The Inapproximability of Illuminating Polygons by α-Floodlights

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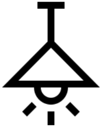
Qatar University

CCCG, 2015

	Guarding	Illumination
360°		
$0^\circ < \alpha < 360^\circ$		

	Guarding	Illumination
360°		
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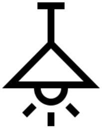
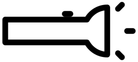
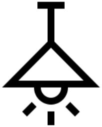
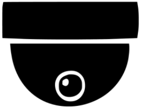
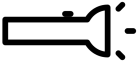
	Guarding	Illumination
$360°$		
$0° < \alpha < 360°$		

Guarding & Illumination

Motivation

	Guarding	Illumination
360°		
$0° < \alpha < 360°$		

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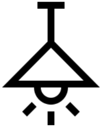
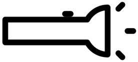
360°		
$0^\circ < \alpha < 360^\circ$		

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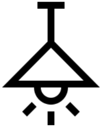
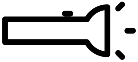
360°		
$0^\circ < \alpha < 360^\circ$	 	

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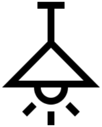
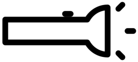
$360°$	?	
$0° < \alpha < 360°$		

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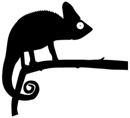
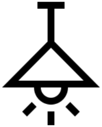
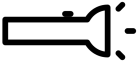
360°		
$0° < \alpha < 360°$	 	

Preliminaries

Definition (α-Floodlight)

An α-floodlight at point p, with orientation θ, is the infinite wedge $W(p, \alpha, \theta)$ bounded between the two rays $\overrightarrow{v_l}$ and $\overrightarrow{v_r}$ starting at p with angles $\theta \pm \frac{\alpha}{2}$. In a polygon P, a point q belongs to the α-floodlight if $\overline{pq}$ lies entirely in both P and $W(p, \alpha, \theta)$.

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Problem (Polygon Illumination by α-Floodlights (PFIP))

Given a simple polygon P with n sides, a positive integer m and an angular aperture α, determine if P can be illuminated by at most m α-floodlights placed in its interior.

Brief History

Problem (Art Gallery Problem)

Let P be a simple polygon without holes. Find the minimum subset S of the vertices of P such that the interior of P is visible from S.

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[2] Eidenbenz, S., Stamm, C., and Widmayer, P. (2001). Inapproximability results for guarding polygons and terrains.
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- $O(\log \log OPT)$-approximation[3].

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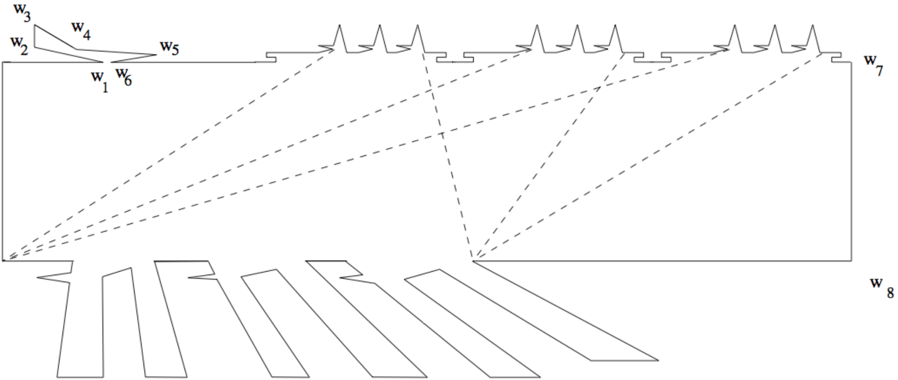
Reduction from 3SAT [Eidenbenz et al.]

