbf{M}^T\mathbf{M}$, where

$$\mathbf{M}=\begin{bmatrix}\mathbf{X} & \mathbf{A}+\mathbf{E}\\ \mathbf{D} & \mathbf{0}\\ \mathbf{0} & \lambda\mathbf{I}\end{bmatrix}.$$

Special structure should be exploited.

RSTLN for $p = \infty$

The optimal function value in Step 4.1 is $\bar{\sigma}$, where $\bar{\sigma}$ is determined from the linear program

$$\begin{array}{lccccc} \min\limits_{\Delta\boldsymbol{\alpha},\Delta\mathbf{x},\bar{\sigma}} & \bar{\sigma} & & & & \\ \text{subject to} & -\bar{\sigma}\mathbf{e}_m & \leq & \mathbf{X}\Delta\boldsymbol{\alpha} + (\mathbf{A}+\mathbf{E})\Delta\mathbf{x} - \mathbf{r} & \leq & \bar{\sigma}\mathbf{e}_m \\ & -\bar{\sigma}\mathbf{e}_q & \leq & \mathbf{D}\Delta\boldsymbol{\alpha} + \mathbf{D}\boldsymbol{\alpha} & \leq & \bar{\sigma}\mathbf{e}_q \\ & -\bar{\sigma}\mathbf{e}_n & \leq & \lambda\Delta\mathbf{x} + \lambda\mathbf{x} & \leq & \bar{\sigma}\mathbf{e}_n \end{array}$$

where $\mathbf{e}_k \in \mathbf{R}^{\mathbf{k}\times\mathbf{1}}$ is a vector of ones.

RSTLN for $p = 1$

$\bar{\sigma}$ is determined by

$$\min_{\Delta\boldsymbol{\alpha},\Delta\mathbf{x},\bar{\sigma}} \quad \bar{\sigma} = \sum_{i=1}^{m} \bar{\sigma}_{1_i} + \sum_{i=1}^{q} \bar{\sigma}_{2_i} + \sum_{i=1}^{n} \bar{\sigma}_{3_i}$$

$$\begin{array}{lccccc} \text{subject to} & -\bar{\boldsymbol{\sigma}}_1 & \leq & \mathbf{X}\Delta\boldsymbol{\alpha} + (\mathbf{A}+\mathbf{E})\Delta\mathbf{x} - \mathbf{r} & \leq & \bar{\boldsymbol{\sigma}}_1 \\ & -\bar{\boldsymbol{\sigma}}_2 & \leq & D\Delta\boldsymbol{\alpha} + D\boldsymbol{\alpha} & \leq & \bar{\boldsymbol{\sigma}}_2 \\ & -\bar{\boldsymbol{\sigma}}_3 & \leq & \lambda\Delta\mathbf{x} + \lambda\mathbf{x} & \leq & \bar{\boldsymbol{\sigma}}_3 \end{array}$$

where $\bar{\boldsymbol{\sigma}}_1 \in \mathbf{R}^{m\times 1}$, $\bar{\boldsymbol{\sigma}}_2 \in \mathbf{R}^{q\times 1}$, and $\bar{\boldsymbol{\sigma}}_3 \in \mathbf{R}^{n\times 1}$.

Convergence of RSTLN for $p = 1$ or $p = \infty$

As for the STLN problem, the function minimized in RSTLN is nonconvex so that there is no guarantee that the RSTLN algorithm converges to a global minimum.

For the $p = 2$ norm case the Gauss-Newton theory is applicable: a suitable line search method can be used to guarantee convergence to a local minimizer from any starting point.

The Plan

- The deconvolution model
- How we solve it: 1, 2, and ∞ norms
- Savings when the 2-norm is used
- Exploiting sparsity structure
- Results
- Conclusions

How can we make these calculations fast?

- For 1-norm or ∞-norm: open question.
- For 2-norm: Mastronardi, Lemmerling, and Van Huffel present an algorithm for solving fast STLS problems when $\mathbf{A}$ is Toeplitz.
- We have extended their method to
  - RSTLS for Toeplitz matrices.
  - RSTLS for a broader class of matrices.

Generators for $\mathbf{M}^T\mathbf{M}$

Our first tool is the derivation of a generator for the matrix $\mathbf{M}^T\mathbf{M}$ when $\mathbf{M}$ has low displacement rank.

The Displacement Rank of $\mathbf{M}^T\mathbf{M}$

Suppose that $\mathbf{M}$ has low displacement rank relative to the matrices $\mathbf{Z}_1 \in \mathcal{R}^{(m+p)\times(m+p)}$ and $\mathbf{Z}_2 \in \mathcal{R}^{(n+p)\times(n+p)}$, which means that if we define

$$\mathbf{N} \equiv \mathbf{M} - \mathbf{Z}_1\mathbf{M}\mathbf{Z}_2^T ,$$

then rank($\mathbf{N}$) $= \rho_1$, which is small relative to $n+p$.

