AMSC/CMSC 660 Quiz 2 , Fall 2006

1. (10) Suppose we have factored the $m \times n$ matrix $\mathbf{A} = \mathbf{QR}$ ($m \geq n$), and let $\hat{\mathbf{x}}$ be the solution to the least squares problem

$$\min_{\mathbf{x}} \|\mathbf{Ax} - \mathbf{b}\|.$$

Show that $\|\mathbf{A}\hat{\mathbf{x}} - \mathbf{b}\|^2 = \|\mathbf{c}_2\|^2$, where $\mathbf{c}_2$ is the vector consisting of the last $m - n$ components of $\mathbf{Q}^*\mathbf{b}$.

2. (10) Write a column-oriented algorithm to solve $\mathbf{Ux} = \mathbf{b}$ where $\mathbf{U}$ is an $n \times n$ nonsingular upper triangular matrix. (If you can't do this, you can get 5 points for any correct algorithm to solve this problem, but you may not use the backslash operator or an inverse matrix.)