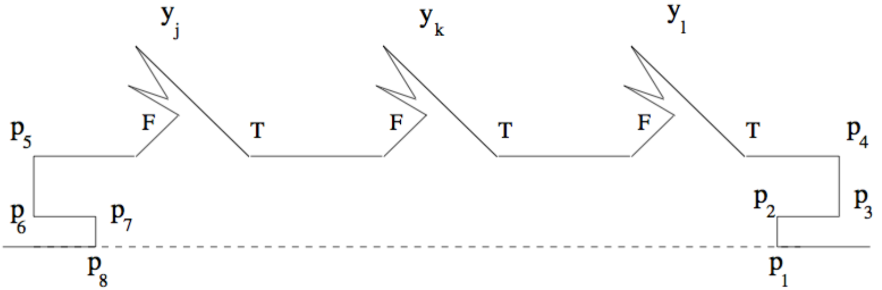
Big Picture



[Fig. 3 in Eidenbenz, ISAAC '98]

Reduction from 3SAT [Eidenbenz et al.]

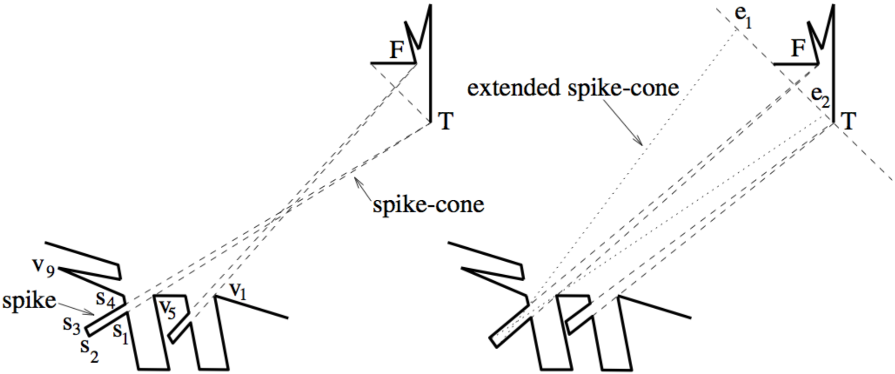
Clauses & Literals



[Fig. 2 in Eidenbenz, ISAAC '98]

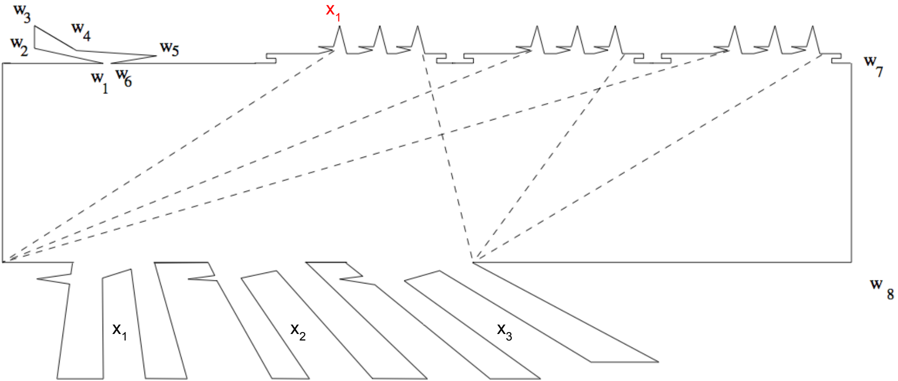
Reduction from 3SAT [Eidenbenz et al.]

Variables & Literals



[Fig. 4 in Eidenbenz, ISAAC '98]

Requirements on Guard Coverage?



[Fig. 3 in Eidenbenz, ISAAC '98]

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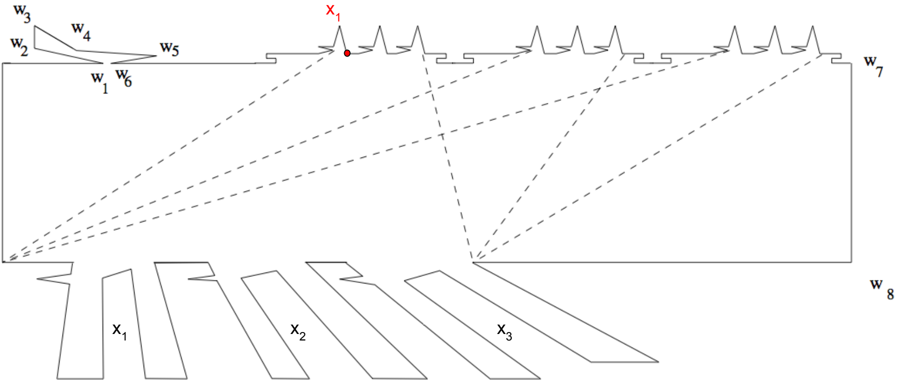