Suppose

$$\tilde{\mathbf{Z}} = \mathbf{Z}_1 + \mathbf{W}$$

is an orthogonal matrix ($\tilde{\mathbf{Z}}^T\tilde{\mathbf{Z}} = \mathbf{I}$), where $\mathbf{W}$ has rank ρ_2, also assumed to be small.

For example, if $\mathbf{E}$ is Toeplitz, let $\mathbf{Z}_1$ be the shift-down matrix with ones on its subdiagonal and zeros elsewhere, and then $\mathbf{W}$ is the matrix with a one in the last position of row 1.

Then $\mathbf{M}^T\mathbf{M}$ also has low displacement rank relative to $\mathbf{Z}_2$:

Theorem: If the rank of $\mathbf{N} \equiv \mathbf{M} - \mathbf{Z}_1\mathbf{M}\mathbf{Z}_2^T$ is ρ_1 and if the orthogonal matrix $\tilde{\mathbf{Z}}$ is equal to $\mathbf{Z}_1 + \mathbf{W}$ where $\mathbf{W}$ has rank ρ_2, then

$$\begin{aligned}\mathbf{M}^T\mathbf{M} - \mathbf{Z}_2\mathbf{M}^T\mathbf{M}\mathbf{Z}_2^T &= -\mathbf{N}^T\mathbf{N} + \mathbf{N}^T(\mathbf{M} - \mathbf{W}\mathbf{M}\mathbf{Z}_2^T) \\ &\quad +(\mathbf{M}^T - (\mathbf{W}\mathbf{M}\mathbf{Z}_2^T)^T)\mathbf{N} \\ &\quad -(\mathbf{W}\mathbf{M}\mathbf{Z}_2^T)^T(\mathbf{W}\mathbf{M}\mathbf{Z}_2^T) \\ &\quad +\mathbf{M}^T(\mathbf{W}\mathbf{M}\mathbf{Z}_2^T) + (\mathbf{W}\mathbf{M}\mathbf{Z}_2^T)^T\mathbf{M}\end{aligned}$$

has rank at most $2(\rho_1 + \rho_2)$.

Example: Applying the result to STLS problem

Example: Suppose $\mathbf{A}$ is Toeplitz, so

$$\mathbf{M} = \begin{bmatrix} \mathbf{X} & \mathbf{A}+\mathbf{E} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$$

$$\begin{aligned}-\mathbf{N}^T\mathbf{N} &= -\mathbf{r}_1\mathbf{r}_1^T - \mathbf{r}_m\mathbf{r}_m^T - \mathbf{e}_1\mathbf{e}_1^T \\ &\quad -(\mathbf{c}_p^T\mathbf{c}_p)\mathbf{e}_{p+1}\mathbf{e}_{p+1}^T + \mathbf{r}_m\mathbf{e}_1^T + \mathbf{e}_1\mathbf{r}_m^T \\ \mathbf{N}^T(\mathbf{M} - \mathbf{W}\mathbf{M}\mathbf{Z}_2^T) & \\ +(\mathbf{M}^T - (\mathbf{W}\mathbf{M}\mathbf{Z}_2^T)^T)\mathbf{N} &= 2\mathbf{r}_1\mathbf{r}_1^T - \mathbf{r}_m\mathbf{e}_1^T + 2\mathbf{e}_1\mathbf{e}_1^T + \mathbf{e}_{p+1}\mathbf{c}_p^T\mathbf{M} \\ &\quad -\mathbf{r}_1\mathbf{e}_{p+1}^T - \mathbf{e}_1\mathbf{r}_m^T + \mathbf{M}^T\mathbf{c}_p\mathbf{e}_{p+1}^T - \mathbf{e}_{p+1}\mathbf{r}_1^T \\ -(\mathbf{W}\mathbf{M}\mathbf{Z}_2^T)^T(\mathbf{W}\mathbf{M}\mathbf{Z}_2^T) &= \mathbf{e}_{p+1}\mathbf{e}_{p+1}^T \\ \mathbf{M}^T(\mathbf{W}\mathbf{M}\mathbf{Z}_2^T) + (\mathbf{W}\mathbf{M}\mathbf{Z}_2^T)^T\mathbf{M} &= \mathbf{r}_1\mathbf{e}_{p+1}^T + \mathbf{e}_{p+1}\mathbf{r}_1^T\end{aligned}$$

