1. (10) Suppose we have used a PECE method with predictor of order 4 (i.e., local error is proportional to $O(h^5)$, where h is the stepsize) and corrector of order 5. We want to keep the local error less than τ. Estimate the local error and explain how to alter the stepsize if necessary to achieve our local error criterion.

Answer:

$$\|y4 - y_{true}\| \approx \|y4 - y5\| \equiv \delta$$

If $\delta > \tau$, reduce h and retake the step:

- perhaps $h = h/2$.

- perhaps $h = h/2^p$ where, since we need $\delta 2^{-4p} \approx \tau$, we define $p = (\log \delta - \log \tau)/(4 \log 2)$.

2. (10) Let

$$y' = y^2 - 5t$$
$$y(0) = 1$$

Apply a PECE scheme to this problem, using Euler and Backward Euler with a stepsize $h = .1$, to obtain an approximation for $y(.1)$.

Answer: Recall that Euler's method is

$$y_{n+1} = y_n + hf(t_n, y_n)$$

and Backward Euler is

$$y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})$$

$$
\begin{aligned}
P: \quad & y = 1 + .1(1^2) = 1.1 \\
E: \quad & f = (1.1)^2 - .5 = .71 \\
C: \quad & y = 1 + .1 * .71 = 1.071 \\
E: \quad & f = (1.071)^2 - .5
\end{aligned}
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