AMSC/CMSC 460 Quiz 4 , Fall 2002

1. (10) Suppose we are given 20 points $0 = t_1 < t_2 < \ldots < t_n = 1$. Write the formula for the composite trapezoidal rule with $n-1$ panels that uses these points to approximate

$$\int_0^1 t^3 dt\,.$$

Answer:

$$Q = \sum_{i=1}^{19} \frac{t_i^3 + t_{i+1}^3}{2}(t_{i+1} - t_i)$$

For computation, we would want to reorder this so that function values were reused rather than recomputed:

$$Q = \frac{t_1^3}{2}(t_2 - t_1) + \sum_{i=2}^{19} \frac{t_i^3}{2}(t_{i+1} - t_{i-1}) + \frac{t_{20}^3}{2}(t_{20} - t_{19})$$

2. (10) Let

$$I = \int_0^1 f(t)\,dt\,.$$

Suppose we approximate I by a rule of the form

$$Q(f) = \omega_1 f(t_1) + \omega_2 f(t_2)\,.$$

Write down conditions to make this rule exact for polynomials of degree 3 or less.

Answer:

$$\begin{aligned}
\int_0^1 1dt = 1 &= \omega_1 + \omega_2 \\
\int_0^1 tdt = 1/2 &= \omega_1 t_1 + \omega_2 t_2 \\
\int_0^1 t^2 dt = 1/3 &= \omega_1 t_1^2 + \omega_2 t_2^2 \\
\int_0^1 t^3 dt = 1/4 &= \omega_1 t_1^3 + \omega_2 t_2^3
\end{aligned}$$