Steiner Point Reduction in Planar Delaunay Meshes

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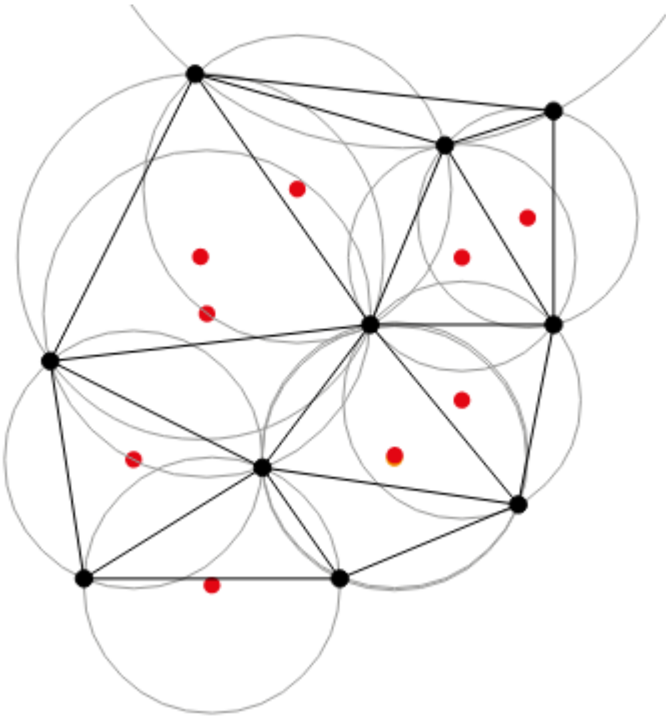
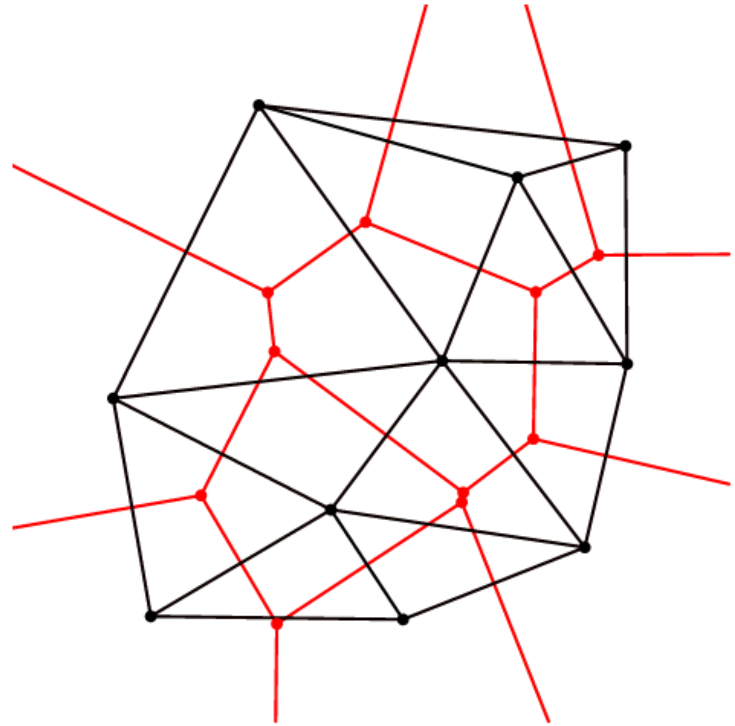
Outline

- Background
- Motivation
- Proposed Method
- Sample Results
- Discussion
- Extras

Background: Delaunay Triangulation

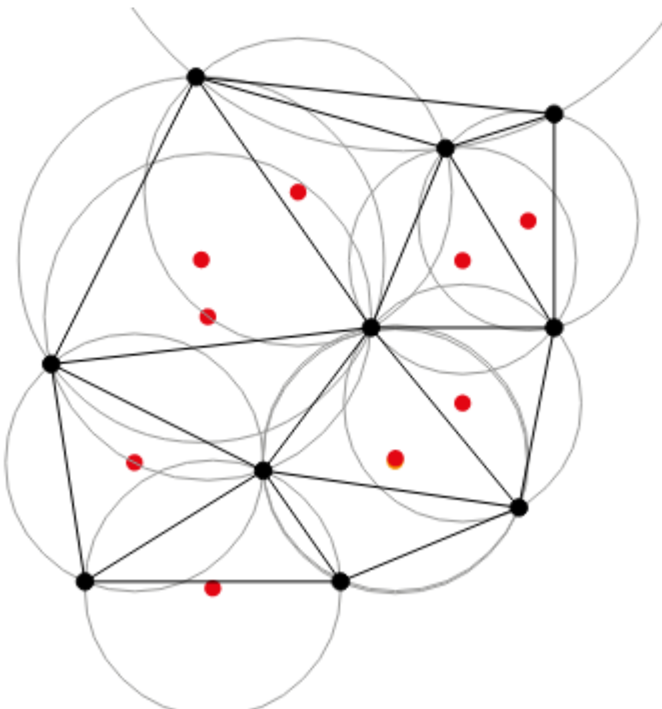
- Empty circumcircles
- Maximizes minimum angle
- Unique if points in general position

