

AMSC 607 / CMSC 764 Homework 1, Fall 2010

20 points

Due September 14, before class begins.

Let  $f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$ . This is called the **Rosenbrock banana function**, and there is a good picture of it at [http://en.wikipedia.org/wiki/Rosenbrock\\_function](http://en.wikipedia.org/wiki/Rosenbrock_function).

1a. (5) Compute the gradient  $\mathbf{g}(\mathbf{x})$  and the Hessian  $\mathbf{H}(\mathbf{x})$ .

1b. (5) Verify that  $\mathbf{x}^* = [1, 1]^T$  is a local minimizer of  $f$ .

1c. (5) Use MATLAB to compute 5 iterates from Newton's method for minimizing  $f(\mathbf{x})$  starting at  $\mathbf{x} = [-1, -2]^T$ . (The next iterate is  $\mathbf{x} = \mathbf{x} - \mathbf{H} \setminus \mathbf{g}$  where  $\mathbf{H}$  is the Hessian at the current point and  $\mathbf{g}$  is the gradient.) Plot the contours of the function using `ezcontour` on the domain `[-1.5;2;-3;3]`. Plot the iterates on the same plot.

1d. (5) Compute the norm of the error  $\mathbf{x} - \mathbf{x}^*$  at each iteration and determine whether the rate of convergence is quadratic. Justify your answer.

**A few tips:**

- If MATLAB reports that the error is 0.0000, that just means that it is smaller than  $10^{-4}$ . Use `format short e`, for example, to force it to show what the number really is.
- The `hold on` statement will allow you to plot the iterates on the contour plot.
- MATLAB has a symbolic arithmetic package that can be used to solve 1a. See the example at <http://www.mathworks.com/help/toolbox/symbolic/diff.html>.
- **Submitting your homework:** Please provide a listing of your MATLAB code, a printout of your plot, and your written answers.
  - If you think you might be late to class, you can hand in homework to me in an earlier class or office hours, or you may slide it under my door before 9:15 on the due date.
  - Do not slide it under my door after I leave for my 9:30 class on the due date; I will get it late.
  - Do not use my first-floor mailbox for homework submission; I will get it late.
  - Email submission is acceptable, but files must be pdf or plain text (.txt or .m). It is ok to bundle them using tar, zip, or gzip. I will not accept any Microsoft-formatted files such as .doc, .xls, or .dll.