Adding these terms together, we obtain

$$\begin{aligned}\mathbf{M}^T\mathbf{M} - \mathbf{Z}_2\mathbf{M}^T\mathbf{M}\mathbf{Z}_2^T &= \mathbf{e}_1\mathbf{e}_1^T + \mathbf{r}_1\mathbf{r}_1^T - \mathbf{r}_m\mathbf{r}_m^T(1 - \mathbf{c}_p^T\mathbf{c}_p)\mathbf{e}_{p+1}\mathbf{e}_{p+1}^T \\ &\quad +\mathbf{e}_{p+1}\mathbf{c}_p^T\mathbf{M} + \mathbf{M}^T\mathbf{c}_p\mathbf{e}_{p+1}^T \\ &= \mathbf{e}_1\mathbf{e}_1^T + \mathbf{r}_1\mathbf{r}_1^T - \mathbf{r}_m\mathbf{r}_m^T - \mathbf{M}^T\mathbf{c}_p(\mathbf{M}^T\mathbf{c}_p)^T/\gamma^2 \\ &\quad +(\gamma\mathbf{e}_{p+1} + \mathbf{M}^T\mathbf{c}_p/\gamma)(\gamma\mathbf{e}_{p+1} + \mathbf{M}^T\mathbf{c}_p/\gamma)^T\end{aligned}$$

where $\gamma^2 = (1 - \mathbf{c}_p^T \mathbf{c}_p)$.

A BTTB matrix $\mathbf{A}$ gives a similar derivation.

We now know how to determine ρ vectors $\mathbf{g}_i$ so that

$$\mathbf{M}^T\mathbf{M} - \mathbf{Z}_2\mathbf{M}^T\mathbf{M}\mathbf{Z}_2^T = \sum_{i=1}^{\rho} s_i \mathbf{g}_i \mathbf{g}_i^T$$

where s_i equals plus or minus 1.

When $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are shift-down matrices, Chun, Kailath, Lev-Ari showed that this implies that

$$\begin{aligned} \mathbf{M}^T\mathbf{M} &= \sum_{i=1}^{\rho} s_i \mathbf{L}_i \mathbf{L}_i^T \\ &= \begin{bmatrix} \mathbf{L}_1 & \dots & \mathbf{L}_\rho \end{bmatrix} \mathbf{S} \begin{bmatrix} \mathbf{L}_1^T \\ \vdots \\ \mathbf{L}_\rho^T \end{bmatrix} \end{aligned}$$

where $\mathbf{S} = \text{diag}(s_i)$ and $\mathbf{L}_i$ is the lower triangular Toeplitz matrix with first row equal to $\mathbf{g}_i^T$. We now generalize this result somewhat.

Theorem: If $\mathbf{Z}_1$ is nilpotent, then

$$\mathbf{A} - \mathbf{Z}_1\mathbf{A}\mathbf{Z}_2^T = \mathbf{g}\mathbf{h}^T$$

if and only if

$$\mathbf{A} = \mathbf{L}_1(\mathbf{g})\mathbf{L}_2^T(\mathbf{h})$$

where

$$\mathbf{L}_i(\mathbf{x}) = \begin{bmatrix} x & \mathbf{Z}_i\mathbf{x} & \dots & \mathbf{Z}_i^{n+p-1}\mathbf{x} \end{bmatrix}.$$

