AMSC/CMSC 460 Quiz 7 , Fall 2002

1. (10) Let

$$f(x) = x^2 - 4\,.$$

Suppose our guess at the zero of this nonlinear equation is 1.5. If we apply Newton's method, what would the next guess be?
Answer: We compute $f'(x) = 2x$. Newton's method says

$$\begin{aligned} x &\leftarrow x - \frac{f(x)}{f'(x)} \\ &= x - \frac{x^2 - 4}{2x} \\ &= 1.5 - \frac{1.5^2 - 4}{3} \\ &= 2.0833\ldots \end{aligned}$$

2. (10) Suppose we solve a linear system $Ax = b$ using Matlab's backslash command

$$x = A\backslash b\,.$$

We compute

$$r = b - Ax$$

and find that

$$\|r\|_1 = 5 \times 10^{-9}$$

and the condition number of A (in the 1-norm) is 6000. Give a bound on

$$\frac{\|x_{true} - x\|_1}{\|x_{true}\|_1}$$

in terms of A and b.
Answer:

$$\frac{\|x_{true} - x\|_1}{\|x_{true}\|_1} \leq \kappa(A)\frac{\|r\|_1}{\|b\|_1} = \frac{6000 \times 5 \times 10^{-9}}{\|b\|_1}\,.$$