CMSC498K Homework 2 Solutions

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Problem 1

We assume each of the 20 misprints is equally likely to be on any of the 300 pages. Consider page 1. The probability that a given misprint is on page 1 is $1/300$, so the probability of no misprint on page 1 is $(299/300)^{20}$ and the probability of exactly one misprint on page 1 is $20(1/300)(299/300)^{19}$ (since that misprint could be any of the 20). Thus, the probability of more than one misprint on page 1 is

$$1 - (299/300)^{20} - 20(1/300)(299/300)^{19} \approx .00203 \ .$$

Problem 2

We assume that defects in different parts, even in the same batch, are independent (not a good assumption). The probability that a given part is good is $.96$, so the probability that all 25 in the batch are good is $.96^{25} \approx .360$.

Problem 3

For $i = 1, \ldots, 1000$, let X_i be a random variable that is 1 if coin i is heads and 0 if it is tails. Apply the two-sided Chernoff bound with $\mu = 500$ and $\delta = .2$:

$$P(X \leq 400 \text{ or } X \geq 600) = P(|X - 500| \geq 100) = P(|X - \mu| \geq \delta\mu) \leq 2e^{-\mu\delta^2/3} = 2e^{-500(.2)^2/3} \approx .00255 \ .$$

Problem 4

Let d_v be the degree of the node v, and let S be the set containing v and its d_v neighbors. By symmetry, the smallest-labeled node of S is equally likely to be any of the $d_v + 1$ nodes, with probability $1/(d_v + 1)$ each. Thus, the probability that v is selected is $1/(d_v + 1)$.

Two adjacent nodes cannot be selected because, if they were, each would have a strictly smaller label than the other, which is impossible.

Problem 5

- Let G be a triangle. One of the groups contains at most one sensor, and that sensor cannot monitor all three targets on its group's turn.
- A partition is good if and only if every target is monitored by both a group-A sensor and a group-B sensor, i.e., if every edge connects a vertex of A and a vertex of B. We see that a good partition for this problem is just a bipartition of G. We can find a bipartition (if one exists) using the simple depth-first search algorithm. **NOTE added – run BFS, and use distance labels mod 2 to compute the bipartition.**
- If we look at a particular target, the probability that its two sensors choose different groups is $1/2$. Thus, the expected number of targets covered in both time slots is half the total number of targets.