AMSC 607 / CMSC 764 Advanced Numerical Optimization
Fall 2006
Homework 2

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1. (10) Consider the Huang family of quasi-Newton formulas:

$$B^{(k+1)} = B^{(k)} - \frac{B^{(k)}s^{(k)}s^{(k)T}B^{(k)}}{s^{(k)T}B^{(k)}s^{(k)}} + \phi(s^{(k)T}B^{(k)}s^{(k)})u^{(k)}u^{(k)T}$$

where ϕ is a scalar between 0 and 1 and

$$\begin{aligned} s^{(k)} &= x^{(k+1)} - x^{(k)}\,, \\ y^{(k)} &= g^{(k+1)} - g^{(k)}\,, \\ u^{(k)} &= \frac{y^{(k)}}{y^{(k)T}s^{(k)}} - \frac{B^{(k)}s^{(k)}}{s^{(k)T}B^{(k)}s^{(k)}}. \end{aligned}$$

(a) Show that this formula satisfies the secant condition.

(b) Show that if $B^{(k)}$ is positive definite, then $B^{(k+1)}$ is, too.

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2. Consider the trust region method obtained by using $g(x)$ and $H(x)$ to form a quadratic model

$$f(x+p) \approx q(p) = f(x) + p^T g + \frac{1}{2}p^T H p$$

and then computing the search direction p from

$$\min_{\|p\|_2 \le h} q(p)\,.$$

2(a) (5) Show that if H is positive definite, then the direction p is downhill.

2(b) (5) Suppose that the solution p to the minimization problem satisfies $\|p\|_2 < h$. In this case, give a formula for p.

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3. (10) Consider the function

$$f(x,y) = \sin(x/y) - x^2 y.$$

Evaluate

$$\frac{\partial f}{\partial x} \text{ and } \frac{\partial f}{\partial y}$$

using the forward (bottom-up) mode of automatic differentiation.

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4. (10) Consider the function

$$f(x,y) = \sin(x/y) - x^2 y.$$

Evaluate

$$\frac{\partial f}{\partial x} \text{ and } \frac{\partial f}{\partial y}$$

using the backward (top-down) mode of automatic differentiation.

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5. (5) Consider the problem

$$\min_{x \geq 0} f(x)$$

where $x \in \mathcal{R}^5$. Suppose we have a current guess

$$\hat{x} = [0, 5, 3, 0, 2]^T$$

for the solution and that we have computed the corresponding Lagrange multipliers

$$\hat{\lambda} = [1, -2, 3, -4, 5]^T .$$

Write the first order necessary conditions for optimality. For each condition, indicate whether the point $\hat{x}, \hat{\lambda}$ satisfies, violates, or possibly satisfies it.

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6. (10) Consider the linear programming problem

$$\begin{array}{rcl} \min\limits_{x} & & c^T x \\ Ax & = & b \\ x & \geq & 0 \end{array}$$

where A is an $m \times n$ matrix with $m < n$.

Show that if there exists a solution to this problem, then there is a solution x_{opt} that has at most m nonzero components.

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7. (15) Write a MATLAB function to minimize a given function f subject to linear equality constraints $Ax = b$. Your function should use the feasible direction formulation, using the QR factors to get a basis for the feasible directions, along with MATLAB's `fminunc`. It should be written to find a local minimizer for arbitrary functions f and arbitrary numbers of variables and constraints. The user should be given the option of providing a starting point.

Test your function on the problem

$$\min_{x} x_1^2 x_2^3 + 4x_1^2 x_3^2 + x_2^4 x_3^2 + 3x_1 x_2 + 4x_2 x_3 + 5x_1 x_3 + x_1 x_3$$

subject to the constraint

$$x_1 + x_2 + x_3 = 3,$$

starting with the point $x = [-1, 5, -1]^T$. Print out the sequence of iterates and document your function well.