An Example of Gauss Elimination

Original matrix:

4	4	8
2	8	7
1	3	6

An Example of Gauss Elimination

Original matrix:

I	4	4	8
II	2	8	7
III	1	3	6

Step 1: Put zeros in first column

	4	4	8
-1/2 I + II:	0	6	3
-1/4 I + III:	0	2	4

An Example of Gauss Elimination

Current matrix:

I	4	4	8
II	0	6	3
III	0	2	4

Step 2: Put zeros in second column

	4	4	8
	0	6	3
-1/3 II + III:	0	0	3

What Gauss Elimination Accomplished:

4	4	8
2	8	7
1	3	6

4	4	8
0	6	3
0	0	3

But Notice: The multipliers are important, too:

$$\begin{bmatrix} 4 & 4 & 8 \\ 2 & 8 & 7 \\ 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

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A = L U

How this solves linear systems:

$$\begin{bmatrix} 4 & 4 & 8 \\ 2 & 8 & 7 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

$A x = b$

$LU x = b$

Step 1: Solve $L y = b$

Step 2: Solve $U x = y$

Solve L y = b

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/4 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$

Solve $L y = b$

1	0	0	y_1		4
1/2	1	0	y_2	=	5
1/4	1/3	1	y_3		5

$1\ y_1 = 4$

$y_1 = 4$

Solve $L y = b$

1	0	0	y_1		4
1/2	1	0	y_2	=	5
1/4	1/3	1	y_3		5

$1/2\ y_1 + y_2 = 5, \quad y_1 = 4$

$y_2 = 3$

Solve $L y = b$

1	0	0	y_1		4
1/2	1	0	y_2	=	5
1/4	1/3	1	y_3		5

$1/4\ y_1 + 1/3\ y_2 + y_3 = 5, \quad y_1 = 4,\ y_2 = 3$

$y_3 = 3$

Result: $L y = b$

1	0	0	4		4
1/2	1	0	3	=	5
1/4	1/3	1	3		5

Now solve U x = y

$$\begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

Solve U x = y

$$\begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

$3\ x_3 = 3$

$x_3 = 1$

Solve U x = y

$$\begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

$6\ x_2 + 3\ x_3 = 3, \quad x_3 = 1$

$x_2 = 0$

Solve U x = y

$$\begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

$4\ x_1 + 4\ x_2 + 8\ x_3 = 4, \quad x_3 = 1, x_2=0$

$x_1 = -1$

Result: U x = y

$$\begin{bmatrix} 4 & 4 & 8 \\ 0 & 6 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

Result: A x = b

$$\begin{bmatrix} 4 & 4 & 8 \\ 2 & 8 & 7 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix}$$