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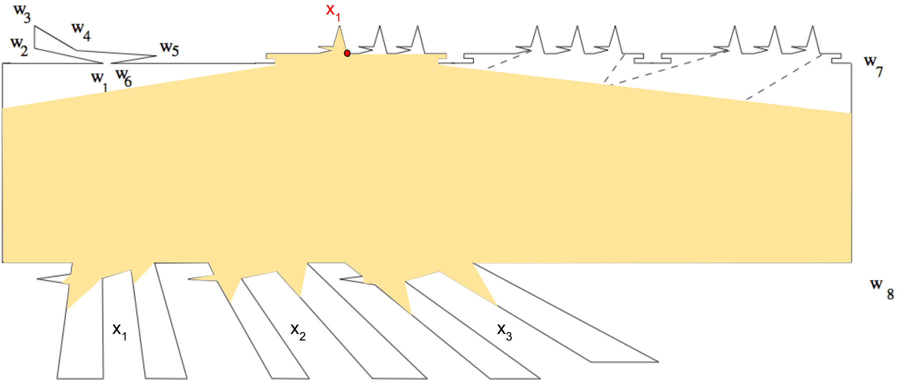

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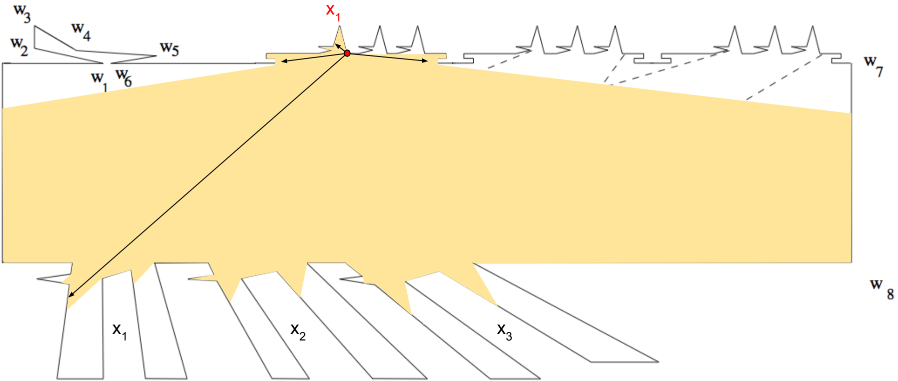

