AMSC/CMSC 660   Quiz 5   ,   Fall 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

1. (10) Mark each statement True or False.

1a. ⎯⎯⎯⎯⎯⎯⎯⎯ Applying the finite difference method to the problem

$$u'' = 10u + e^t u',$$

with $u(0) = u(2) = 5$, gives a linear system of equations to solve.

1b. ⎯⎯⎯⎯⎯⎯⎯⎯ ODE solvers try to keep the global error smaller than a user-provided tolerance.

1c. ⎯⎯⎯⎯⎯⎯⎯⎯ An ODE problem can be stiff for some values of t and unstable for others.

1d. ⎯⎯⎯⎯⎯⎯⎯⎯ An ODE involving 4-th derivatives of a function $y(t)$ can be expressed as a system of 4 differential equations involving only first derivatives.

1e. ⎯⎯⎯⎯⎯⎯⎯⎯ Gear methods should not be used for unstable ODEs.

2. Define the Hamiltonian

$$H(q, p) = \frac{1}{2m}p^2 + \frac{k}{2}(q - L)^2,$$

where m, k, and L are constants.

2a. (7) Write the Hamiltonian system corresponding to H. Recall that the system is

$$\boldsymbol{y}' = \boldsymbol{D}\nabla_{\boldsymbol{y}}H(\boldsymbol{y}),$$

where $\boldsymbol{D}$ is a block-diagonal matrix with blocks equal to

$$\boldsymbol{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

2b. (3) What does the Hamiltonian tell you about the solution $p(t), q(t)$?