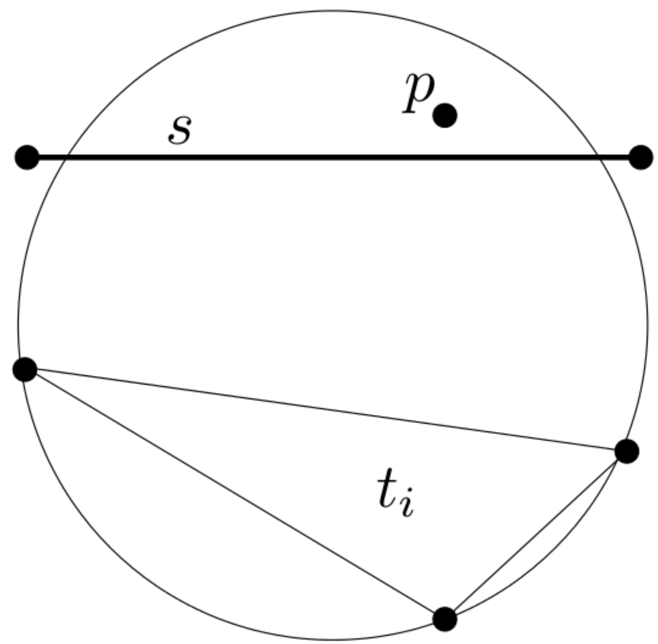



(Left) A Deluanay Triangulation. (Right) Corresponding Voronoi Diagram (constant weighted dual).
[Source: Wikipedia]

Background: Constrained DT

- ~~Empty circumcircles~~ (Not *true* Delaunay)
- Fewer elements
- Better bounds (min. angles + grading)





(Left) A Deluanay Triangulation with empty circumcircles. (Right) Circumcircle of t_i contains point p, which is not to the interior of t_i. [Source: Wikipedia (left), Chrisochoides et al.(right)]

Background: Meshing Algorithms

- Input: (P, [S]), α. Output: good [conformal] Delaunay complex.
- Ruppert's Delaunay Refinement Algorithm (DR) [1]
  - *" … perhaps the first theoretically guaranteed meshing algorithm to be truly satisfactory in practice"* [2].
  - Halts for an angle constraint of up to 20.7^{o} [1], 26.45^{o} [3].
- Chew's Second Algorithm [4]
  - Terminates with minimum angle up to 26.57^{o} [2], 28.6^{o} [5].

[1] Ruppert. A Delaunay Refinement Algorithm for Quality 2-Dimensional Mesh Generation. J. of Alg., 1995.

[2] Shewchuk. Delaunay refinement algorithms for triangular mesh generation. Comput. Geom. Theory Appl., 2002.

[3] Miller, Pav, and Walkington. When and why Delaunay refinement algorithms work. Int. J. Comp. Geom. Appl., 2005.

[4] Chew. Guaranteed-quality mesh generation for curved surfaces. Symp. Comput. Geom., 1993.

[5] Rand, Where and How Chew's Second Delaunay Refinement Algorithm Works, Canad. Conf. Comp. Geom., 2011.

Background: Why Triangulate?

- Complicated object -> collection of simple objects

- Interpolation
  - Graphics rendering

- Finite element analysis
  - Convergence and accuracy
  - Condition number of the linear system

Motivation

- Reduce mesh size (number of points)
- Retain angle bounds
- Preserve all features

Proposed Method: Sifting

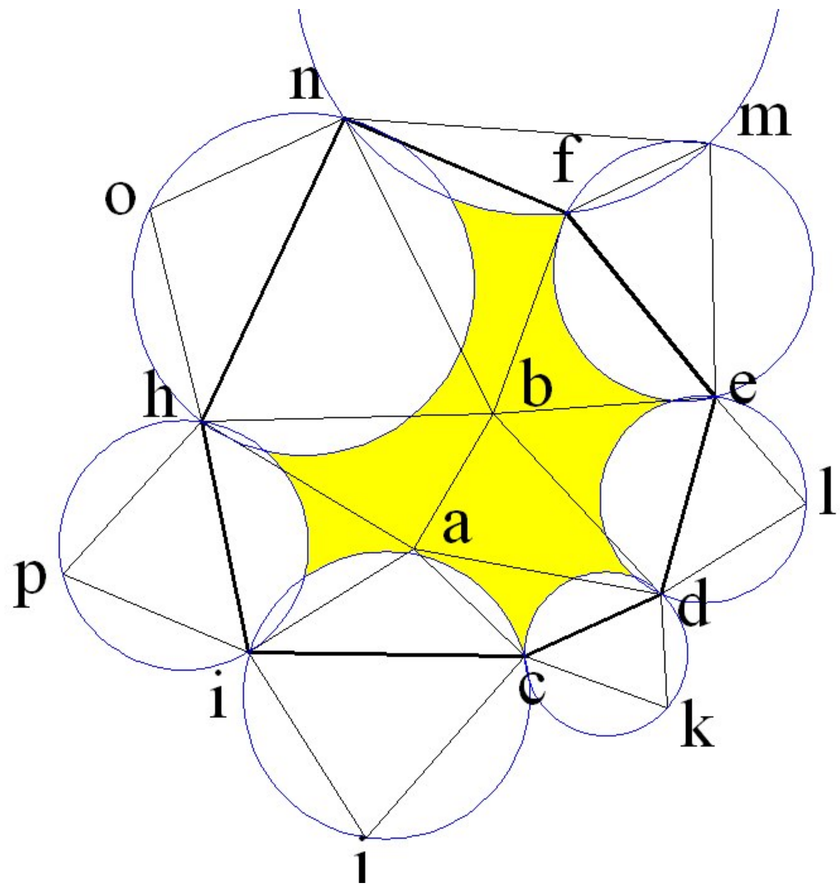
1. Replace an edge (2 points) with 1 point