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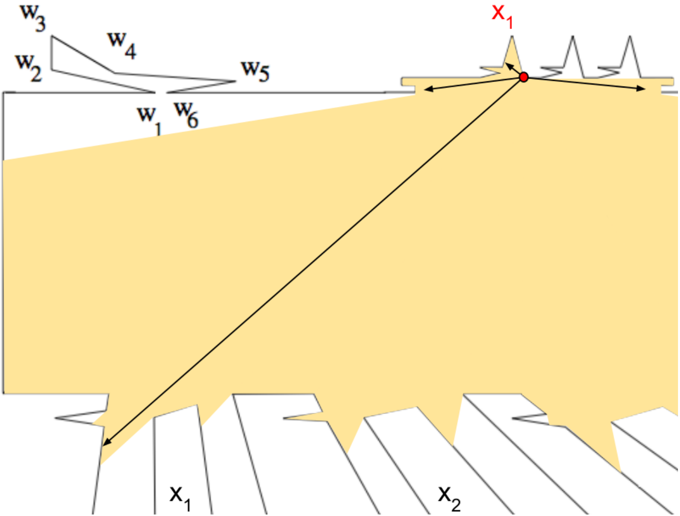
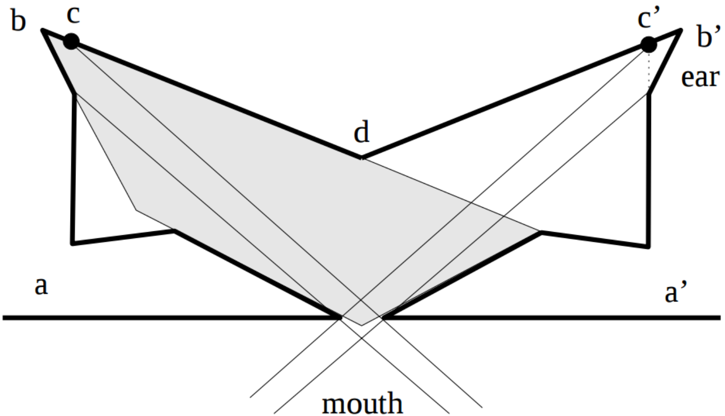


[Fig. 3 in Eidenbenz, ISAAC '98]

Beam Machine

Culberson and Reckhow, FOCS'88 & JAlg'94



[Fig. 7 in Eidenbenz and Widmayer, SICOMP '03]

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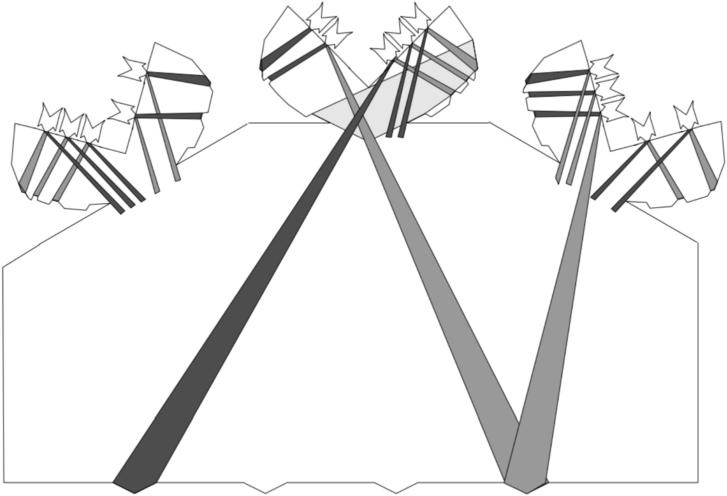
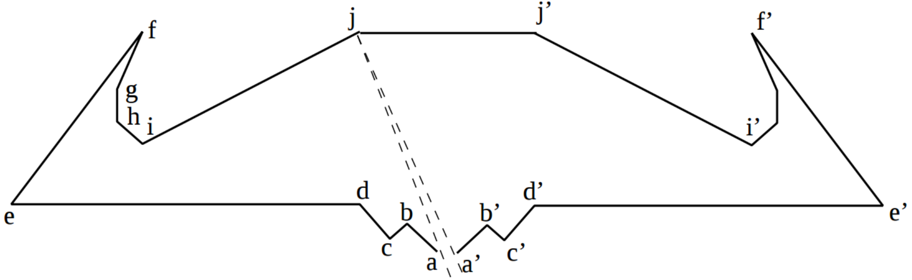


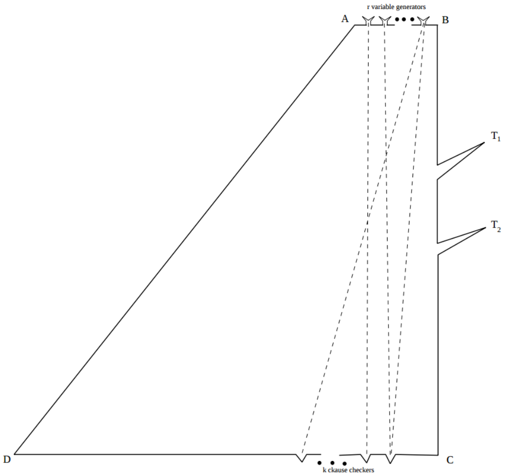
[Fig. 9 in Eidenbenz and Widmayer, SICOMP '03]



