Two Matrix Problems and Many Matrix Algorithms

In this homework, you will look for an efficient way to solve two matrix problems.

The first problem, from Boeing, is a finite element approximation of a pde model used in structural engineering.

The second problem concerns an interaction matrix for the coauthors of a mathematician named Paul Erdős. Each row and column corresponds to one coauthor. For example, row 11 is for James M. Anderson. There is a 1 in column j of this row if Anderson collaborated with the jth coauthor of Erdős, and a 0 otherwise.

Obtaining the data: The matrices `msc10848` and erdos991 can be obtained from Tim Davis' website `http://www.cise.ufl.edu/research/sparse/matrices`. (Use their "browse" command.) It is easiest to download the `.mat` files, which can then be loaded into MATLAB with the command `load msc10848` or `load Erdos991`. The matrix is called `Problem.A` and is in sparse format.

CHALLENGE 1 *In this challenge we solve the linear system* $\mathbf{Ax} = \mathbf{b}$ *where* $\mathbf{A}$ *is the matrix from* `msc10848` *and* $\mathbf{b}$ *is the vector with each entry equal to 1.*

1a. Make a table of the number of nonzeros in the Cholesky factorization of `msc10848` *using the original ordering, reverse Cuthill-McKee (*`symrcm`*), column count (*`colperm`*), and approximate minimum degree (*`symamd`*). Use each of the algorithms to solve the linear system. In each case, compute the time for solution (including time for ordering) and the norm of the difference between computed solution and the solution using the original ordering. (These should all be very small!) Also display the four matrices and their Cholesky factors using* `spy`*.*

1b. Use `pcg` *to solve the linear system. Set* `TOL = 1.e-5` *and* `MAXIT = 1000`*. First use it with no preconditioning. Then try with the cholesky-*∞ *preconditioning (*`cholinc(A,'inf')`*). Then try two other preconditioners, chosen by you to try to produce a solution quickly. (Note: There are lots of possibilities. Reordering* $\mathbf{A}$ *won't help the time for pcg with no preconditioning, but it will change the times with preconditioning. You could also experiment with different options to* `cholinc`*.) Produce a table of number of* `pcg` *iterations, solution times (including formation of preconditioner), and difference between the computed solution and the first solution from 1a.*

1c. Discuss your results.

POINTER 1 Matrix models
We have seen that finite element models are the most commonly used methods for solving differential equations, so efficient solution of them is obviously important.
Interaction matrices are also quite important in applications. For example, if the rows and columns represent websites and the entries represent link *or* no link *between the sites, the resulting matrix can be used in information retrieval applications. Interaction matrices are also used to analyze social interactions. In criminal investigations (e.g., the Enron scandal) they are used to identify interactions between individuals who might be engaged in conspiracies.*

POINTER 2 Irrelevant note
The Wikipedia article about Paul Erdős is interesting.
Paul Erdős' friends developed the concept of the Erdős number. Erdős is 0, coauthors of Erdős are 1, their coauthors are 2, etc. My Erdős number is 3. It is believed that 90 % of research-active mathematicians worldwide have Erdős numbers less than 8.

CHALLENGE 2 *Use* `eigs` *to find the smallest 6 eigenvalues of the matrix in* `Erdos991`.

Hint: If λ is an eigenvalue of $\mathbf{A}$, then $\lambda + \sigma$ is an eigenvalue of $\mathbf{A} + \sigma\mathbf{I}$. Choose (by trial-and-error) a value of σ so that `eigs` *finds the eigenvalues of $\mathbf{A} + \sigma\mathbf{I}$ corresponding to the smallest eigenvalues of $\mathbf{A}$. Justify your choice of σ.*

POINTER 3 Confession

Interaction networks are analyzed by computing eigen-information for the corresponding matrix. For example, the eigenvector corresponding to the largest eigenvalue can be used to partition the coauthors into two groups (those with positive entries in the eigenvector and those with negative) for which interactions within group are stronger than interactions between groups.

As far as I know, the values of the 6 smallest eigenvalues don't reveal anything about the interaction network, so Challenge 2 is rather artificial.

On the other hand, for a finite element model, these eigenvalues do hold useful information (about vibration frequencies for a structure, for example), so the technique used in Challenge 2 is useful in such contexts.
