Homology Localization by Hierarchical Blowups

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- Hierarchical Cover
  \[ C_h \rightarrow \cdots \rightarrow C_1 = C \]

- Mayer-Vietoris Blowup
  \[ \mathcal{K}^C = \{(\sigma, s) \mid \sigma \in \mathcal{K}, s \subseteq C, \sigma \in \cap s\} \]

- Hierarchical Blowup
  \[ \mathcal{K}^{\mathcal{H}_1} = \mathcal{K}^{C_1} \]
  \[ \mathcal{K}^{\mathcal{H}_{i+1}} = (\mathcal{K}^{\mathcal{H}_i})^{\mathcal{C}_{i+1}} \]

- Range Queries?

\[ H(K_1) \rightarrow \cdots \rightarrow H(K_i) \rightarrow \cdots \rightarrow H(K_n) \]
\[ \uparrow \pi^*_1,1 \quad \uparrow \pi^*_1,i \quad \uparrow \pi^*_1,n \]

\[ H(K_1^{\mathcal{H}_1}) \rightarrow \cdots \rightarrow H(K_i^{\mathcal{H}_1}) \rightarrow \cdots \rightarrow H(K_n^{\mathcal{H}_1}) \]
\[ \uparrow \pi^*_2,1 \quad \uparrow \pi^*_2,i \quad \uparrow \pi^*_2,n \]

\[ H(K_1^{\mathcal{H}_2}) \rightarrow \cdots \rightarrow H(K_i^{\mathcal{H}_2}) \rightarrow \cdots \rightarrow H(K_n^{\mathcal{H}_2}) \]