AMSC/CMSC 661 Quiz 8 , Spring 2005

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. During the quiz you may use your textbook, my notes, and your own notes. No communication with others and no calculators or other electronic devices are permitted.

Name ____

1. (10) Consider the equation

$$u_t + 2u_x - 2u_y + 6u = 5(x + y + t)^2$$

where the domain Ω is the unit circle, and $u : \Omega \times \mathcal{R}_+ \to \mathcal{R}$. Draw the unit circle, and mark on it the points that define the inflow boundary for this problem. Justify your answer.

2. (10) Consider the problem

$$u_{tt} - c^2 \Delta u = e^{-i\omega t} f(x)$$

with initial conditions $u(x, 0) = u_t(x, 0) = 0$ for $x \in \Omega \subset \mathcal{R}^d$.
a. Assume that the solution is of the form $u = e^{-i\omega t} z(x)$. Substitute this solution into the equation to obtain a problem of the form of the Helmholtz equation

$$-\Delta z - \kappa^2 z = g$$

with z given on the boundary of Ω. How are g and κ defined?
b. Let $\kappa = 4$ and let Ω be the square $(-1, 1) \times (-1, 1)$. Suppose we make a Galerkin finite element approximation to this problem. This gives us a linear system of equations to solve. In Homework 3, you solved a similar system of equations using conjugate gradients. Why can't conjugate gradients be used on our new linear system?