AMSC/CMSC 660 Quiz 3 , Fall 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name _____

1. (10) Give as much information as you can about the location of the eigenvalues of the matrix

$$\begin{bmatrix} 1.0 & -0.2 & 0.3 & 0.1 & 0.1 \\ -0.2 & 2.0 & 0.8 & -0.1 & 0.8 \\ 0.3 & 0.8 & 2.2 & 0.2 & 0.1 \\ 0.1 & -0.1 & 0.2 & 0.5 & -0.1 \\ 0.1 & 0.8 & 0.1 & -0.1 & 1.5 \end{bmatrix}.$$

2. (10) One way to solve the least squares problem

$$\min_{\boldsymbol{x}} \|\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}\|_2^2,$$

where $\boldsymbol{A}$ and $\boldsymbol{b}$ are given, is to set the gradient of the function to zero, obtaining the relation

$$\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}^T\boldsymbol{b}.$$

2a (3) Show that the matrix $\boldsymbol{A}^T\boldsymbol{A}$ is symmetric. (It is also positive definite, but you do not need to show this.)

2b (7) If $\boldsymbol{A}$ is $m \times n$ $(m > n)$ and full rank, how many multiplications does it take to form and solve the linear system $\boldsymbol{A}^T\boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}^T\boldsymbol{b}$?