AMSC/CMSC 661 Quiz 3 , Spring 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no calculators, cellphones, or any other electronic devices, and don't communicate with other students. You may use the Larsson&Thomèe textbook, anything taken from the course website, and your own notes.

Name _____

1. (10) Let $\Omega = (0, 1)$. Give the variational form (weak form) of the problem

$$-u''(x) = f(x), \quad x \in \Omega,$$

$$u(0) = 0, \qquad u'(1) = 0.$$

You will need the integration-by-parts formula

$$\int_0^1 v'w\,dx = vw\,|_0^1 - \int_0^1 vw'\,dx.$$

Your final result should look like

$$a(u, \phi) = (f, \phi)$$

for all ϕ in an appropriately chosen space. Tell me what that space is, and how $a(u, \phi)$ is defined.

2. Let Γ be the unit circle and Ω be its interior. For each of these problems, use either the maximum principle or the minimum principle (verifying the hypotheses) to give a bound on the solution to the differential equation.

2a. (5)

$$u_{xx} + u_{yy} = 1 - x^2 - y^2, \ \ (x, y) \in \Omega,$$
$$u(x, y) = 0, \ \ (x, y) \in \Gamma.$$

2b. (5)

$$-w_{xx} - w_{yy} + 5w = 1 - x^2 - y^2, \ \ (x, y) \in \Omega,$$
$$w(x, y) = 5, \ \ (x, y) \in \Gamma.$$