2. Constrain the region of valid replacement points

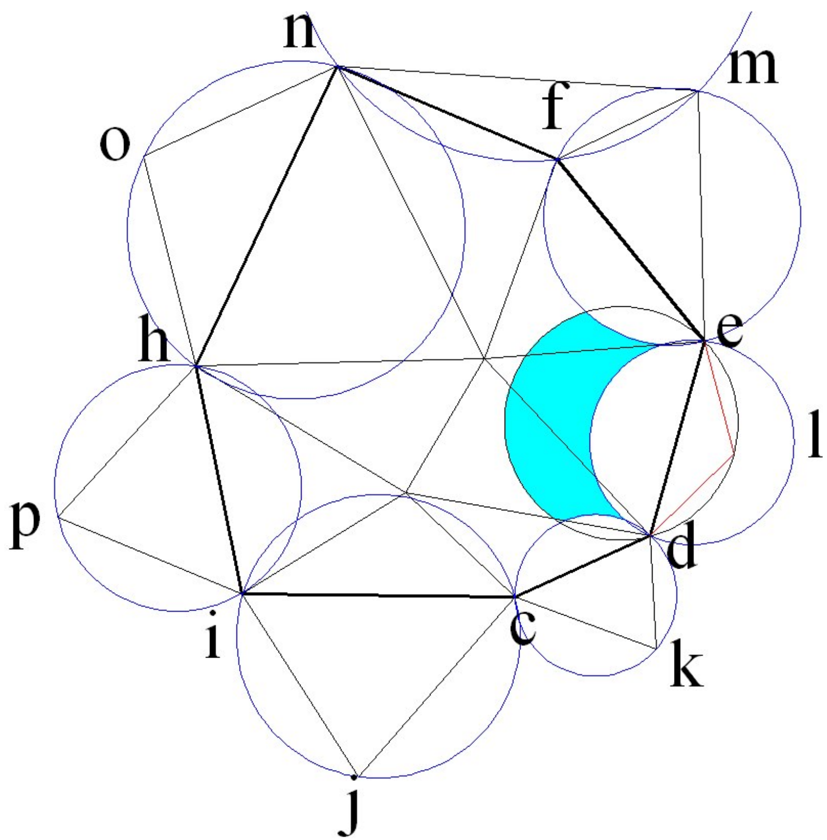
3. Sample uniformly from this region

4. Repeat until no more sifting is possible

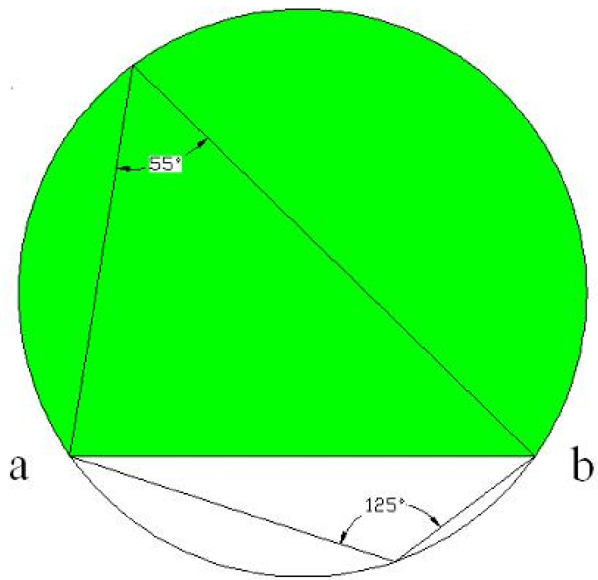
Constraint #1: Neighboring Circumcircles

