AMSC/CMSC 460 Quiz 6 , Fall 2007

1. (10) Suppose we use an Adams PECE schme to solve a differential equation $y' = f(t, y)$ and obtain $y^P_{n+1} = 1.2450$ and $y^C_{n+1} = 1.2430$. Suppose the error formula for the predictor is $\frac{5h^3}{12} y^{(3)}(\eta)$ and for the corrector is $\frac{h^4}{24} y^{(4)}(\xi)$. What can you say about the error? (Give an unambiguous statement about what error you mean and what your estimate of it is.)

Answer:
The error formulas tell us that

$$
\begin{aligned}
y_{n+1} - y^P_{n+1} &= \frac{5h^3}{12} y^{(3)}(\eta), \\
y_{n+1} - y^C_{n+1} &= \frac{h^4}{24} y^{(4)}(\xi).
\end{aligned}
$$

We can't compute the left or right-hand sides, but if we subtract the two equations we obtain

$$
y^C_{n+1} - y^P_{n+1} = \frac{5h^3}{12} y^{(3)}(\eta) - \frac{h^4}{24} y^{(4)}(\xi),
$$

and the left-hand side is computable. Assuming that the error in the corrector is much smaller than the error in the predictor, we have

$$
|y_{n+1} - y^P_{n+1}| \approx |y^C_{n+1} - y^P_{n+1}| = 1.2450 - 1.2430 = .002.
$$

This is an approximation to the **local error** in the predictor: i.e., the difference between the true solution and the predicted value, assuming that all of the values used in the predictor formula are correct.

2a. (5) Give an important advantage of PECE Adams methods over Runge-Kutta methods.

Answer:
PECE uses two function evaluations, **regardless of the order of the method**. Runge-Kutta uses more function values than this for formulas of order greater than 2.

2b. (5) Consider the differential equation $\mathbf{y}' = \mathbf{f}(t, \mathbf{y})$, with $\mathbf{y} : \mathcal{R}^1 \rightarrow \mathcal{R}^2$. Given t_1 and $\mathbf{y}(t_1)$, how do you test whether the differential equation is stiff at t_1?

Answer:
Since $\mathbf{y}$ has two components, we need to

- Compute the 2×2 matrix of partial derivatives of $\mathbf{f}$ with respect to the two components of $\mathbf{y}$.
- Evaluate the matrix at t_1 and $\mathbf{y}(t_1)$.
- Evaluate the two eigenvalues of the matrix.

If the real parts of the eigenvalues are negative, and one is much less than the other, then the equation is stiff at t_1.