AMSC/CMSC 661 Quiz 6 , Spring 2005

1. (10) Suppose you want to solve a linear system of equations $Ax = b$, $n = 100,000$, and have your choice of 2 preconditioners for GMRES:

- $M_1$ creates a $\hat{G}_1$ with 5 small clusters of eigenvalues. 2 seconds and $10^6$ storage locations are required to form $M_1^{-1}z$ for an arbitrary vector $z$.
- $M_2$ creates a $\hat{G}_2$ with 10 small clusters of eigenvalues. 1 second and $5 \times 10^5$ storage locations is required to form $M_2^{-1}z$ for an arbitrary vector $z$.

It takes .25 seconds and $10^6$ storage locations to form $Az$. Which preconditioner would you advise using in order to compute an approximate solution to the problem? Why?

**Answer:** First, note that $\hat{G} = I - M^{-1}N = M^{-1}(M - N) = M^{-1}A$, so each multiplication by $\hat{G}$ requires one multiplication by $A$ and one solve of a linear system involving $M$.

Consider $M_1$. There are 5 clusters of eigenvalues, so we expect a good answer in 5 iterations. Each iteration costs 2.25 sec (since the GMRES overhead will be microseconds at most when $n = 10^5$), so the time will be about 11.25 sec. The storage required will be about $10^6$ for multiplication by $A$, $10^6$ for multiplication by $M_1^{-1}$, and $5 \times 10^5$ for the $P$ matrix.

Consider $M_2$. There are 10 clusters of eigenvalues, so we expect a good answer in 10 iterations. Each iteration costs 1.25 sec, so the time will be about 12.5 sec. The storage required will be about $10^6$ for multiplication by $A$, $5 \times 10^5$ for multiplication by $M_1^{-1}$, and $10 \times 10^5$ for the $P$ matrix.

So $M_1$ should be faster, and $M_2$ should require less storage. $M_1$ is overall probably the better choice.

2. (10) The following is the Lanczos algorithm, closely related to CG. As usual, $A$ is a matrix with $nz$ nonzero elements and $b$ is a vector of length $n$. How much storage does the algorithm use? How many multiplications and divisions does it perform? (Express your answers in terms of the parameters $n$, $k$, and $nz$.)

```
         | r=b;
n mult   | beta = norm(b);
         | vsav=[];
         | beta0=beta;
         | v=zeros(n,1);
         |
         | for i=1:k,
         |
         |    vold=v;
n div.   |    v=r/beta;
nz mult  |    av=A*v;
n mult   |    alpha=v'*av;
2n mult  |    r=av-alpha*v-beta*vold;
n mult   |    beta=norm(r);
         |
         |    vsav=[vsav v];
         |
         |    alphasav(i)=alpha;
         |    betasav(i)=beta;
         |
         | end
```

**Answer:** $n + (5n + nz)k$ multiplications and divisions.

Storage:

- `A` takes $3nz$ locations (Matlab sparse matrix).
- `b, r, v, vold, av` each take $n$ locations.
- `vsav` takes $nk$ locations.
- `alphasav, betasav` each take $k$ locations.

Total: $3nz + (k + 5)n + 2k + O(1)$ locations.