| $\Delta$ MSC | /CMSC          | 660 | Quiz 3 |   | Fall 2003 |
|--------------|----------------|-----|--------|---|-----------|
| AMDO         | $O_{\rm MISO}$ | UUU | Quiz 5 | • | ran zuus  |

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, communication with others, scratchpaper, etc.

| Name           |  |  |
|----------------|--|--|
|                |  |  |
| Student number |  |  |

1. (10) Suppose we have a matrix A of dimension  $n \times n$  of rank n-1. Give two numerically stable ways to find a vector z so that Az = 0.

## 2. (10) Recall the Gram-Schmidt algorithm:

Set 
$$r_{11} = ||a_1||$$
.

Set 
$$q_1 = a_1/r_{11}$$
.

for 
$$k = 1, ..., n - 1$$
,

Set 
$$q_{k+1} = a_{k+1}$$
.  
for  $i = 1, ..., k$ ,

for 
$$i = 1, \ldots, k$$

$$r_{i,k+1} = q_{k+1}^T q_i$$

$$r_{i,k+1} = q_{k+1}^T q_i q_{k+1} = q_{k+1} - r_{i,k+1} q_i$$

end for

$$r_{k+1,k+1} = ||q_{k+1}||$$

$$r_{k+1,k+1} = ||q_{k+1}||$$
  
 $q_{k+1} = q_{k+1}/r_{k+1,k+1}$ 

end for

Show that  $q_i^T q_k = 0$  for i < k.