AMSC/CMSC 661 Quiz 9 , Spring 2005

1. (5) Consider the function

$$f(x) = \sin(\pi x)$$

expressed as

$$f(x) = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} F[m,k]\psi(2^m x - k)$$

where

$$\psi(x) = \begin{cases} 1 & \text{if } 0 \le x < 1/2 \\ -1 & \text{if } 1/2 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

What is $F[2,1]$?

Answer:

$$\begin{aligned} F[2,1] &= 2^2 \int_{-\infty}^{\infty} \sin(\pi x)\psi(2^2 x - 1)dx \\ &= 4\int_{1/4}^{1/2} \sin(\pi x)\psi(4x-1)dx \\ &= 4\int_{1/4}^{3/8} \sin(\pi x)dx - 4\int_{3/8}^{1/2} \sin(\pi x)dx \\ &= \frac{-4}{\pi}\cos(\pi x)|_{1/4}^{3/8} + \frac{4}{\pi}\cos(\pi x)|_{3/8}^{1/2} \\ &= \frac{4}{\pi}(-\cos(3\pi/8) + \cos(\pi/4) + \cos(\pi/2) - \cos(3\pi/8)). \end{aligned}$$

2 (5) Using the data returned from PDEtool by `getpetuc`, how can you find the maximum length of the side of a triangle in the triangulation?

Answer: From solution to Homework 2: The maximum side of the ith triangle can be computed by

```
x = p(1,t([1 2 3 1],i)); % x coordinates of vertices
y = p(2,t([1 2 3 1],i)); % y coordinates of vertices
len = sqrt((x(1:3)-x(2:4)).^2 + (y(1:3)-y(2:4)).^2);
maxlen = max(len);
```

Take the maximum over all triangles.

3. (5) Suppose you are computing a finite element approximation to a self-adjoint elliptic partial differential equation and you decide to use an iterative method to solve the linear system. Which of these two algorithms would you use: conjugate gradients or GMRES? Why?

Answer: From Homework 3: Our matrix will be symmetric and positive definite, so cg will be more efficient in storage and (almost always) in time than GMRES.

4. (5) Consider the problem

$$-u''(x) = f(x) \text{ for } x \in (0,1)$$

with $u(0) = u(1) = 0$ and f computed so that the true solution is

$$u(x) = \begin{cases} x(1-x)e^x & \text{if } x \leq 2/3 \\ x(1-x) & \text{if } x > 2/3 \end{cases}$$

Will a Galerkin finite element approximation with piecewise linear elements and mesh size $h = .0001$ give a good approximation to u? Justify your answer.

Answer: From Homework 1 solution: The finite element equations are derived from the weak formulation of our problem, and when we use integration by parts, we leave off the boundary term that we would have gotten at $x = 2/3$, so our equations are wrong. Therefore, as $h \to 0$, we do not converge to this function.