AMSC/CMSC 661 Quiz 5 , Spring 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no calculators, cellphones, or any other electronic devices, and don't communicate with other students. You may use the Larsson&Thomèe textbook, anything taken from the course website, and your own notes.

Name ____

1. (6) The following is the CG algorithm for solving the linear system $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$. As usual, $\boldsymbol{A}$ is a symmetric positive definite matrix with nz nonzero elements and $\boldsymbol{b}$ is a vector of length n. How much storage does the algorithm use? How many multiplications and divisions does it perform? (Express your answers in terms of the parameters n, K, and nz.)

Let $\boldsymbol{r} = \boldsymbol{b}$, $\boldsymbol{x} = \boldsymbol{0}$, and let $\gamma = \boldsymbol{r}^T\boldsymbol{r}$, $\boldsymbol{p} = \boldsymbol{r}$.
for $k = 1, \ldots, K$

$\boldsymbol{q} = \boldsymbol{A}\boldsymbol{p}$

$\alpha = \gamma/(\boldsymbol{p}^T\boldsymbol{q})$

$\boldsymbol{x} = \boldsymbol{x} + \alpha\boldsymbol{p}$

$\boldsymbol{r} = \boldsymbol{r} - \alpha\boldsymbol{q}$

$\hat{\gamma} = \boldsymbol{r}^T\boldsymbol{r}$

$\beta = \hat{\gamma}/\gamma$, $\gamma = \hat{\gamma}$

$\boldsymbol{p} = \boldsymbol{r} + \beta\boldsymbol{p}$

end (for k)

2a. (6) Suppose we use CG to solve the linear system $\widehat{\boldsymbol{G}}\boldsymbol{x}^* = \boldsymbol{c}$, and we know that all of the eigenvalues of $\widehat{\boldsymbol{G}}$ are in the interval $[1, 10^4]$. Give an upper bound on the number of iterations it will take to reduce the error

$$\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^*\|_{\widehat{\boldsymbol{G}}}$$

by a factor of 10^5.

2b. (8) Apply one step of SOR ($\omega = 1.5$) to the linear system

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

with a starting guess of $x_1^{(0)} = x_2^{(0)} = 1$. What is $\boldsymbol{x}^{(1)}$? Will the iteration converge to the true solution? Justify your answer.