AMSC/CMSC 660 Quiz 4 , Fall 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name _____

1. (10) Suppose we have a singular value decomposition of the $m \times n$ matrix $\boldsymbol{A}$ as $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$. Suppose that $m > n > d = rank(\boldsymbol{A})$.

1a. How many of the singular values are positive?

1b. Give an orthogonal basis for the range of $\boldsymbol{A}$.

1c. Give an orthogonal basis for the nullspace of $\boldsymbol{A}^T$.

1d. What is the largest eigenvalue of $\boldsymbol{A}^T\boldsymbol{A}$?

1e. What is the smallest eigenvalue of $\boldsymbol{A}^T\boldsymbol{A}$?

2. (10) Consider Volterra's model of rabbits and foxes:

$$\begin{aligned} \frac{dr}{dt} &= 2r - \alpha r f, \\ \frac{df}{dt} &= -f + \alpha r f, \end{aligned}$$

and let $\alpha = 0.01$.

At the point $f(t) = 20$, $r(t) = 5$, is the system of ODEs stable or unstable? Justify your answer. (Hint: Using the Gerschgorin Theorem might save you some work.)