Your Homework Assignment
CMSC/AMSC 661 Spring 2005
Waves
Homework 4 Part 2, Due May 12

The Model

The model for this problem is the wave equation

$$u_{tt} - c^2 u_{xx} = f$$

where $(x, u(x,t))$ gives the coordinates of a string at time t, and $f(x,t)$ defines the force exerted on the string at x at time t. ($x \in \mathcal{R}$)

The constant c is the speed of propagation of waves in the medium.

In order to make the solution unique, we specify initial conditions $u(x,0)$ and $u_t(x,0)$ and boundary conditions $u(0,t) = u(1,t) = 0$. The boundary conditions say that the string's position is fixed at both endpoints.

To discretize, we use 2nd order central differences for u_{tt} and u_{xx}, and for stability we keep $c\Delta t/\Delta x < 1$. Let $u_\ell^m \approx u(\ell\Delta x, m\Delta t)$. To start the method when $t = 0$, we need values for u_ℓ^{-1}, which we can obtain from the centered difference approximation

$$\frac{u_\ell^1 - u_\ell^{-1}}{2\Delta t} \approx u_t(x_\ell, 0).$$

Problem 3. (20) Write a well-documented Matlab function `[u,M,flag]=wavesolve(c,tmax,nt,nx,u0,ut0,f)` to solve the wave equation, where

- c is the speed of propagation of the wave.
- $0 \le t \le tmax$. $tmax$ should be a positive integer. Either flag an error or do something else reasonable if it is not.
- $\Delta t = 1/nt$ is the stepsize in t.
- $\Delta x = 1/nx$ is the stepsize in x.
- `flag` is an error flag, 0 if no error occurs and defined by your documentation otherwise. (Flag errors 1–5 for: $c, tmax, nt, nx \le 0$ or stability condition violated.)
- $x \in [0,1]$, and $u(0,t) = u(1,t) = 0$ for $t \ge 0$.
- `f` is a pointer to a function that evaluates $f(x,t)$.
- `u0` is a column vector of function values for $u(x_\ell, 0)$, with $x_\ell = \ell\Delta x$ and $\ell = 1, \ldots, nx - 1$.
- `ut0` is a column vector of function values for $u_t(x_\ell, 0)$, with $x_\ell = \ell\Delta x$ and $\ell = 1, \ldots, nx - 1$.

(continued)

- u is an array, with the computed value of $u(\ell\Delta x, (m-1)\Delta t)$ stored in position (ℓ, m).
- M is a Matlab movie displaying the solution. (Use `getframe` to make the movie, and use `axis` to make sure that each plot has the same axes.)

Try your function on three types of problems:

- On a piano or dulcimer, the string is struck with a hammer to make it vibrate. We'll approximate striking at $x = 1/4$ by

$$\begin{aligned} u(x,0) &= 0, \\ u_t(x,0) &= -\frac{1}{4} \text{ if } \ x = 1/4 \textit{ and } 0 \textit{ otherwise} \\ f(x,t) &= 0, \\ c &= 1. \end{aligned}$$

  (Assume that $x = 1/4$ is a mesh point.)

- On a guitar or harp, the string is plucked. We'll approximate plucking at $x = 1/2$ by

$$\begin{aligned} u(x,0) &= \min(2x, 2(1-x)), \\ u_t(x,0) &= 0, \\ f(x,t) &= 0, \\ c &= 1. \end{aligned}$$

- (Forced vibration) Let $\omega = 2\pi$ and set

$$\begin{aligned} u(x,0) &= 0, \\ u_t(x,0) &= 0, \\ f(x,t) &= (\sin\omega t)(\sin 2\pi x), \\ c &= 1. \end{aligned}$$

Describe each solution in words. (For ease of submission, insert these descriptions as comments to your Matlab function.)

Submission instructions: Submit your Matlab file as a plain-text email attachment. I will grade your submission by

- Looking at the code, to see that the algorithm is correct and well documented.
- Checking the documentation for the description of the three sample solutions.
- Running your code on some test problems.

Therefore, make sure that your inputs and outputs match the specifications exactly.