Corollary: If $\mathbf{Z}_1$ is nilpotent, then

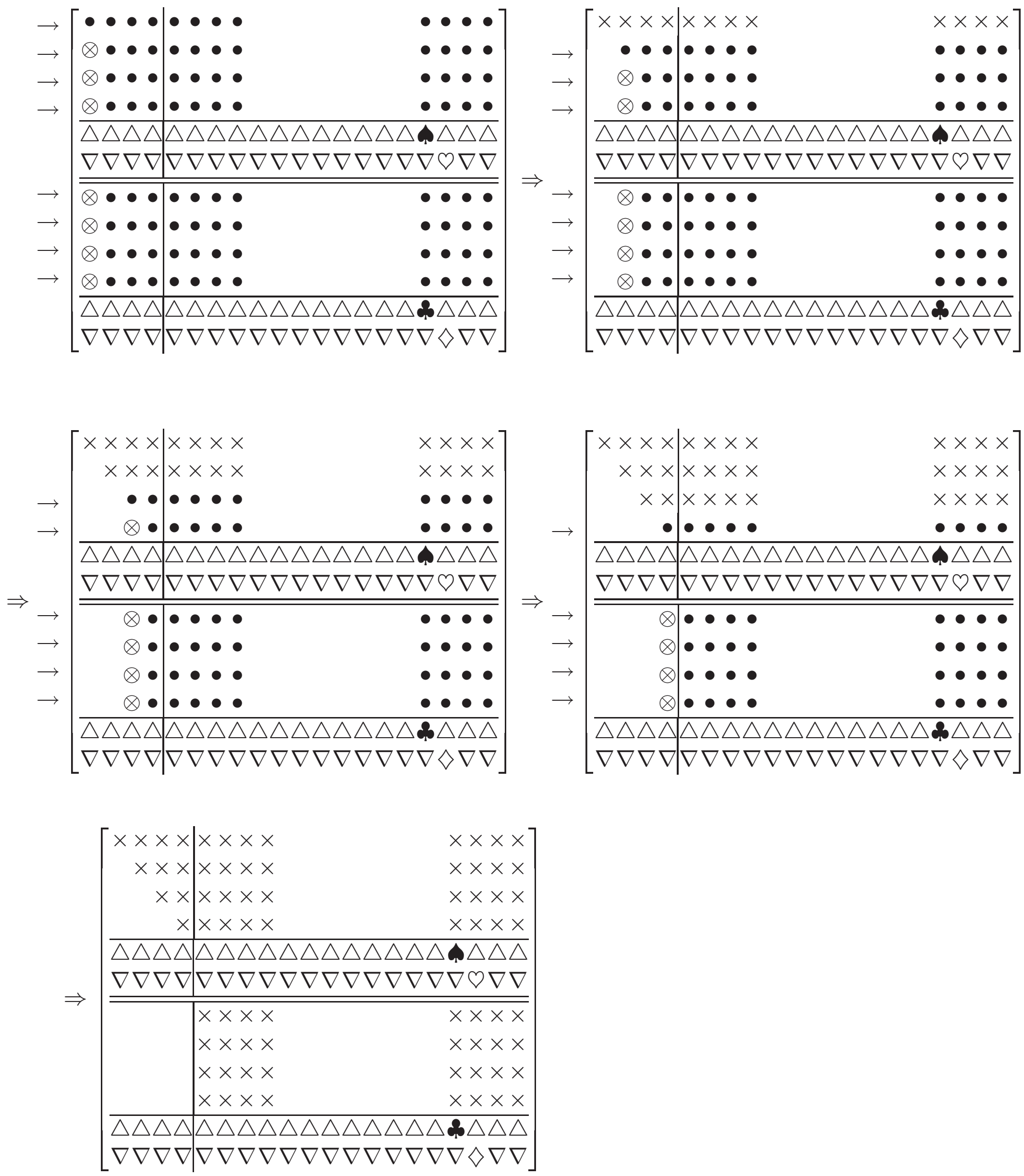
$$\mathbf{A} - \mathbf{Z}_1\mathbf{A}\mathbf{Z}_2^T = \sum_{i=1}^{\rho} \mathbf{g}_i \mathbf{h}_i^T$$

