"How Professor Eduard Stiefel Got to NBS-INA-UCLA in August 1951"
Notes from the address of John Todd to the Latsis Symposium 2002: "Iterative solvers for Large Linear Systems: celebrating 50 years of the conjugate gradient method," held at ETH Zürich, Switzerland, 18-21 February 2002. John Todd California Institute of Technology

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### 1 Introduction

Mr. Chairman, Ladies and Gentlemen, Meinen Damen and Herren. First of all let me thank the organizers for inviting me to speak on this important occasion. As you will see, it was my late wife Olga Taussky who should have talked, but I lost her seven years ago.

When I chose my title, I soon found that there was a remarkable parallelism between ETH and the National Bureau of Standards (NBS), and so I arranged a surveying with pages devoted to ETH and NBS which will be supplemented by more detail as necessary.

### 2 Olga and Professor Stiefel (1929-1932)

The story begins just over 72 years ago. Olga has finished her dissertation - apart from the oral by Christmas 1929 and having become interested in Topological Algebra decided to spend her last student semester in Zürich, where an uncle lived. Here are her own words.

"My last semester at the university was spent in Zürich. My thesis was completed by that time. I was even allowed to lecture about it on the weekly colloquium in the department in Zürich. I attended courses by Speiser, Fueter, M. Gut, Plancherel and Pölya. The latter gave me some good advice concerning lecturing. He attended my colloquium lecture and did not approve of my style. I am grateful to him to this day. I still had my oral examinations in Vienna looming over me, but during my semester in Zürich, I somehow relaxed for the first time in many years." [1]

Pölya's recipe was as follows: The first quarter should be understandable to absolutely everyone, the second quarter should include kind words about your friends (particularly those in the audience) and then it doesn't matter what you say in the last half hour. I adjust this by adding, sit down after a quarter hour.

Although Olga's thesis was on Algebraic Number Theory (Classfield Theory), she had also obtained results in Topological Algebra and approached Professor Heinz Hopf for advice on how to prove a

converse of one of them. He told her to get in touch with Stiefel. This meeting only took place twenty years later at the NBS Institute for Numerical Analysis at the University of California at Los Angeles (NBS-INA-UCLA).

Conversations at that time suggest that they were both in Göttingen in 1932. Olga was editing Hilbert's Collected Works, and Stiefel was on study trips to Hamburg and Göttingen. Actually, Stiefel recalled going on a ski trip with Olga, but I believe that Olga never skied with Stiefel or with anyone else.

### 3 Gene H. Golub

But I am going too fast. There was another global event in 1932, which was the birth of Gene Golub, who is being honored at this conference on the occasion of his 70th birthday. I have pictures here which were prepared by my good companion Rosemary Lonergan. One was taken in Pasadena in May 2001 when I had my 90th birthday. You see the president of Cal Tech, David Baltimore, with me and Gene Golub. Now Rosemary was not present at the birth of Gene Golub, but the other picture shows what she thinks he looked like.

# 4 Olga and Professor Stiefel (1932-1952)

Olga spent 1932-1934 in Vienna as an Assistant mainly to Hans Hahn and Karl Menger. In 1934-35 she was with Emmy Noether in Bryn Mawr College, and in 1935-37 she was at Girton College, Cambridge. Up to that time, her family were in Vienna, and she visited them from time to time, often stopping at Zürich on the way. She usually found that Stiefel was on military service.

Then in 1937 she came to work in London, in 1938 we were married, and in 1939 the war began, with little chance of contact at that time.

The story begins again with the founding of the National Applied Mathematics Laboratories of the NBS in 1947 by John Curtiss, followed by the foundation of the Institute of Applied Mathematics at ETH a year later.

Both Olga and I were invited to spend the Academic Year 1947-8 at NBS-INA-UCLA, for during the war years each of us had gotten involved in computation, and both of us were in contact with the USA (and other) mathematicians who were in England. In actual fact we only spent about 6 months in California since it took time to get buildings ready. Our first quarter was spent at the NBS in Washington, D.C. and the second at the Princeton Institute for Advanced Study (IAS) with John von Neumann. In the fall of 1948 we returned to London to decide whether we would emigrate. Because of this, Olga missed a chance of meeting Stiefel during the visit of the ETH team to the USA. In 1948-9 actually this team met with the New York Works Progress Administration (NY WPA) group and with NBS Director E. U. Condon in Washington but apparently did not get to California where John H. Curtiss was Acting Director of the INA. We came back in the fall of 1949, J.T. as Chief of the Computation Laboratory (CL) and Olga as a consultant to John Curtiss in the Division Office. Things went very well. The Electronic Computer Laboratory built the Standards Electronic Automatic Computer (SEAC) [15] for installation in the Computation Laboratory. F.L. Alt and J.T. first ran problems on it on Good Friday, 1950. The INA also flourished with Barkley Rosser as Director in 1950. Another type of machine, the Standards Western Automatic Computer (SWAC), was designed and built by Harry Huskey.

