

Due in class: March 6.

If you cannot come up with algorithms that run in the required time, then provide (correct) slower algorithms for partial credit. Write your answers using *pseudo-code* in the same style as the textbook. These make the algorithm description precise, and easy to read (as opposed to code in C or some other language).

Please also provide a proof of correctness.

- (1) Suppose that a document contains 300 pages and contains 20 misprints. What is the probability that there is more than one misprint on a particular page?
- (2) Suppose 4% of the parts made in a factory are defective. Suppose we ship out a batch of 25 parts. What is the probability that there are no defective parts in this batch?
- (3) Suppose we toss 1000 fair coins. Use Chernoff bounds to derive an upper bound on the probability that we either get less than 400 heads or more than 600 heads is small.
- (4) Suppose we randomly label n nodes of a graph using labels in $\{1, \dots, n\}$. Suppose we now select a node if its label value is smaller than the labels of all its neighbors. What is the probability that a node v is selected? Also prove that two adjacent nodes cannot be selected.
- (5) Suppose we have a collection of sensors that monitor some targets. Assume that each target can be monitored by exactly two sensors. Construct a graph $G = (V, E)$ where nodes in V correspond to sensors, and corresponding to each target there is an edge in the graph connecting the sensor nodes. Our goal is to put the sensors into an on-off cycle to save battery power. Lets assume that we partition the sensors into two groups A and B , such that each sensor is in exactly one group and the groups are disjoint. Sensors in group A are switched on for some time, and then they are shut off and sensors in group B are switched on. Some targets are always monitored since one of the two sensors is always on.
 - Give an example to show that it may be the case that no matter how we come up with a partitioning of the sensors in some cases there is no schedule that always monitors all targets.
 - If the graph has the property that such a schedule exists then develop an algorithm to find it.
 - Suppose we partition the sensors randomly. In other words, each sensor joins partition A with probability 0.5 and B with probability 0.5. What is the expected number of targets that are covered in both time slots?