AMSC/CMSC 661 Quiz 5 , Spring 2005

1a. (2) In Graph A, which node will be ordered first in the minimum degree algorithm?

Answer: d

1b. (2) How many elements that are zero in the lower triangle of B will be nonzero in its Cholesky factor? (In other words, how much fill-in will occur?)

Answer: The profile contains two zero elements below the diagonal: (4,2) and (4,3). Only one, (4,2) actually fills in, since (1,3) is zero.

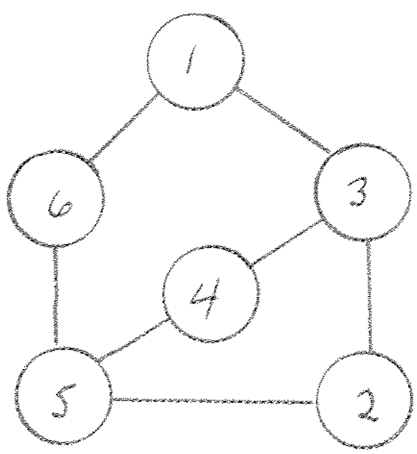
1c. (2) Give a basis (any basis) for $\mathcal{K}_2(D, c)$.

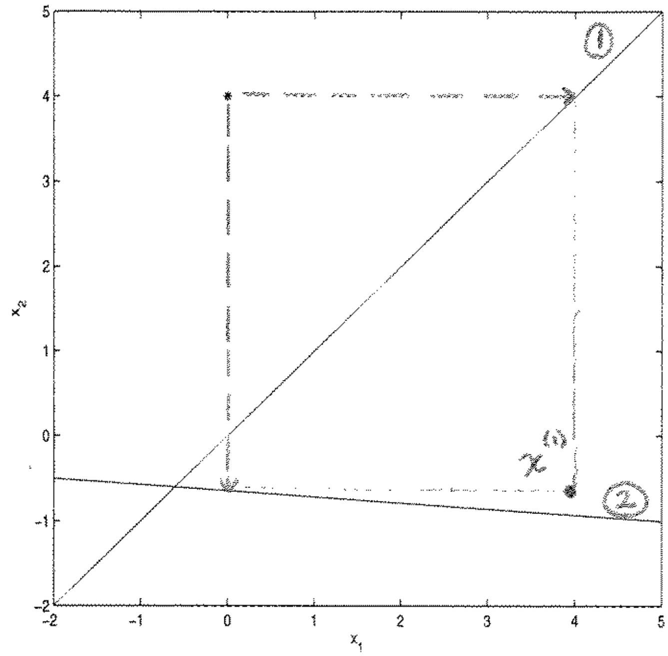
Answer:

$$c = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad Dc = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}.$$

(A side note: For this matrix, $\mathcal{K}_4(D, c)$, which usually has dimension 4, has

1d. (4) Draw the graph for matrix E.
Answer: There is a cycle created by 2, 3, 4, and 5, and then edges between 5 and 6, 6 and 1, and 1 and 3.





2a. (5) Suppose $n = 2$ and our linear system can be graphed as in the figure. The first equation is the line with positive slope. Draw the next Jacobi iterate using the point marked with a star as $x^{(0)}$. Does the iteration depend on the ordering of the equations?

Answer: See graph.
Yes, the iteration does depend on the ordering. (This can be seen from the example above; the Jacobi iterate would be off the graph, far to the left, if the 2nd equation were ordered first.)

2b.(5) Apply SOR ($\omega = .5$) to the linear system

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

with a starting guess of $x_1^{(0)} = x_2^{(0)} = 0$. What is $x^{(1)}$? Will the iteration converge to the true solution? Justify your answer.

Answer: For Gauss-Seidel,

$$\begin{aligned} x_1^{(1)} &= (4 - x_2^{(0)})/2 = 2, \\ x_2^{(1)} &= (-1 - x_1^{(1)})/3 = -1, \end{aligned}$$

so the SOR iterate is

$$(1-\omega)x^{(0)} + \omega x_{GS}^{(1)} = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}.$$

SOR does converge for this matrix; this is guaranteed by the SIM handout, since the matrix in the problem is symmetric positive definite (and also irreducible and diagonally dominant).