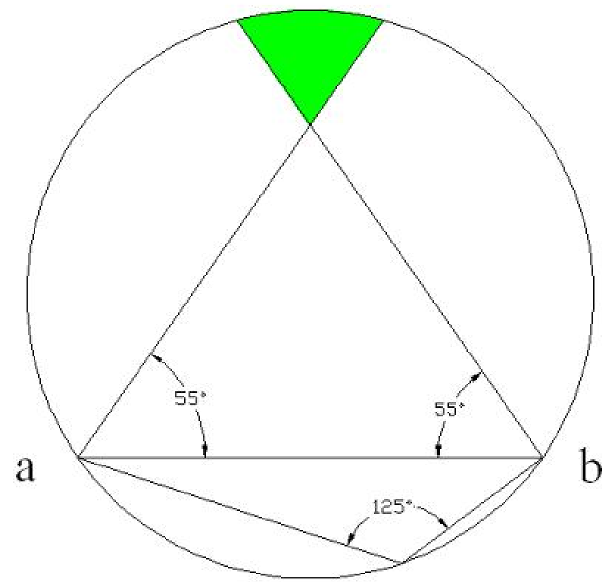


Constraint #2: No Thin Triangles

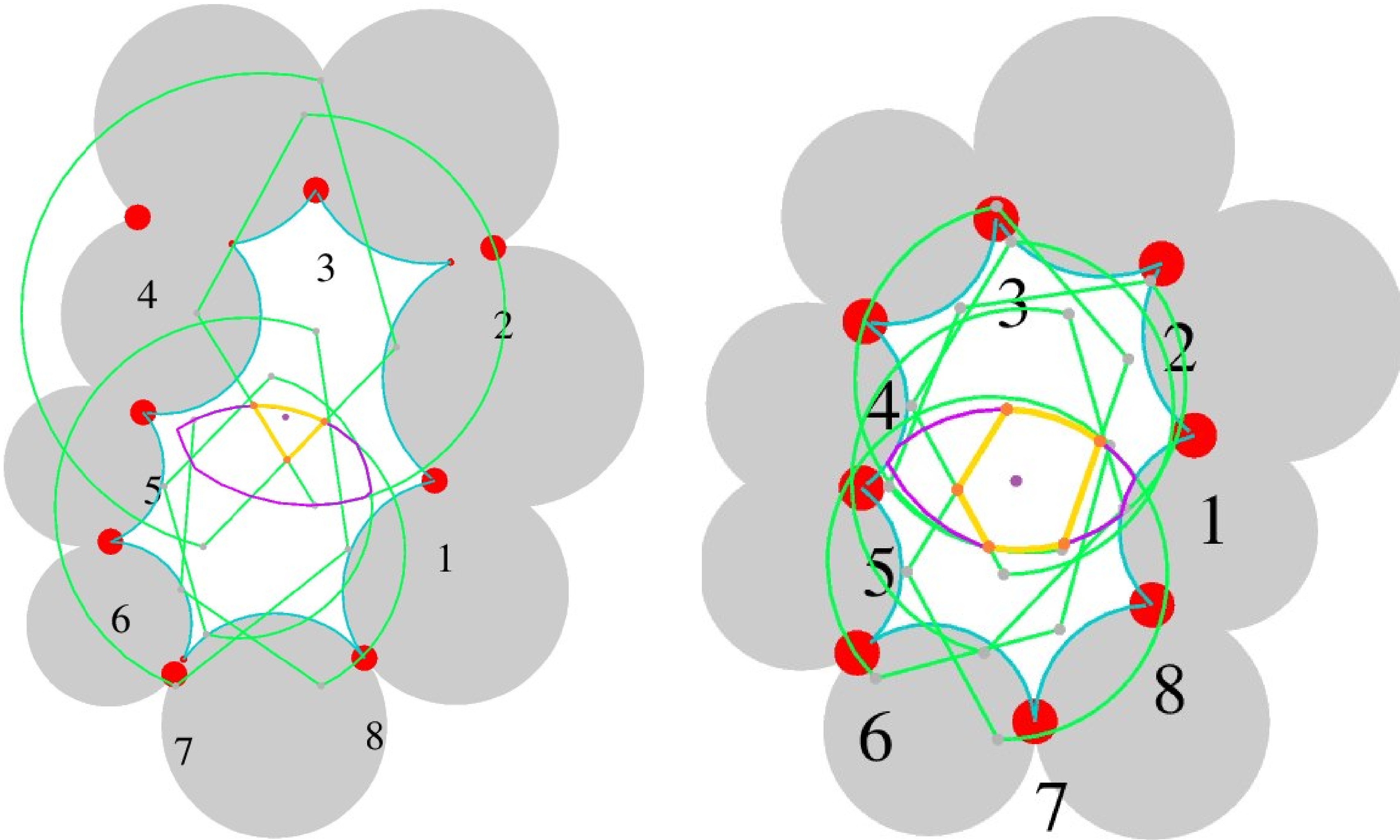


Constraint #2: No Thin Triangles (Cont.)

