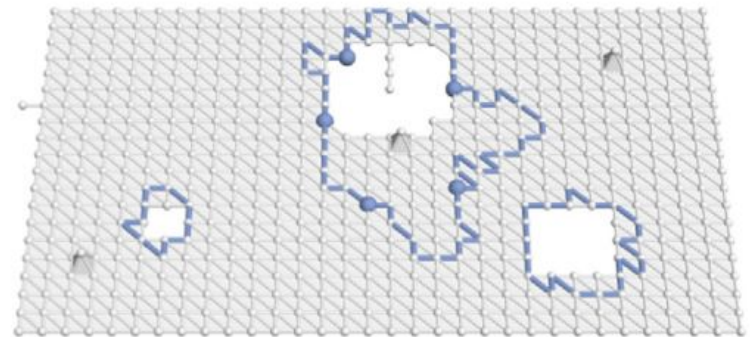
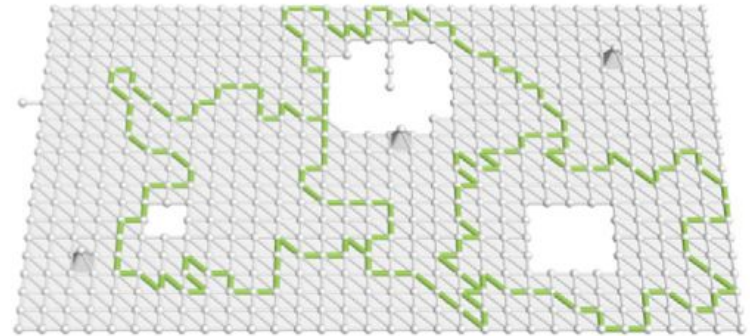
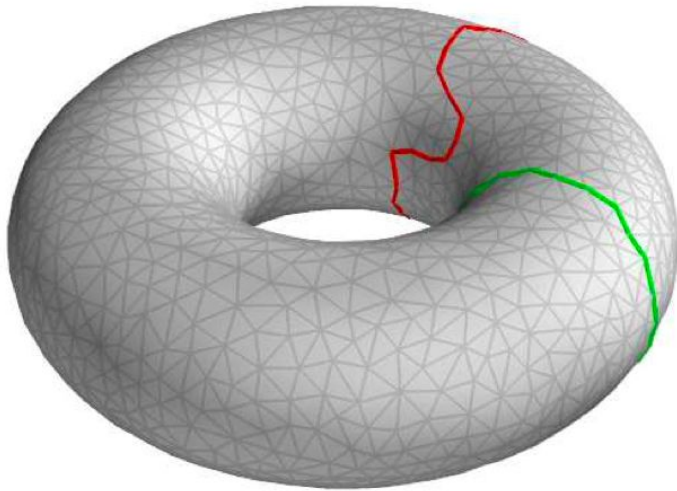


Homology Localization by Hierarchical Blowups

Ahmed Abdelkader

Department of Computer Science
University of Maryland, College Park

Homology Localization?



