AMSC/CMSC 660 Quiz 9 , Fall 2006

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name _____

1. (10) Recall that a Hamiltonian system is a system of ODEs for which there exists a scalar Hamiltonian function $H(\mathbf{y})$ so that $\mathbf{y}' = \mathbf{D}\nabla_{\mathbf{y}} H(\mathbf{y})$ where $\mathbf{D}$ is a block-diagonal matrix with blocks equal to

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Derive the Hamiltonian system for

$$H(\mathbf{y}) = \frac{1}{2}y_1^2 + \frac{1}{2}y_2^2 + \frac{1}{2}y_3^2 + \frac{1}{2}y_4^2 + \frac{1}{2}y_1^2 y_2^2 y_3^2 y_4^2.$$

2. (10) Suppose we have used the Adams-Bashforth and Adams-Moulton methods of order 3 to form two estimates of $y(t_{n-1})$, the solution to a differential equation. These formulas are:

$$\begin{aligned} y_{n+1}^{ab} &= y_n + \frac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2}) \; error : \frac{3h^4}{8}y^{(4)}(\eta). \\ y_{n+1}^{am} &= y_n + \frac{h}{12}(5f_{n+1} + 8f_n - f_{n-1}) \; error : -\frac{h^4}{24}y^{(4)}(\eta). \end{aligned}$$

How would you estimate the local error in the Adams-Moulton formula? How would you use that estimate to change h in order to keep the estimated local error less than a user-supplied local error tolerance τ without taking steps smaller than necessary?