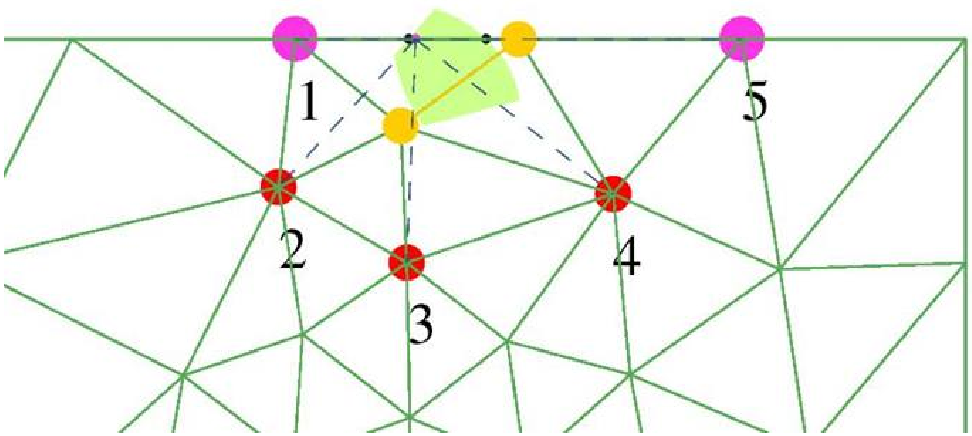


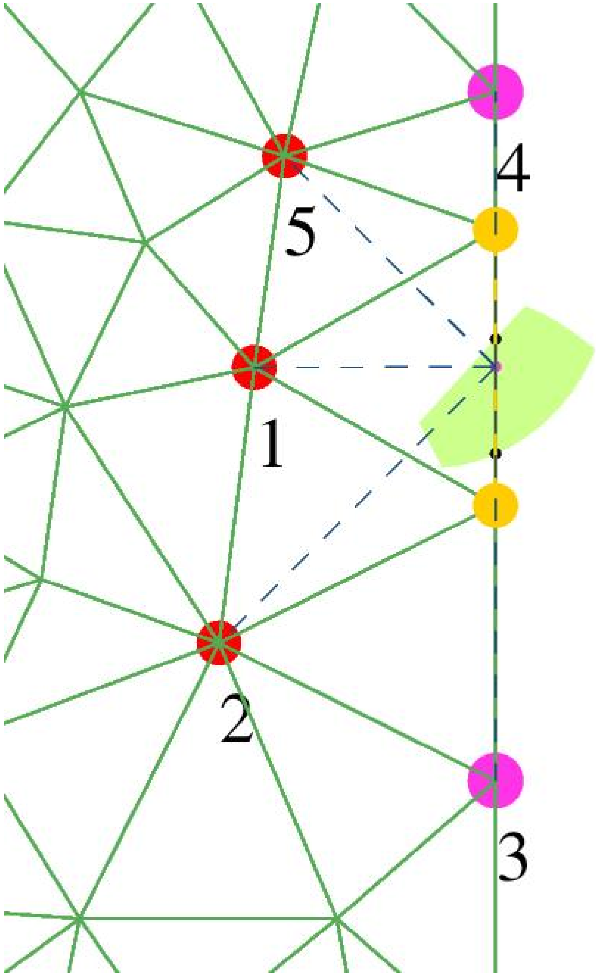


Examples



Constraint #3: Boundary Segments



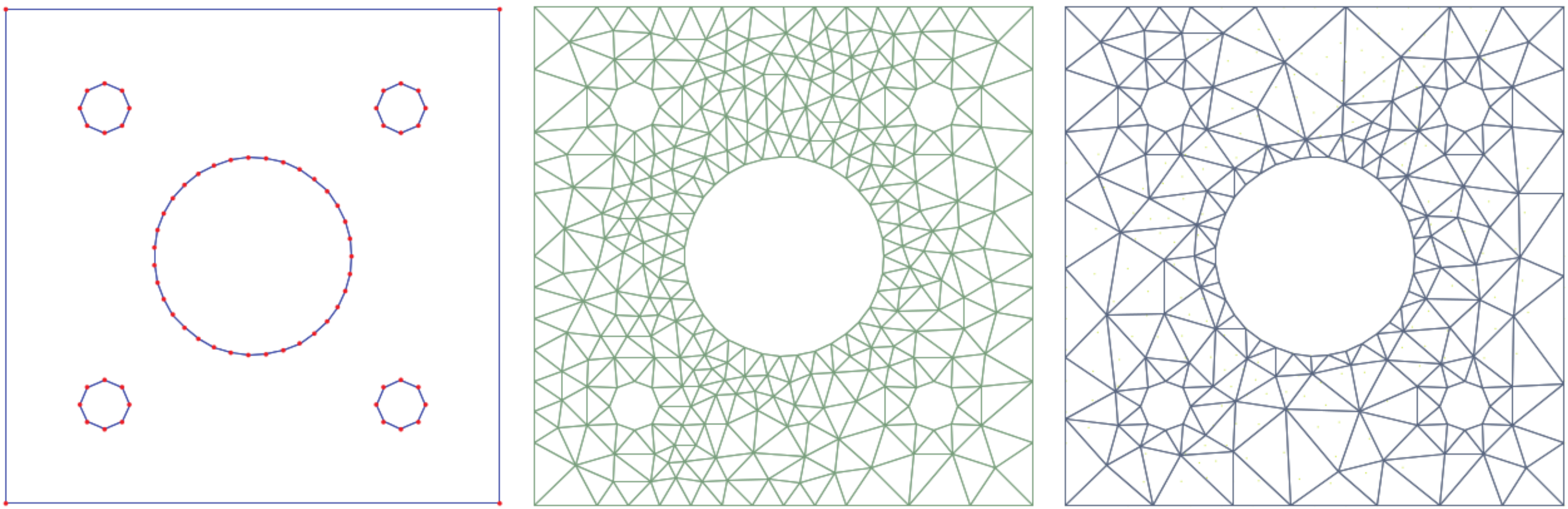


Quantitative Results

./triangle -q		20		30		35	
B5	71	139	17%	197	29%	326	43%
Spiky	229	330	54%	505	59%	715	56%
Dolphin	260	471	31%	865	49%	3409	78%