Now in 1951 was the Jubilee of NBS, and among the celebrations was to be a special Symposium at INA to coincide with the demonstration of SWAC. Olga was ordered to plan this Symposium. She realized that this was the chance to meet Stiefel and she invited him to visit Washington, D.C. and

INA. The Proceedings of the Symposium were published as NBS-AMS 29 [2], and among the other speakers were George Forsythe, A. M. Ostrowski, R. A. Frazer, Magnus Hestenes, Alfred Brauer, Gaetano Fichera, F. Rellich, Alexander Weinstein, Wallace Givens, and Alston Householder. Olga and Stiefel settled down in a Visitor Office in the CL in Washington and after talking over 20 years of mathematics, Stiefel produced a new proof of Olga's Cauchy-Riemann theorem of 1939 using group representation theory. I remember that as Stiefel was leaving his temporary office for Pasadena, Ostrowski came in to take it over for a few days (until he moved out to Pasadena). He scolded Stiefel and me for leaving him an untidy desk!

As far as I can remember, I do not think they returned to consider the original 1930 problem. The problem was solved later in [18].

It is interesting to compare the separate accounts of Hestenes and Stiefel in original their papers on the conjugate gradient method with that in their joint paper [17] and also with the account written later by Hestenes in [4], as a preliminary to the current papers presented at the Latsis Symposium [9].

# 5 Chronological Survey: ETH

ETH founded 1855

Eduard L. Stiefel 21 April 1909 - 25 November 1979

1928 ETH
1932 Hamburg, Göttingen
1935 Dissertation (Heinz Hopf)
1942 Habilitation
1948 Full Professor, Institute for Applied Mathematics

October 1948 - March 1949: ETH Team in US Published NBS AMS 49 (E.L. Stiefel., P. Henrici, H. Rutishauser.) 1952 Cauchy-Riemann work

### 6 Chronological Survey: NBS

NBS founded 3 April 1901

Magnus R. Hestenes 1906 - 31 May 1991

Olga Taussky August 30, 1906 - October 7, 1995

1925 Koemer Schule, Linz
1930 Vienna - Zurich University
1930 Dissertation (P. Furtwangler)
1930 Vienna
1931 Göttingen (Hilbert edition)
1932 Zurich ICM (International Congress of Mathematicians)

1932-34 Vienna (Hans Hahn and Karl Menger) 1934-5 Bryn Mawr College (Emmy Noether) 1935-37 Girton College, Cambridge 1937-46 University of London 1939-Cauchy-Rieman equation paper 1947-8 NBS-INA-UCLA 1948 Princeton IAS (von Neumann)

1947 E.U. Condon fourth Director of NBS National Applied Mathematical Laboratories, J.H. Curtiss, Chief

Good Friday, 1950 (NBS-INA-UCLA, CL, SEL (Statistical Engineering Laboratory), MDL (Machine Development Laboratory)) SEAC, (F.L. Alt, J.T.)

August 1951 NBS Jubilee Symposium 24-25 SWAC NBS AMS 29 (ed. Paige and Olga Taussky) [2] NBS AMS 39 (ed. Olga Taussky) [3]

1952-54 BATTERY ADDITIVE SCANDAL, [4, pp.36-37] [16] ending Department of Defense support to NBS and thus the funding for the INA. Transfer to D.C. as Section 11.1 (Numerical Analysis)

1955 ERMETH

1959 Numerische Mathematik founded, with R. Sauer, A.S. Householder, A. Walther, E.L. Stiefel, J.T. as Founding Editors. On behalf of them (I'm the only surviving one) I thank all who have worked on it.

1964 NBS AMS 55 Abramowitz, Stegun editors [5]

1988 NBS renamed the National Institute for Standards and Technology

### 7 Cauchy-Riemann Equations

 $\operatorname{Let}$ 

$$\begin{split} w &= w(z) = u + iv, \, z = x + iy, \\ \ell_1 &= \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0, \, \ell_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0 \end{split}$$

Differentiating and subtracting

$$\Delta u = \frac{\partial \ell_1}{\partial x} + \frac{\partial \ell_2}{\partial y} = 0, \ \Delta v = \frac{\partial \ell_2}{\partial x} - \frac{\partial \ell_1}{\partial y} = 0$$

so that the real and imaginary parts of w are harmonic, satisfying the Laplace equation.