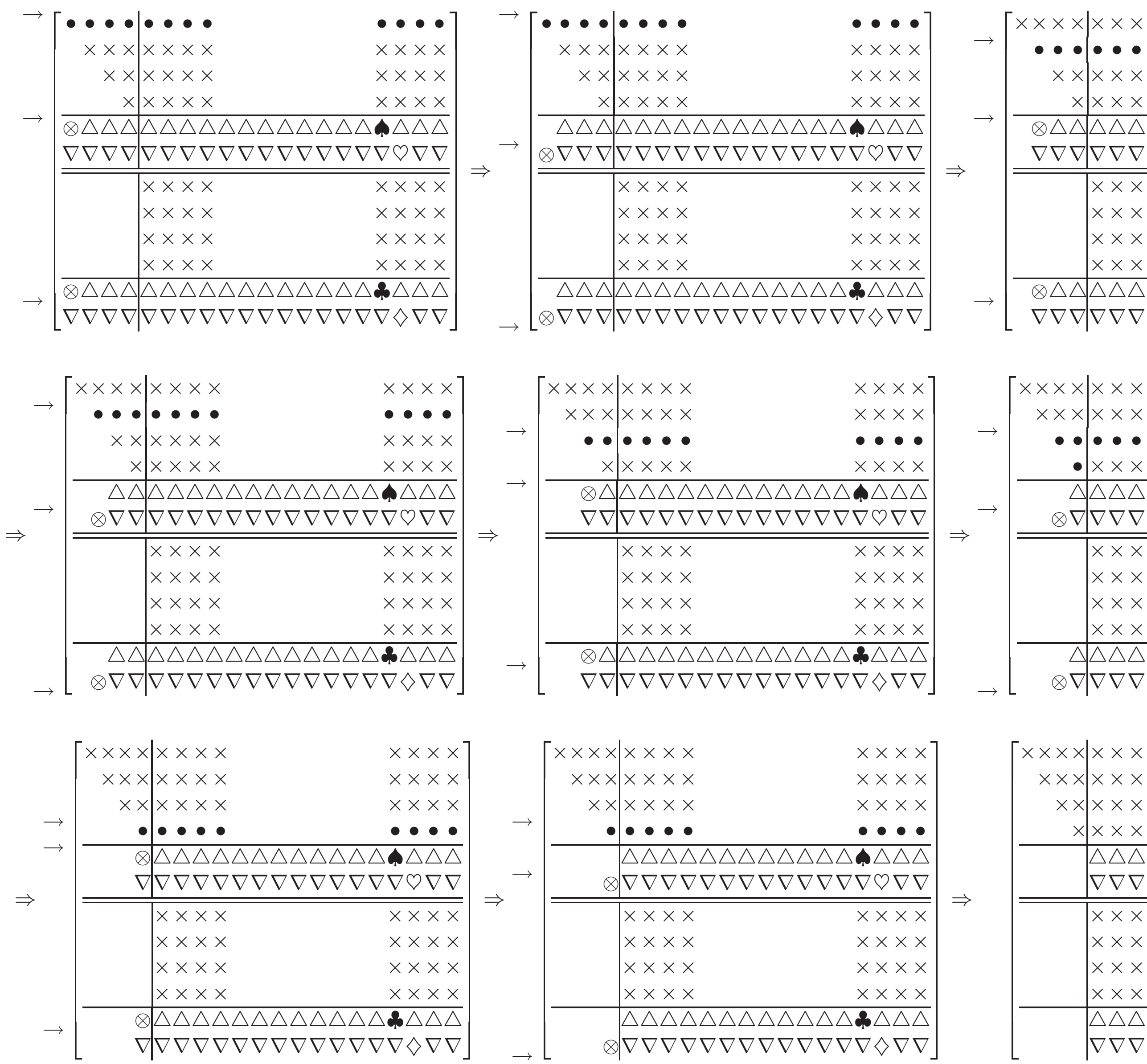
if and only if

$$\mathbf{A} = \sum_{i=1}^{\rho} \mathbf{L}_1(\mathbf{g}_i)\mathbf{L}_2^T(\mathbf{h}_i).$$

The Plan

- The deconvolution model
- How we solve it: 1, 2, and ∞ norms
- Savings when the 2-norm is used
- Exploiting sparsity structure
- Results
- Conclusions





The Plan

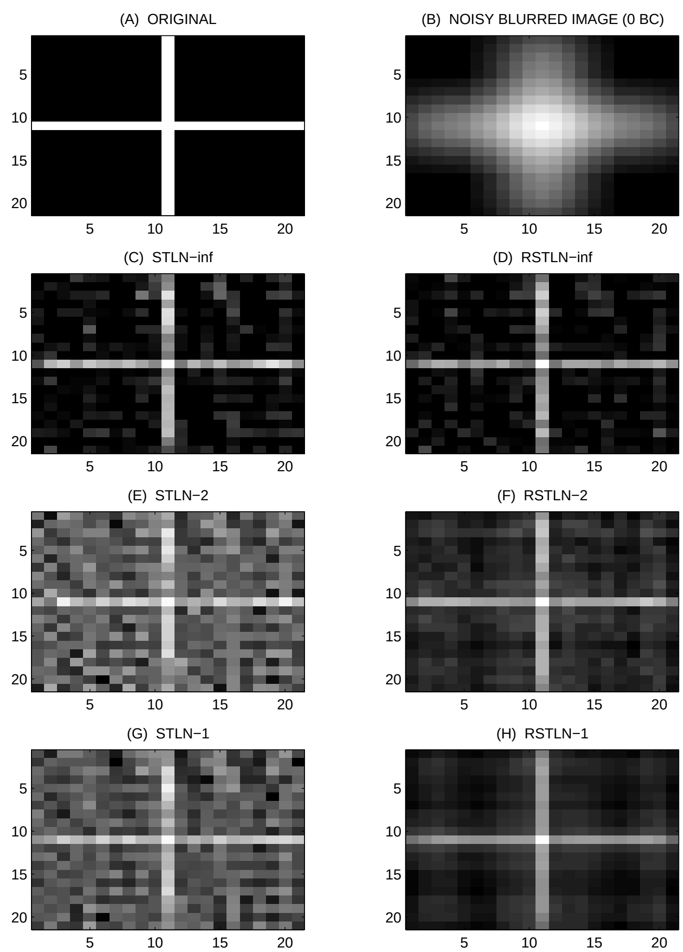
- The deconvolution model
- How we solve it: 1, 2, and ∞ norms
- Savings when the 2-norm is used
- Exploiting sparsity structure