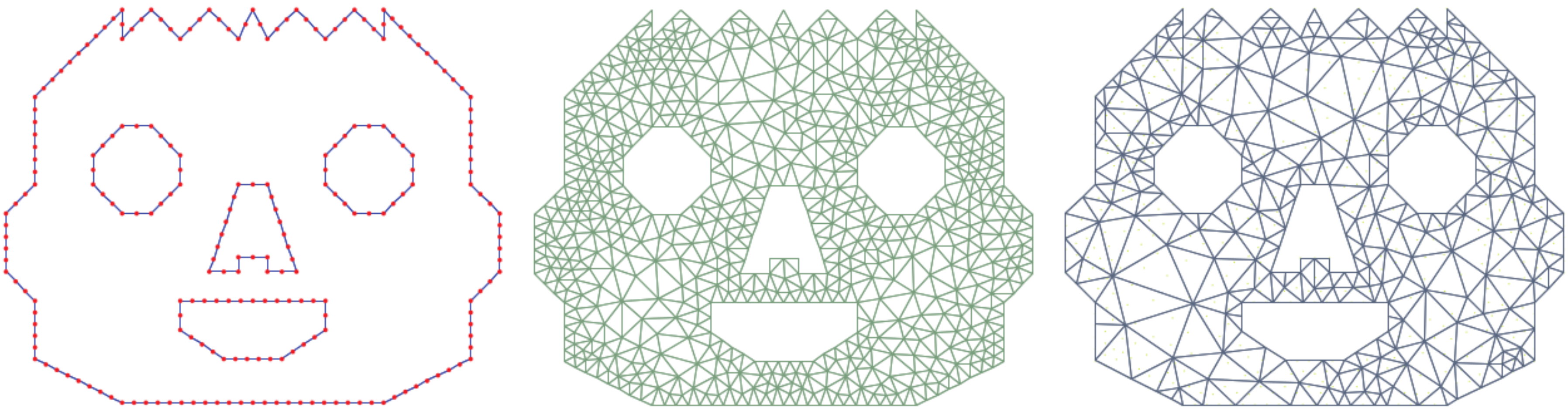
model_name | #input points | (#Triangle's output size, reduction ratio)

Sensational Results: B5 Model



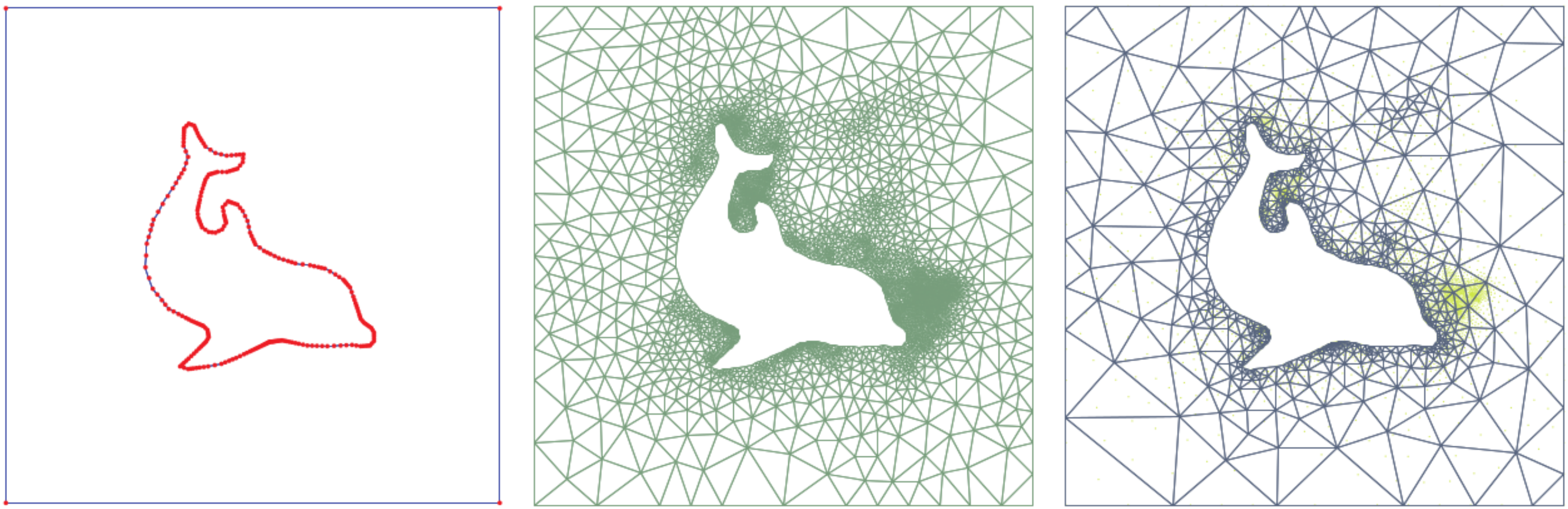
(a) B5 (71) (b) Triangle (326) (c) Sifted (43%)

Sensational Results: Spiky Model



(d) Spiky (229) (e) Triangle (715) (f) Sifted (56%)

Sensational Results: Dolphin Model



(g) Dolphin (260)   (h) Triangle (3409)   (i) Sifted (78%)

Discussion

- Sifting order
  - Edges chosen at random
- Random Sampling
  - Replacement points chosen at random (from region)
- Runtime
  - Expected number of “sifting attempts” per edge
- Quantify improvement?