[Fig. 1 in Bagga, Gewali and Glasser, CCCG '94]

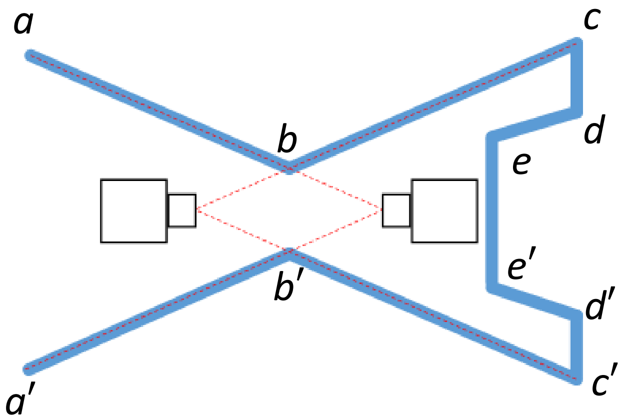
Beam Machine

Bagga, Gewali and Glasser, CCCG'96



[Fig. 5 in Bagga, Gewali and Glasser, CCCG '94]

Floodlight Gadget



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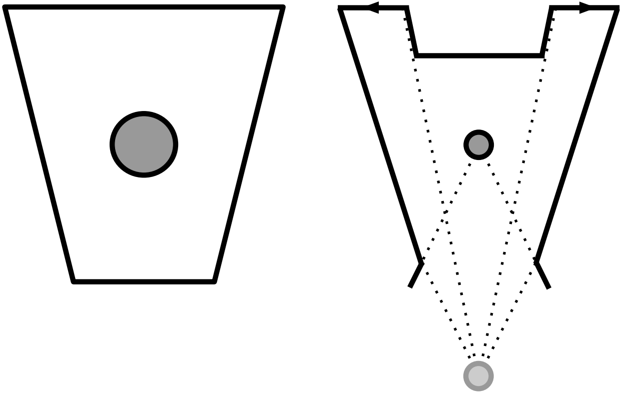
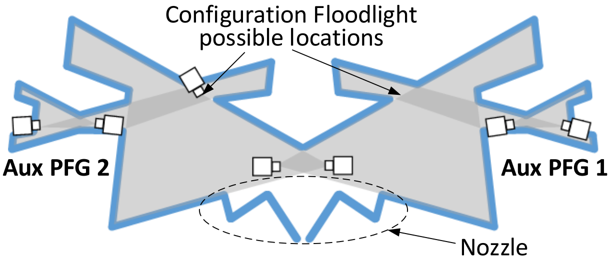


New Beam Machine



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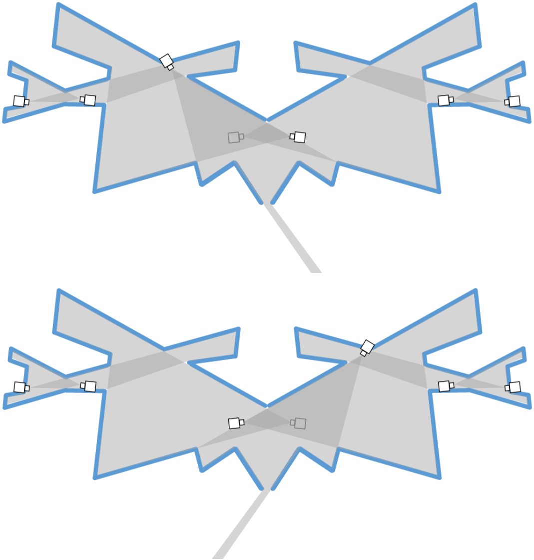

