AMSC/CMSC 661 Quiz 4 , Spring 2010

1. Consider solving the equation

$$- \bigtriangledown \cdot \bigtriangledown u = f$$

in Ω, with $u = 0$ on the boundary of Ω, using piecewise linear finite elements, which gives us a linear system of equations $\mathbf{AU} = \mathbf{g}$. The domain Ω (see figure) has been divided into 26 triangles.

Let P be the point where triangles 3 and 7 intersect, and let Q be the point where triangles 14 and 20 intersect.

Let ϕ_P be the basis function that is 1 at P, and let ϕ_Q be the basis function that is 1 at Q.

a. (3) What is the dimension of the matrix $\mathbf{A}$?

Answer: 9×9, since there are 9 nonboundary nodes.

b. (3) One of the matrix entries is equal to $a(\phi_P, \phi_Q)$. Which triangles are used in computing this entry?

Answer: 5 and 7. Since ϕ_P is nonzero only on triangles 3,5,6,7 and ϕ_Q is nonzero only on triangles 4,5,7,13,14,20,21, the product of their gradients can be nonzero only on 5 and 7.

c. (3) How many nonzeros are in the row of the matrix corresponding to the equation $a(u_h, \phi_P) = (f, \phi_P)$?

Answer: 4, corresponding to basis functions ϕ_P, ϕ_Q, the function for the vertex shared by triangles 2 and 9, and the function for the vertex shared by triangles 6 and 13.

d. (3) How many triangles are used in computing (f, ϕ_P)?

Answer: 4. The triangles are 3, 5, 6, and 7.

2. (8) Draw an inadmissible triangulation of the unit square and explain what makes it inadmissible.

Answer: Any triangulation in which two of the triangles share a partial edge is inadmissible. The simplest inadmissible triangulation is constructed by drawing one the the diagonals of the square, and half of the other diagonal.