AMSC/CMSC 661 Quiz 3 , Spring 2005

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. During the quiz you may use your textbook, my notes, and your own notes. No communication with others and no calculators or other electronic devices are permitted.

Name ____

1. Consider solving the equation

$$-\bigtriangledown \cdot \bigtriangledown u = f$$

in Ω, with $u = 0$ on the boundary of Ω, using piecewise linear finite elements, which gives us a linear system of equations $\mathbf{AU} = \mathbf{g}$. The domain Ω (see figure) has been divided into 26 triangles.
Let P be the point where triangles 3 and 7 intersect, and let Q be the point where triangles 14 and 20 intersect.
Let ϕ_P be the basis function that is 1 at P, and let ϕ_Q be the basis function that is 1 at Q.

a. (2) What is the dimension of the matrix $\mathbf{A}$?

b. (2) One of the matrix entries is equal to $a(\phi_P, \phi_Q)$. Which triangles are used in computing this entry?

c. (2) How many nonzeros are in the row of the matrix corresponding to the equation $a(u_h, \phi_P) = (f, \phi_P)$?

d. (2) How many triangles are used in computing (f, ϕ_P)?

2. (10) Consider the differential equation

$$-5u_{xx} - u_{yy} + \pi^2 u = -3\pi^2$$

on the unit square $\Omega = (0,1) \times (0,1)$ with boundary conditions $u(x,y) = x^2 + y^3$ on the boundary. Without using a Green's function or an explicit solution to the problem, tell me about the solution:
(3 pts) Does it exist?
(2 pts) Is it unique?
(5 pts) What are upper and lower bounds on the solution?
Justify each of your answers by citing a theorem and verifying its hypotheses.