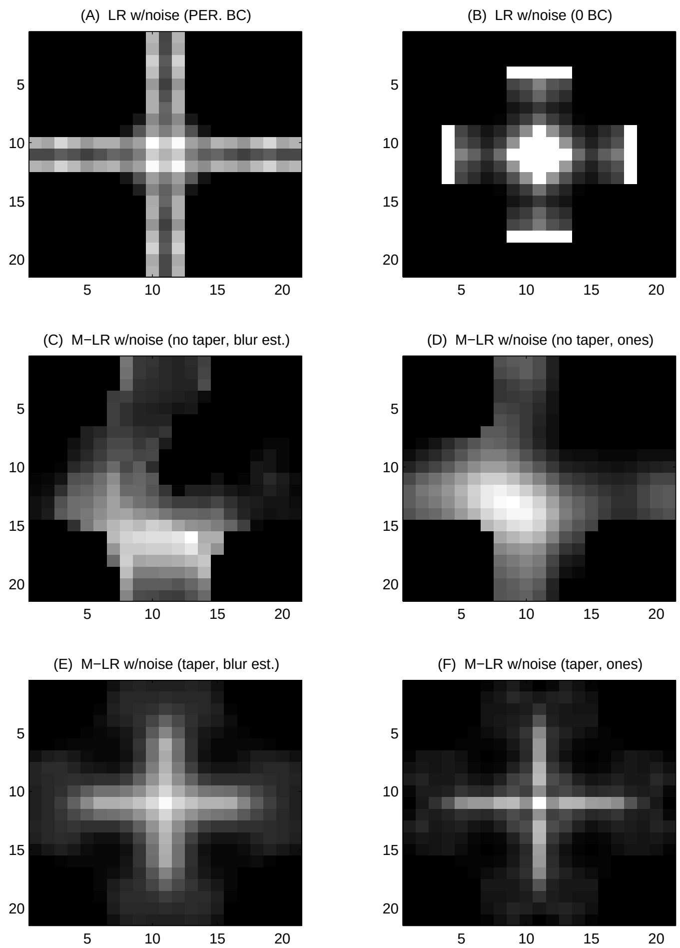
- Results
- Conclusions

Test 1

Our first test consists of a cross of size 21×21. The true PSF is a Gaussian blur with variance 2.5, truncated to a support of size 11×11.

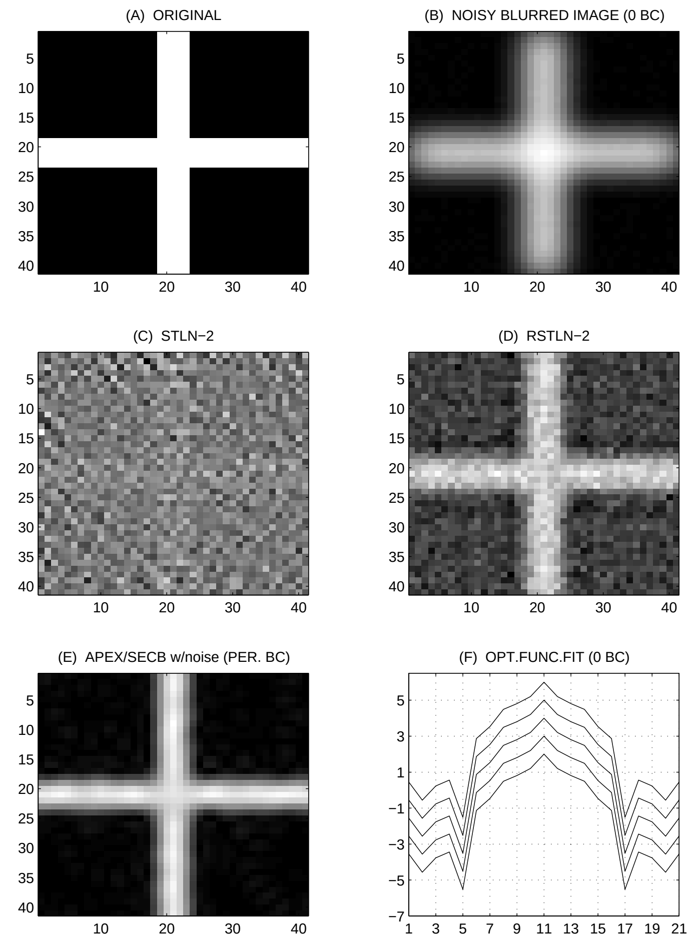
This resulted in $pert(\mathbf{A}) = 3.99 \times 10^{-2}$. Furthermore, 11-bit noise was added to the blurred image, resulting in $pert(\mathbf{b}) = 1.10 \times 10^{-3}$.





