AMSC/CMSC 660 Quiz 6 , Fall 2008

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name _____

1. (10) Write MATLAB statements to apply 5 steps of Newton's method to the problem

$$\begin{aligned} x^2y^3 + xy &= 2\,, \\ 2xy^2 + x^2y + xy &= 0\,, \end{aligned}$$

starting at the point $x = 5$, $y = 4$.

2. Consider solving a nonlinear system of equations using the limited memory quasi-Newton method using Broyden's update formula with $\boldsymbol{B}^{(0)} = \boldsymbol{I}$:

$$\boldsymbol{B}^{(k+1)} = \boldsymbol{B}^{(k)} + \frac{(\boldsymbol{y}^{(k)} - \boldsymbol{B}^{(k)}\boldsymbol{s}^{(k)})\boldsymbol{s}^{(k)T}}{\boldsymbol{s}^{(k)T}\boldsymbol{s}^{(k)}}.$$

As an example, let $k = 2$.

(a) (3) Let

$$(\boldsymbol{B}^{(k+1)})^{-1} = (\boldsymbol{B}^{(k)})^{-1} + \boldsymbol{w}^{(k)}\boldsymbol{u}^{(k)T}.$$

Use the Sherman-Morrison-Woodbury formula

$$(\boldsymbol{A} - \boldsymbol{Z}\boldsymbol{V}^T)^{-1} = \boldsymbol{A}^{-1} + \boldsymbol{A}^{-1}\boldsymbol{Z}(\boldsymbol{I} - \boldsymbol{V}^T\boldsymbol{A}^{-1}\boldsymbol{Z})^{-1}\boldsymbol{V}^T\boldsymbol{A}^{-1}$$

to write formulas for the vectors $\boldsymbol{w}^{(k)}$ and $\boldsymbol{u}^{(k)}$.

(b) (3) What vectors would you store in order to be able to form $(\boldsymbol{B}^{(3)})^{-1}\boldsymbol{v}$ for an arbitrary vector $\boldsymbol{v}$?

(c) (4) How many floating-point multiplications would it take to form $(\boldsymbol{B}^{(3)})^{-1}\boldsymbol{v}$?