AMSC/CMSC 660 Quiz 9 , Fall 2004

1. (10) Consider the problem

$$\min_x x_1^2 + 5x_2^2$$

subject to $x_1 \geq 0$, $x_2 \geq 0$, and $x_1 + 2x_2 = 4$. Use feasible directions and a barrier method to formulate this problem as an unconstrained optimization problem. (2 points for a particular solution to the equality constraint, 2 points for determining the space spanned by the feasible directions, 6 points for the unconstrained problem.)

Answer:

One particular solution to the equality constraint: $[4, 0]^T$.

A basis for the nullspace of the matrix: $[2, -1]^T$.

General solution to equality constraint:

$$x = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} z$$

where z is an arbitrary scalar.

The problem becomes

$$\min_z (4 + 2z)^2 + 5(-z)^2$$

subject to $4 + 2z \geq 0$, $-z \geq 0$, so the barrier problem that is equivalent to the original problem is

$$\min_z (4 + 2z)^2 + 5(-z)^2 - \mu \log(4 + 2z) - \mu \log(-z)$$

2. (Max points = 10)
2a. (2) Give two pitfalls of symbolic computation.

Answer: Can be expensive in time and space; can produce a formula for the answer that cannot be evaluated accurately in floating point computation.

2b. (2) What does it mean to say that a problem is ill-conditioned?

Answer: A small change in the data may cause a large change in the answer. (Note that this definition applies to *any* problem, not just one from linear algebra.)

2c. (3) Suppose a colleague comes to you with a spectroscopy problem, and you can think of 5 ways to model the experimental error. How would you decide which to use?

Answer: Ideally: have the colleague supply sample data from the spectrometer for which the answer is known. Then try your ideas on the data and see which works best.
If that is not possible, find out what the colleague believes the machine is doing, generate sample data using those assumptions, and see which works best.

2d. (2) When you type `help alg` in Matlab, the result is the beginning set of comments in the file `alg.m`. Give two kinds of information that should be included in these comments.

Answer: Describe the input, output, purpose, method, author and date, and a reference to learn more about the algorithm.

2e. (3) Suppose you solve the nonlinear equation $f(x) = 0$ using a Matlab routine, and the answers are complex numbers with small imaginary parts. If you know that the true answers are real numbers, what would you do?

Answer: Throwing out the imaginary part or taking $\pm$ the absolute value is dangerous, unless the imaginary part is within the desired error tolerance. It is probably best to try a different method; you have an answer that you believe is close to the true one, so you might, for example, use your favorite algorithm to solve $\min_x (f(x))^2$ using the real part of the previously computed solution as a starting guess.