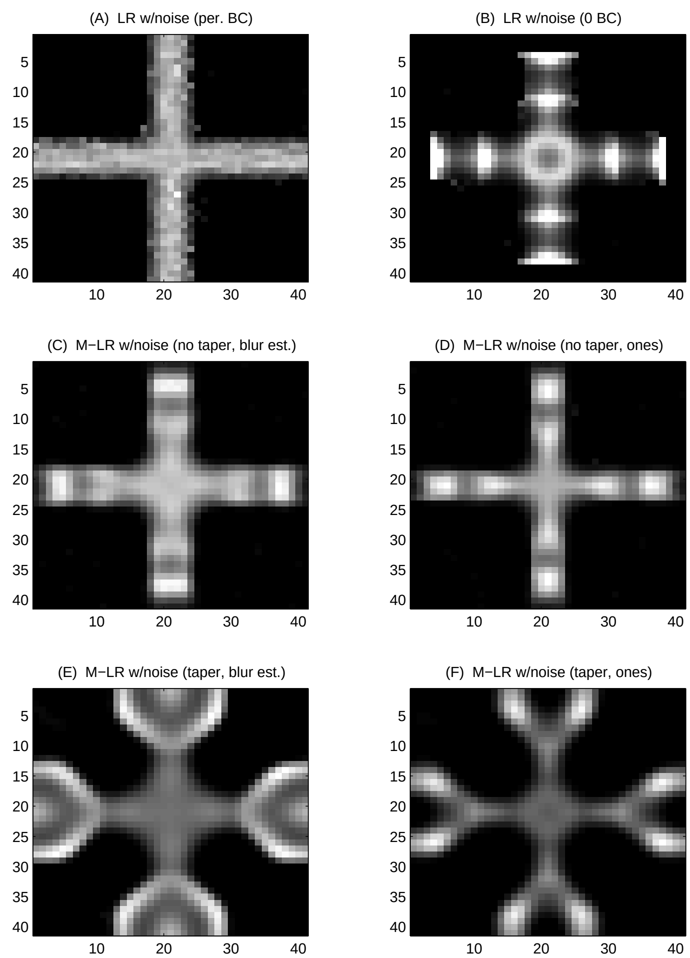
Test 2

Our next test consists of a somewhat broader cross image of size 41×41 with a nonzero cross width of 5. The image was blurred with an 11×11 Gaussian. 8-bit noise was added to the blurred images, resulting in $pert(\mathbf{b}) = 1.05 \times 10^{-2}$ and 9.8×10^{-3}, respectively. The blur estimate was obtained by adding 6-bit noise to the original blur, resulting in $pert(\mathbf{A}) = 3.91 \times 10^{-2}$.



(A) ORIGINAL
(B) NOISY BLURRED IMAGE (0 BC)
(C) STLN−2
(D) RSTLN−2
(E) APEX/SECB w/noise (PER. BC)
(F) OPT.FUNC.FIT (0 BC)

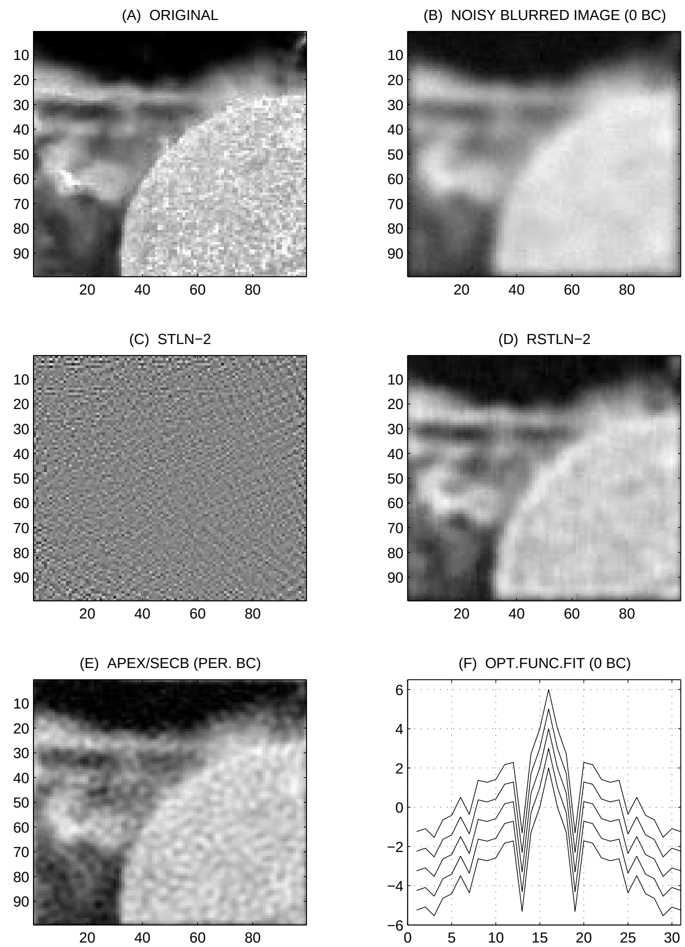
Test Case 2	$err(\mathbf{x})$	$err(\mathbf{A})$	$err(b)$
$p = 2$ STLN	4.2895	4.03e−2	1.03e−2
$p = 2$ RSTLN	0.5885	1.15e+0	9.20e−3

