AMSC/CMSC 661   Quiz 2   ,   Spring 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no calculators, cellphones, or any other electronic devices, and don't communicate with other students. You may use the Larsson&Thomèe textbook, anything taken from the course website, and your own notes.

Name _____

Recall that the derivative of $\ln(x)$ is $1/x$.

1. (10) Let $\bar{\Omega} = [0, 1]$.

1a. (5) Give an example of a function $f(x)$ that is defined on Ω but not in $C(\Omega)$.

Answer: Any function that has at least one point of discontinuity in $(0, 1)$ is acceptable. For example, the function $f(x) = 1$ for $x <= .5$ $f(x) = 2$ for $x > .5$ is discontinuous at $x = .5$.

This was not meant to be a trick question, but four of you found it confusing, and I can understand why. See p. 231: $C(\Omega)$ is the set of functions that are continous on Ω. This is the same as $C^0(\Omega)$ but different from $C^1(\Omega)$.

1b. (5) Let $g(x) = 1/\sqrt{x}$. Is $g(x) \in L_2(\bar{\Omega})$? Justify your answer.

Answer: To be in $L_2(\bar{\Omega})$, we need this integral to be finite:

$$\int_0^1 g(x)^2 \, dx = \int_0^1 \frac{1}{x} \, dx \text{``} = \text{''} \log(x)|_0^1.$$

This is not finite, so $g(x)$ is not in $L_2(\bar{\Omega})$.

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2. (10 points) consider the problem

$$\mathcal{A}u = -(1 - \frac{x^2}{4})u''(x) - \frac{x}{2}u'(x) + \frac{1}{2}u = -2,$$

$$u(0) = 0, \ \ u(1) = 1.$$

Use the theorems in the book/notes to tell me as much as you can about the solution to this problem, without actually solving the problem. Hint: One bound on the solution can be obtained from using the monotonicity property to compare u to the solution to $\mathcal{A}w = f$ where $w(x)$ is an appropriately-chosen constant.

Answer:

- Verify that the assumptions for the Maximum Principle are satisfied, so

$$u(x) \leq \max(u(0), u(1), 0) = 1 \ \ x \in \Omega.$$

- The Minimum Principle can be applied by letting $w = -u$ but gives no new information.

- By Theorem 2.4 (with p.14 of the notes), the solution exists.

- By Cor. 2.2a, the problem has a unique solution.

- By Cor. 2.2b, the problem is stable.

- Let $v(x) = -4$. Then $v(x)$ is the solution to

$$\mathcal{A}v = -(1 - \frac{x^2}{4})v''(x) - \frac{x}{2}v'(x) + \frac{1}{2}v = -2,$$

  with boundary conditions $v(0) = v(1) = -4$. By Cor. 2.2c, the Monotonicity Theorem, $u(x) \geq v(x) = -4$ for $x \in (0,1)$. So now we have upper and lower bounds on the solution.

This is like a problem on 2005 Quiz 1.