Questions?

Thank You!

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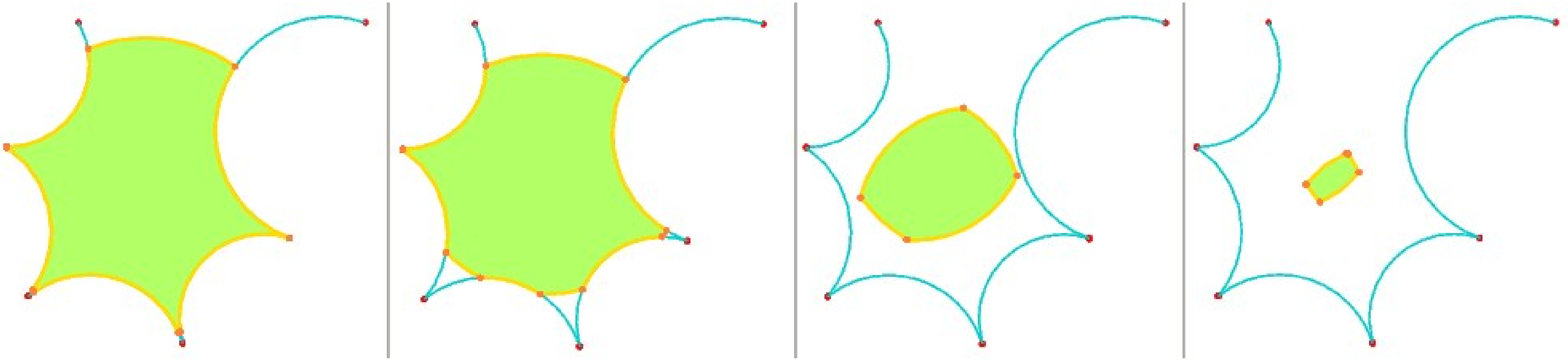
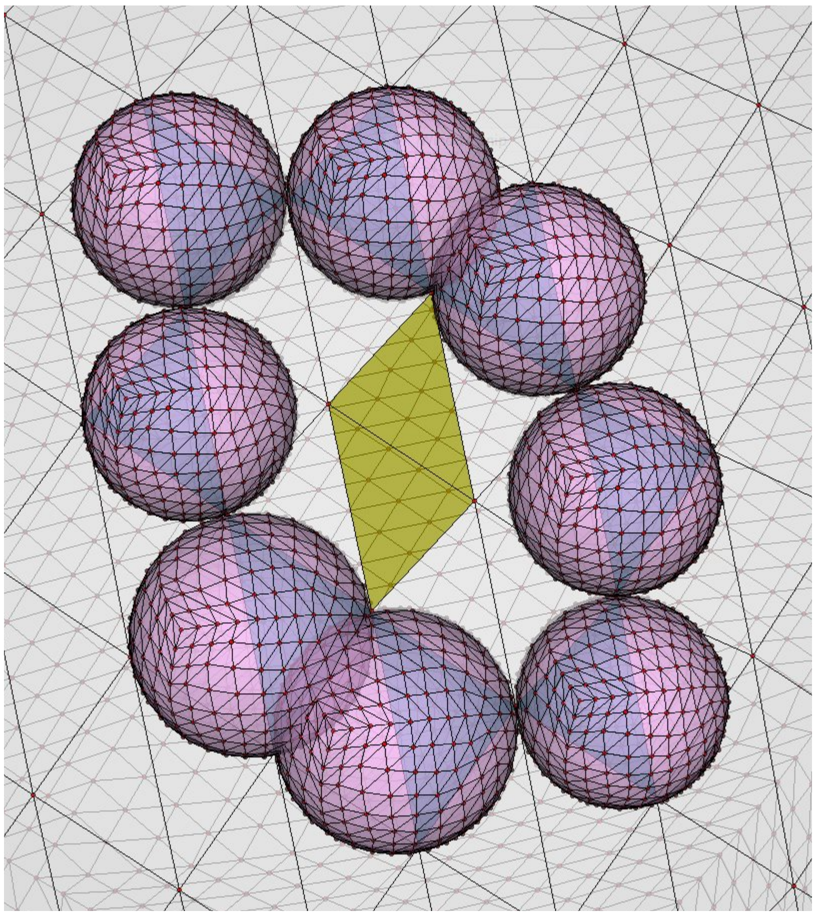
Extra Slide #1: Adaptive Local Max.