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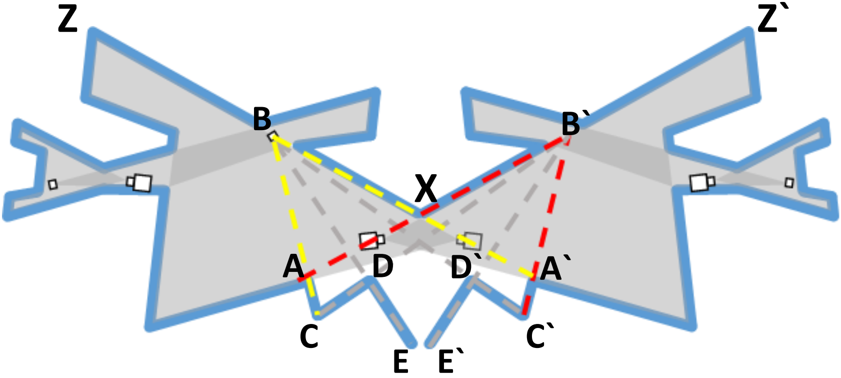

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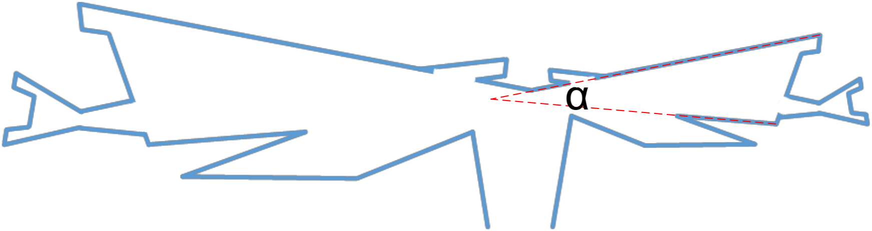

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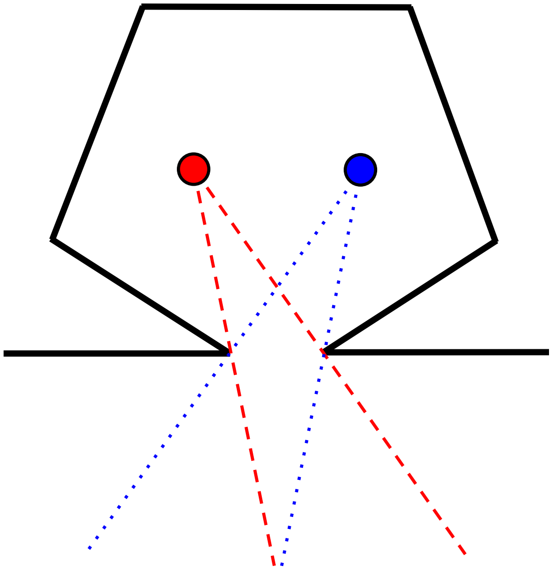
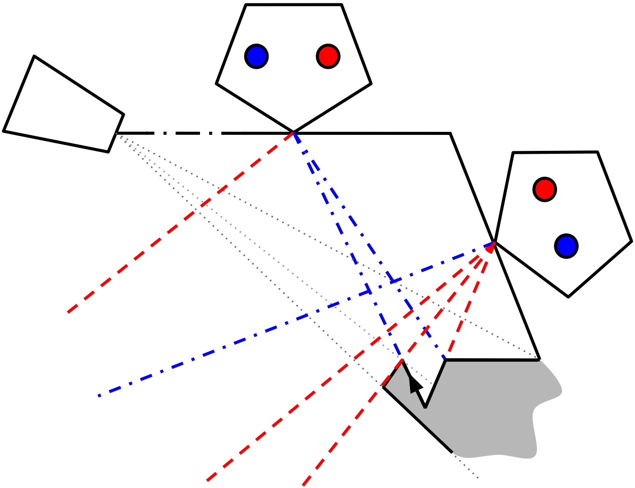
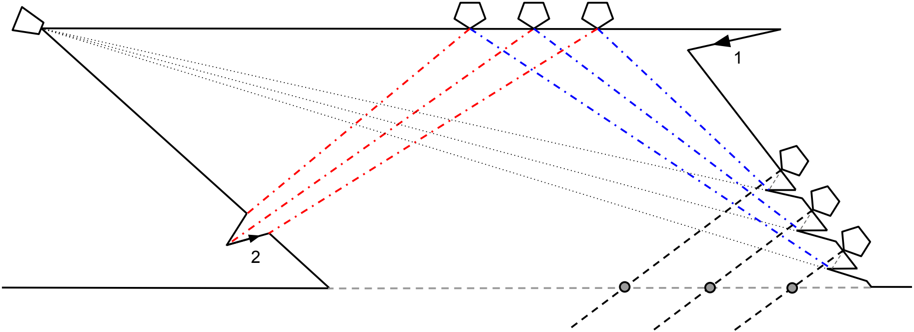


