AMSC/CMSC 660 Quiz 3 , Fall 2006

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name _____

1. (10) Suppose we have factored $\mathbf{A} = \mathbf{LU}$ and now we need to solve a linear system $(\mathbf{A} - \mathbf{ZV}^T)\mathbf{x} = \mathbf{b}$, where $\mathbf{Z}$ and $\mathbf{V}$ have dimension $n \times k$ and k is much less than n. Write MATLAB code to do this accurately and efficiently. You might want to use the Sherman-Morrison-Woodbury formula

$$(\mathbf{A} - \mathbf{ZV}^T)^{-1} = \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{Z}(\mathbf{I} - \mathbf{V}^T\mathbf{A}^{-1}\mathbf{Z})^{-1}\mathbf{V}^T\mathbf{A}^{-1}.$$

(Don't use matrix inverses!)

2. (10) Denote the SVD of the 2×2 matrix $\mathbf{A}$ by $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$.

(a) Express the solution to the linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ as $\mathbf{x} = \alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2$ where $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2]$.

(b) Consider the linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ with

$$\mathbf{A} = \begin{bmatrix} 1+\delta & \delta-1 \\ \delta-1 & 1+\delta \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -2 \end{bmatrix},$$

and $\delta = 0.002$, and suppose we compute the solution to the nearby systems

$$(\mathbf{A} + \mathbf{E}^{(i)})\mathbf{x}^{(i)} = \mathbf{b}$$

for $i = 1, \ldots, 1000$, where the elements of $\mathbf{E}^{(i)}$ are independent and normally distributed with mean 0 and standard deviation $\tau = .0001$. Using part (a), explain why the resulting solutions all fall near a straight line, as shown in the figure.