Due at the start of class Thurs, Apr 1, 2004.
Read Chapter 4 from Harel's book.

Problem 1. Given a graph $G = (V, E)$, how can we check to see if it is 2 colorable? We need to color all the nodes using only two colors, so that adjacent nodes have different colors.

Problem 2. Prove by induction that for all $n \geq 1$.

$$1^3 + 2^3 + \ldots n^3 = (1 + 2 + 3 + \ldots + n)^2$$

Problem 3. Consider the following problem: There is a set of movies $M_1, M_2, \ldots, M_k$. There is also a set of customers, with each one indicating the two movies they would like to see this weekend. (Assume that customer i specifies a subset S_i of the two movies that he/she would like to see. Movies are shown on saturday evening and sunday evening. Multiple movies may be screened at the same time.)

You need to decide which movies should be televised on saturday, and which movies should be televised on sunday, so that every customer gets to see the two movies they desire. The question is: is there a schedule so that each movie is shown at most once? How can we check if such a schedule exists?

(One naive solution would be to show all movies on both days, so that each customer can see one desired movie on each day. We would need k channels if k is the number of movies in this solution – hence if there is a solution in which each movie is shown at most once, we would like to find it.) It is convenient to formulate this problem as a graph problem.

Problem 4. Three couples are going together on a journey. They reach a river which they have to cross. They can cross using a boat that can only hold two people at a time. No husband wishes to leave his wife in the company of other men when he is not present. Construct a graph to show how the transfer can be done.

Problem 5. We need to assign two frequencies to each node of a graph. Adjacent nodes should have different frequencies. In other words if node a has frequencies $\{1, 2\}$ then any node adjacent to a cannot use frequencies from this set. What is the minimum number of frequencies (colors) required to color an odd cycle (such a graph might come from cellular frequency assignments for transmitters on the beltway)?