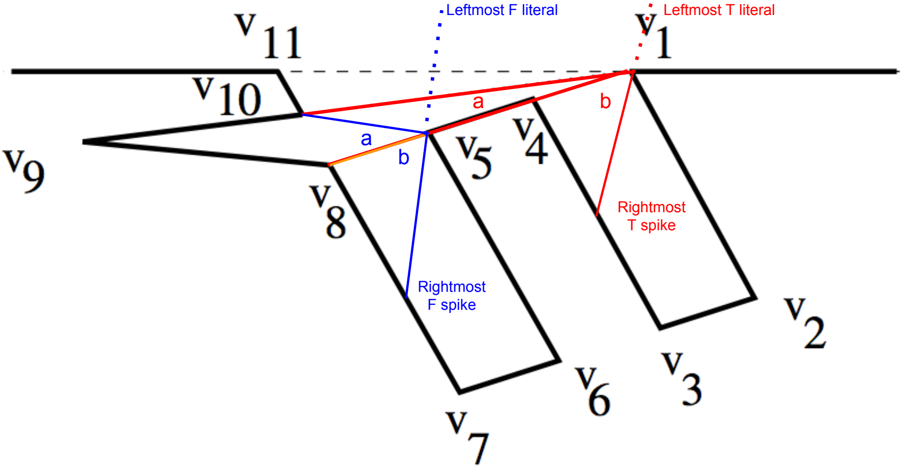
Beam Coupling



New Clause Gadget



New Variable Gadget



[Fig. 1 in Eidenbenz, ISAAC '98]

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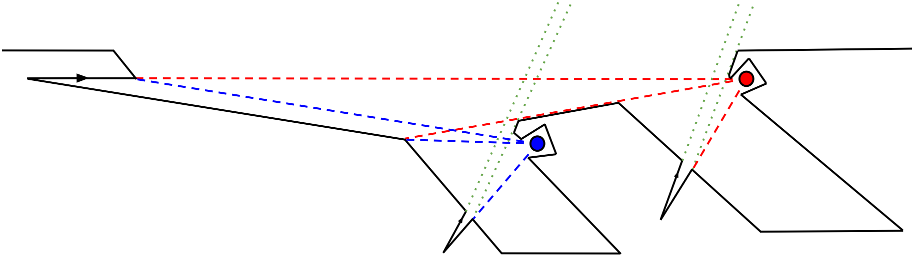
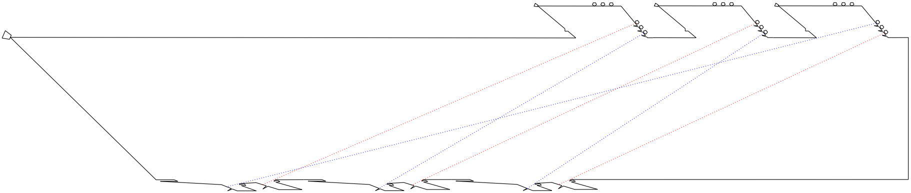


Hardness Results



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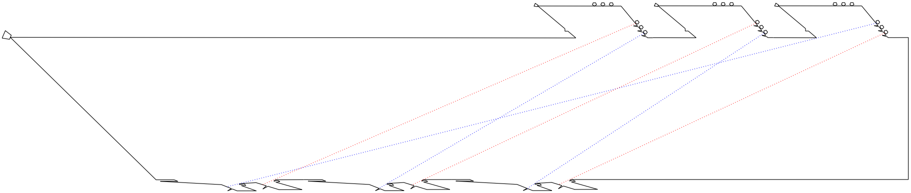

Theorem

PFIP is NP-hard.

