AMSC/CMSC 460 Quiz 3 , Fall 2007

1. (10) Use Simpson's rule to compute an approximation to

$$\int_0^1 e^t dt.$$

(If you can't remember Simpson, composite Trapezoid with 3 panels ($h = 1/3$) is worth 7 points.)

Answer:

Simpson:

$$S = \frac{1}{6}\left[e^0 + 4e^{1/2} + e^1\right].$$

Trapezoidal rule:

$$T = \frac{1}{6}\left[e^0 + 2e^{1/3} + 2e^{2/3} + e^1\right].$$

Numerically, $S = 1.7$, $T = 2.4$, and the true integral is 2.7.

2. (10) Let

$$I(f) = \int_0^1 f(t)dt.$$

Suppose we approximate I by a Gauss-Lobatto rule of the form

$$Q(f) = \omega_1 f(0) + \omega_2 f(t_1) + \omega_3 f(t_2) + \omega_4 f(1).$$

Write down conditions to make this rule exact for polynomials of degree 5 or less.

Answer:

$$\begin{array}{rlccccccc}
\int_0^1 dx & = 1 = & w_1 & + & w_2 & + & w_3 & + & w_4 \\
\int_0^1 x\,dx & = 1/2 = & & & t_1 w_2 & + & t_2 w_3 & + & w_4 \\
\int_0^1 x^2\,dx & = 1/3 = & & & t_1^2 w_2 & + & t_2^2 w_3 & + & w_4 \\
\int_0^1 x^3\,dx & = 1/4 = & & & t_1^3 w_2 & + & t_2^3 w_3 & + & w_4 \\
\int_0^1 x^4\,dx & = 1/5 = & & & t_1^4 w_2 & + & t_2^4 w_3 & + & w_4 \\
\int_0^1 x^5\,dx & = 1/6 = & & & t_1^5 w_2 & + & t_2^5 w_3 & + & w_4
\end{array}$$