CMSC/AMSC 460 Fall 2007
Homework 4
Due Tuesday, October 30, before class begins
15 points

The assignment investigates the stability of two of the ways we learned for polynomial interpolation: the power basis and the Newton basis.

Suppose we want to find a polynomial that interpolates data values f_i ($i = 1, \ldots, n$) at n equally spaced points x_i between 0 and 1.

If we use the power basis to express

$$p(x) = d_1 x^{n-1} + d^2 x^{n-2} + \ldots + d_{n-1} x + d_n,$$

then our problem is to solve $\mathbf{Vd} = \mathbf{f}$, where $\mathbf{V}$ is a Vandermonde matrix.

If we use the Newton basis to express

$$p(x) = c_1 + c_2(x - x_1) + c_3(x - x_1)(x - x_2) + \ldots + c_n(x - x_1)(x - x_2)\ldots(x - x_{n-1}),$$

then our problem is to solve $\mathbf{Bc} = \mathbf{f}$, where $\mathbf{B}$ is a lower triangular matrix. (Note that the solution $\mathbf{c}$ is just the vector of divided differences. This is another way to compute it.)

The accuracy of our coefficients $\mathbf{d}_{computed}$ or $\mathbf{c}_{computed}$ will depend on the condition number of the coefficient matrix ($\mathbf{V}$ or $\mathbf{B}$), where the condition number is defined to be

$$\kappa(\mathbf{A}) = \|\mathbf{A}\|\|\mathbf{A}^{-1}\|.$$

1. Write a Matlab program that computes the condition numbers of $\mathbf{V}$ and $\mathbf{B}$ for $n = 1, \ldots, 20$. Hand in a copy of your program and its output (table and figure). Follow these directions:

1a. (4) Make sure your program generates $\mathbf{V}$ and $\mathbf{B}$ correctly. Have Matlab display the matrices $\mathbf{V}$ and $\mathbf{B}$ for $n = 4$. (This will also help you in debugging.)

1b. (2) Have Matlab make a well-formatted table of the condition numbers.

1c. (2) Have Matlab plot the condition numbers as a function of n (one figure, containing $\kappa(\mathbf{V})$ and $\kappa(\mathbf{B})$ as functions of n). Label the axes and the curves.

1d. (3) Design your program so that it generates $\mathbf{B}$ one column at a time, using a single Matlab statement to generate all of the elements below the main diagonal of the column.

1e. (1) Make sure your program is well documented.

2. (3) Explain from the condition number data why it is better to use the Newton basis rather than the power basis.

Hint: The following Matlab commands may be helpful to you: `linspace`, `vander`, `eye`, `cond`, `semilogy`, `legend`, `xlabel`, `ylabel`, `print`, `disp`, `sprintf`, `diary`.