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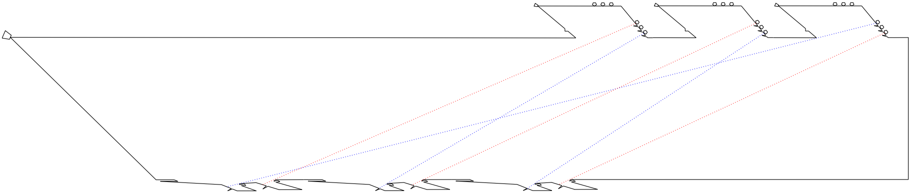

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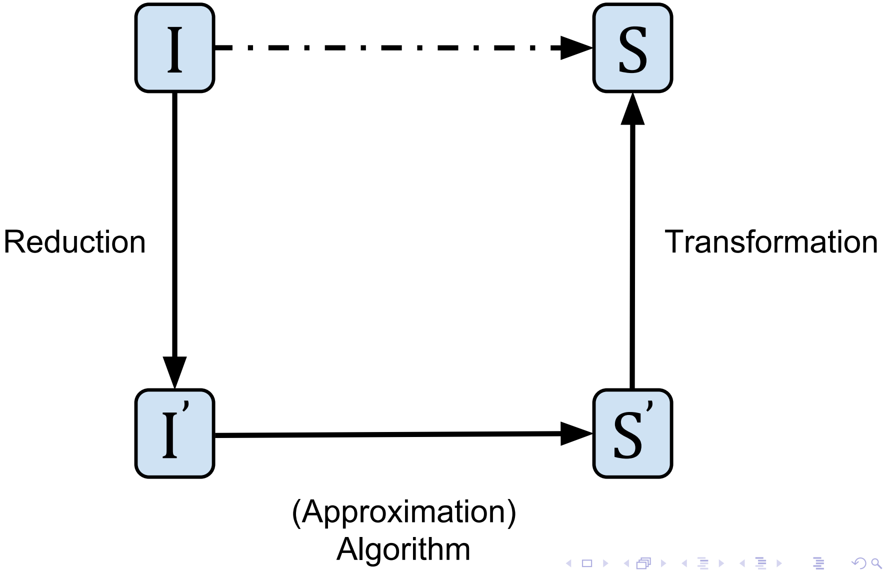
APX-hardness

Gap Preserving Reductions (informal)

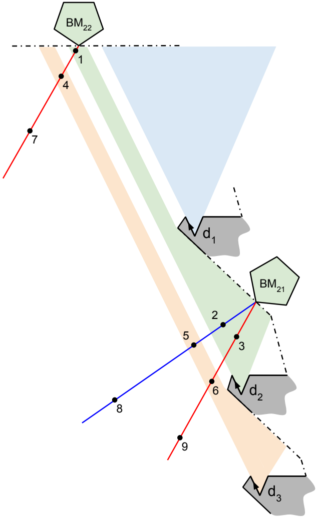


Problem (5-OCC-MAX-3-SAT)

Given a boolean formula Φ *in conjunctive normal form, with* m *clauses and* n *variables, 3 literals at most per clause, and 5 literals at most per variable, find an assignment of the variables that satisfies as many clauses as possible.*

APX-hardness

Transformation Process



APX-hardness

Definition (Flushing Condition)

An α-floodlight is *flush with the vertices of the polygon P* if at least one of $\overrightarrow{v_l}$ or $\overrightarrow{v_r}$ passes through some vertex of P, different from p, such that θ is determined implicitly.

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R-PFIP is APX-hard.

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Thank You

Questions?

akader@cs.umd.edu