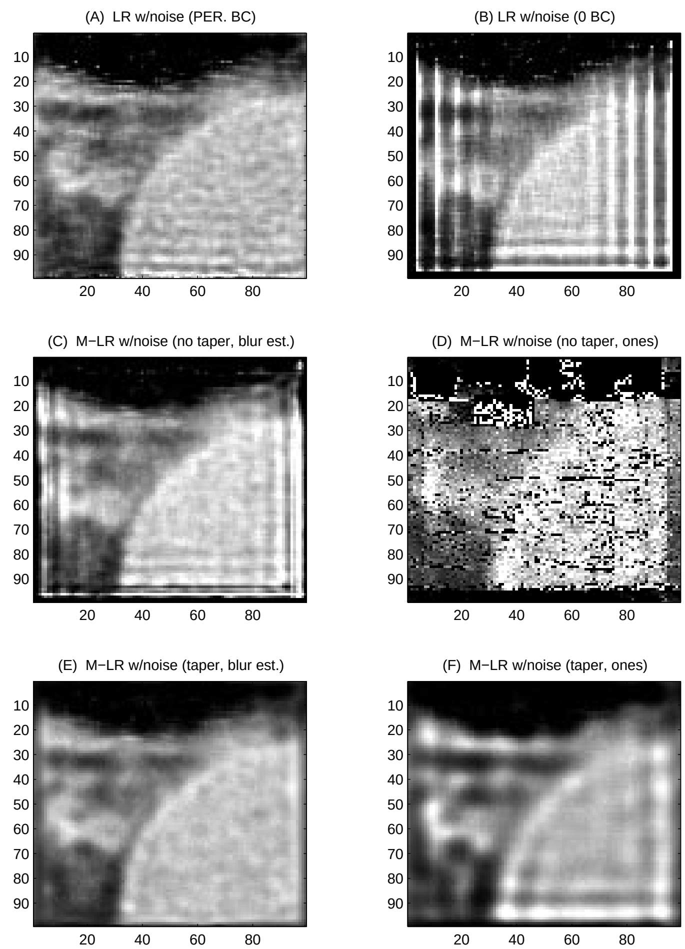


Test 3

Our final comparison test consists of an image obtained from the *NASA Image Exchange* (`http://nix.nasa.gov`). It shows the corona of the sun and a large solar eruption. We truncated the image to size 99×99 and reduced it to gray-scale.

Again, the image was blurred with a Gaussian PSF of size 11×11 in two ways: one assuming zero values for pixels outside the image, and the other assuming a periodic image. 6-bit noise was added to the image after blurring using a zero boundary condition. This resulted in $pert(\mathbf{b}) = 2.20 \times 10^{-2}$. For the periodic image no noise was added to the blurred image. The blur estimate was obtained by adding 6-bit noise to the original blur ($pert(\mathbf{A}) = 2.46 \times 10^{-2}$).





The Plan

- The deconvolution model
- How we solve it: 1, 2, and ∞ norms
- Savings when the 2-norm is used
- Exploiting sparsity structure
- Results
- Conclusions

Summary of results

- The **Blind Lucy-Richardson** method using FFTs is useful only if the original image either was blurred using periodic boundary conditions or if the image has finite support. If it does not satisfy these conditions recovered images often suffer from ringing.

- Like the blind Lucy-Richardson method, Carasso's **APEX/SECB method** requires periodic boundary conditions or finite support. Furthermore, it can only be applied to a restrictive class of PSFs and requires images to belong to a restrictive class.

- In contrast, neither **STLN** nor **RSTLN** imposes any restrictions on the image or PSF and both are effective on small images.

- If the noise is Gaussian, then least squares theory provides ample justification for choosing the 2-norm in RSTLN rather than the 1-norm or infinity-norm. However, in order to take advantage of this theory, the standard deviations of the two error distributions must be known so that the error terms can be balanced. When this data is unavailable, or when the noise distributions are not Gaussian, then the 1-norm or infinity-norm have no theoretical disadvantages. Our experiments show that the 1-norm in particular provides high-quality reconstructions and is not sensitive to outliers in the data.

- For image deblurring, the sparsity as well as the displacement structure needs to be exploited to make the computations practical.

References

Armin Pruessner and Dianne P. O'Leary, "Blind Deconvolution Using a Regularized Structured Total Least Norm Approach," *SIAM J. on Matrix Analysis and Applications*, 24 (2003) 1018-1037.

Anoop Kalsi and Dianne O'Leary, "Algorithms for Structured Total Least Squares Problems with Applications to Blind Image Deblurring," *Journal of Research of the National Institute of Standards and Technology*, to appear.

N. Mastronardi, P. Lemmerling, Anoop Kalsi, Dianne P. O'Leary, and S. Van Huffel, "Regularized Structured Total Least Squares Algorithms for Blind Image Deblurring," Manuscript, July 2003.