Olga's 1939 result [6] still allowed for the possible existence of Cauchy-Riemann (CR) systems of dimension 16,32,..., but soon work of Bott, Milnor and Kervaire ruled these cases out. Stiefel [7] not only gave the exact result but obtained the number of CR systems in each case when the coefficients were real or complex.

Abstract from [7]:

The n linear partial differential equations with constant complex coefficients

$$\ell_j \equiv \sum_{i,k}^n a_{jk}^i \frac{\partial u_k}{\partial x_i} = 0,$$

(j = 1, ..., n) are said to form a system of generalized Cauchy-Riemann equations, if

there exist constants  $f^h_{jk}$  such that

$$\Delta u_j \equiv \sum_{h,k}^n f_{jk}^h \frac{\partial \ell_k}{\partial x_h} \,.$$

It is proved that such systems exist for n = 1, 2, 4, 8 only. In the cases n = 2, 4 there are three essentially inequivalent systems; n = 8, only two. If the coefficients are required to be real, there exist only the classic system of two equations, the two systems of Dirac-Fueter equations, and two systems of eight equations.

#### 8 References and Further Resources

[1] D.J. Albers and G.L. Alexanderson, eds., Mathematical People, Birkhaüser 1985.

[2] L. J. Paige and Olga Taussky, eds., Simultaneous Linear Equations and the Determination of Eigenvalues, National Bureau of Standards Applied Mathematics Series, No. 29. U. S. Government Printing Office, Washington, D. C., 1953.

[3] Olga Taussky, ed., Contributions to the Solution of Systems of Linear Equations and the Determination of Eigenvalues, National Bureau of Standards Applied Mathematics Series, No. 39. U. S. Government Printing Office, Washington, D. C., 1954.

[4] Magnus R. Hestenes and John Todd, NBS-INA – The Institute for Numerical Analysis – UCLA 1947-1954, NIST Special Publication 730, U. S. Government Printing Office, Washington, D. C., 1991.

[5] Milton Abramowitz and Irene A. Stegun, eds., Handbook of Mathematical Functions, National Bureau of Standards Applied Mathematics Series, No. 55 U. S. Government Printing Office, Washington, D. C., 1964.

[6] O. Taussky, "An algebraic property of Laplace's Differential Equation," Quarterly J. Math. (Oxford) 10 (1939) 99-103.

[7] E. Stiefel, "On Cauchy-Riemann Equations in Higher Dimensions," J. Research Nat. Bur. Standards 48 (1952) 395-398.

[8] David R. Lide, ed., A Century of Excellence in Measurements, Standards, and Technology: A Chronicle of Selected NBS/NIST Publications, 1901-2000 NIST Special Publication 958, U. S. Government Printing Office, Washington, D. C., 2001.

http://nvl.nist.gov/pub/nistpubs/sp958-lide/cntsp958.htm

[9] 2002 LATSIS SYMPOSIUM Iterative Solvers for Large Linear Systems: celebrating 50 years of the conjugate gradient method http://www.cg50.ethz.ch/

[10] "In Memoriam: Prof. Dr. Eduard Stiefel 1909-1978," ZAMP 30 (2) 137-403, 1979

[11] M.H. Gutknecht, "The pioneering days of scientific computing in Switzerland," in The History of Scientific Computing, S.G. Nash ed. Addison-Wesley 1990, 301-313.

[12] John Todd, "Numerical Analysis at the National Bureau of Standards," SIAM Review 17 (1975) 361-370.

[13] John Todd, in The History of Scientific Computing, S.G. Nash ed. Addison-Wesley 1990, 301-313.

[14] Honoring the Achievements and Contributions of John Todd. This website contains a biography of John Todd and a transcript of an interview with him by Shirley K. Cohen in 1996, giving background on many of the issues discussed here. http://www.cacr.caltech.edu/Todd/

[15] Information about the SEAC, http://museum.nist.gov/panels/seac/INTROD 1.HTM

[16] Information about the battery additive scandal, http://www.100.nist.gov/phpostwar.htm1953

[17] Magnus R. Hestenes and Eduard Stiefel, "Methods of Conjugate Gradients for Solving Linear Systems," J. Res. Nat. Bur. Standards 49 (1952) 409-436.

[18] M. L. Curtis and J. Dugundji, "Groups which are Cogroups," Proc. Amer. Math. Soc. 22 (1969) 235-237.