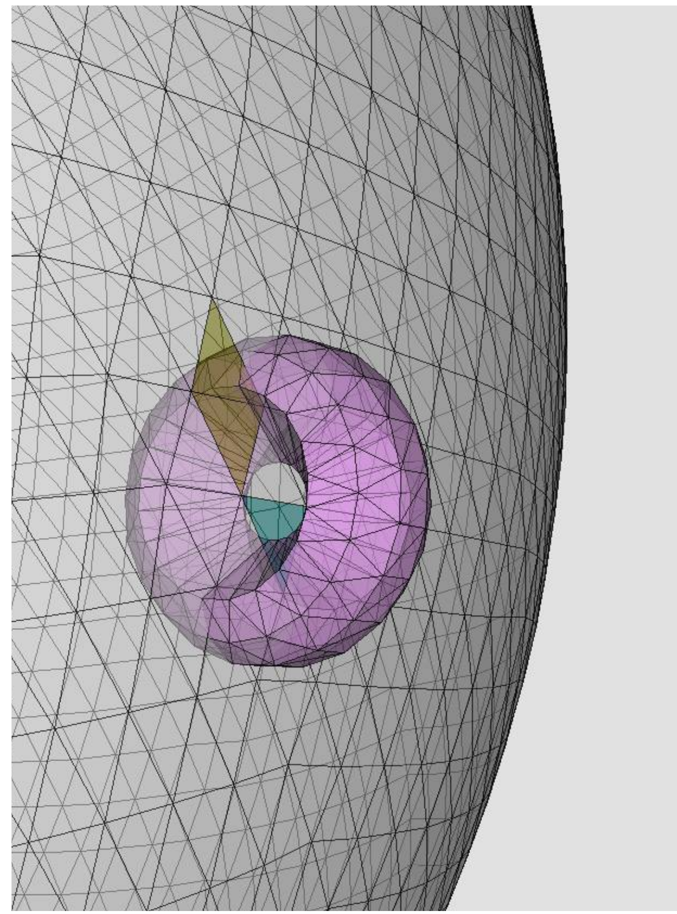


Varying the angle bound used.
Left to right: 20, 30, 40, 50.

Extra Slide #2: Surface Teaser



~1: Neighboring Circumspheres



~2: No Thin Triangular Faces

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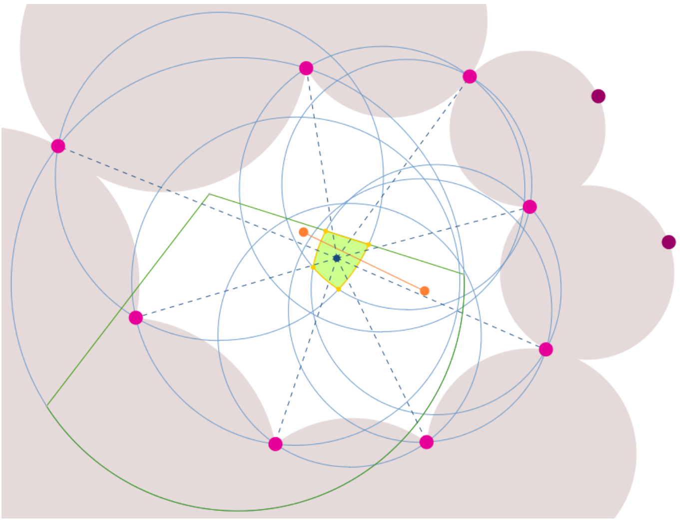
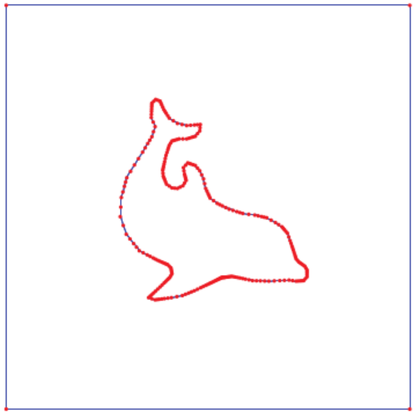
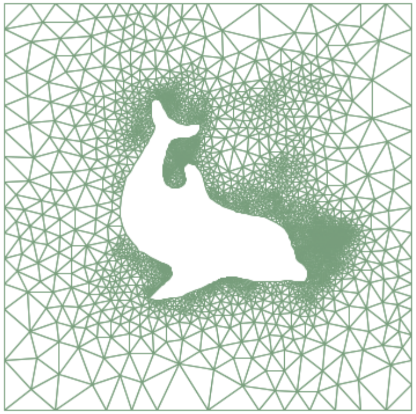
- Goal: reduce the number of points while retaining the angle bounds.
- Local update strategy (Sifting)
  - Remove 2 points
  - Constrain sampling region
    - Neighbor Circumcircles
    - Angle bounds
  - Pick a replacement point
- Example: 78% reduction

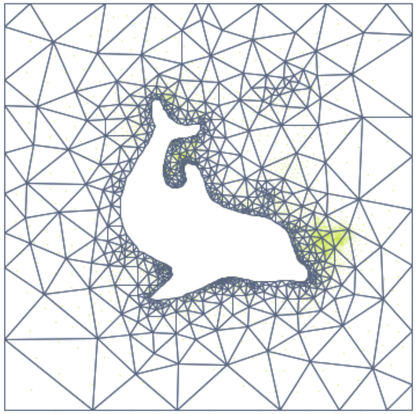




Input Model



Triangle Mesh



Sifted Mesh