

AMSC/CMSC 460      Quiz 3      ,      Fall 2002

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, communication with others, scratchpaper, etc.

Name \_\_\_\_\_

Student number \_\_\_\_\_

1.(10) Suppose that we have a function  $f$  defined on the interval  $[0, 2]$ , and we want to approximate it by a piecewise linear function that is never further than  $10^{-4}$  from  $f$ . If the second derivative of  $f$  is bounded by 36, how many equally-spaced points should be use?

2.(10) Recall the basis that we are using for cubic splines with knots  $x_1 < x_2 < \dots < x_n$ : We will set  $s(x)$  equal to  $s_{i+1}(x)$  on interval  $I_{i+1}$ , where

$$s_{i+1}(x) = m_i \frac{(x_{i+1} - x)^3}{6h_{i+1}} + m_{i+1} \frac{(x - x_i)^3}{6h_{i+1}} + a_i(x - x_i) + b_i$$

for some constants  $m_i$ ,  $m_{i+1}$ ,  $a_i$ , and  $b_i$ , where

- $h_{i+1} = x_{i+1} - x_i$ ,  $i = 1, \dots, n - 1$
- $k_{i+1} = f_{i+1} - f_i$ ,  $i = 1, \dots, n - 1$
- $I_{i+1} = [x_i, x_{i+1}]$ ,  $i = 1, \dots, n - 1$

Write down the conditions that guarantee that  $s''$  is continuous at the knots  $x_2, \dots, x_{n-1}$ .