Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. During the quiz you may use your textbook, my notes, and your own notes. No communication with others and no calculators or other electronic devices are permitted.

Name _

1. Suppose we use a piecewise linear finite element algorithm to compute an approximate solution u_h to an elliptic partial differential equation

 $\mathcal{A}u = f$

in a smooth domain Ω , with u = 0 on the boundary of Ω . List the 4 most important sources of error that make our computed solution u_h^{comp} different from u. (If you list more than 4 sources, I'll give full credit if the most important 4 are in your list.)

2a. Let $\mathcal{A}u = -u_{xx} - u_{yy}$, and verify that for any positive integers j and k,

$$v_{jk}(x,y) = \sin(j\pi x)\sin(k\pi y)$$

is an eigenfunction of \mathcal{A} . What is the corresponding eigenvalue? 2b. Note that $v_{jk} = 0$ on the boundary of the unit square $\Omega = (0, 1) \times (0, 1)$. Consider solving the problem

$$-u_{xx} - u_{yy} = f$$

in Ω , with u = 0 on the boundary of Ω . Express the solution u in terms of the eigenfunctions of \mathcal{A} and give a computable expression for the coefficients of the eigenfunctions.