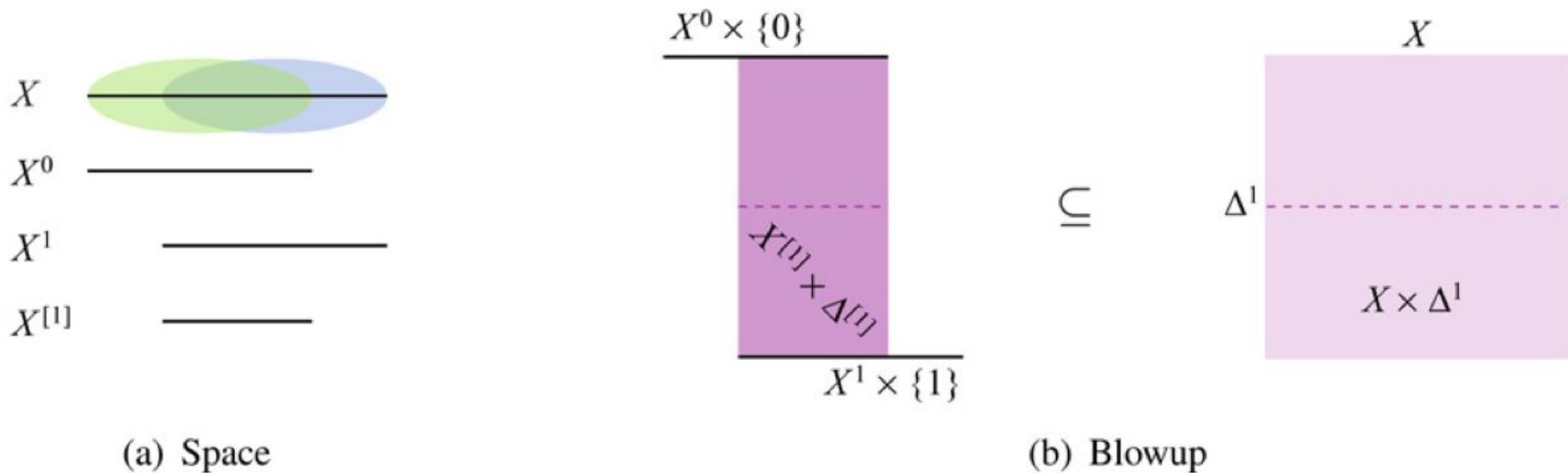
- Optimal Hom. Cycles

[Dey, Hirani, Krishnamoorthy, SIAMJ'11]

- By a cover

[Zomorodian, Carlsson, CGTA'08]

Mayer-Vietoris Blowup

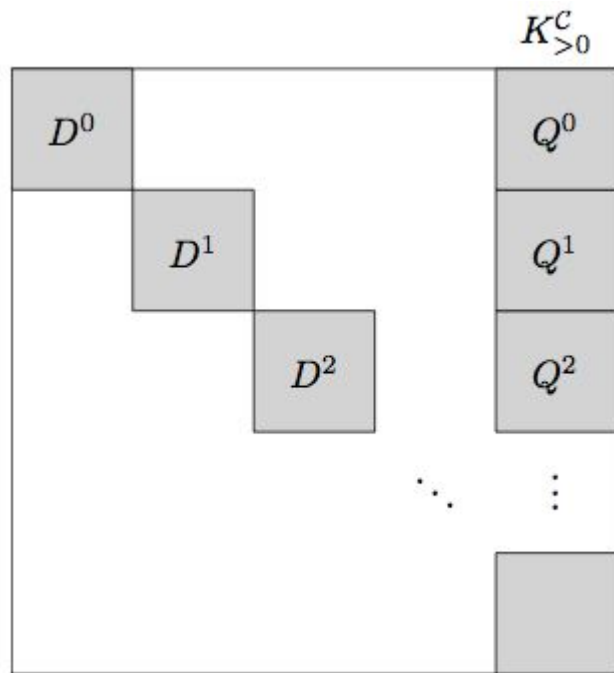


[Zomorodian, Carlsson, CGTA'08]

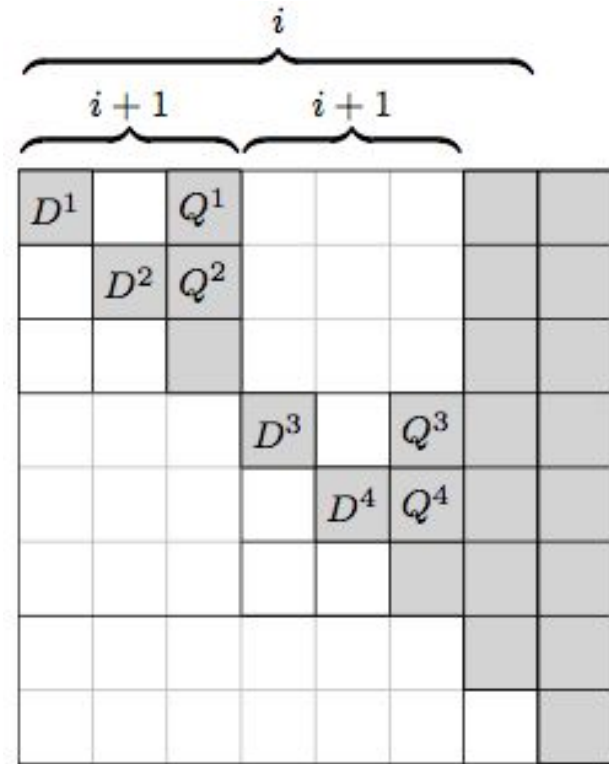
$$\mathcal{K}^{\mathcal{C}} = \{(\sigma, s) \mid \sigma \in \mathcal{K}, s \subseteq \mathcal{C}, \sigma \in \cap s\}$$

$$\pi : \mathcal{K}^{\mathcal{C}} \rightarrow \mathcal{K}$$

Decomposition for Parallel Computation



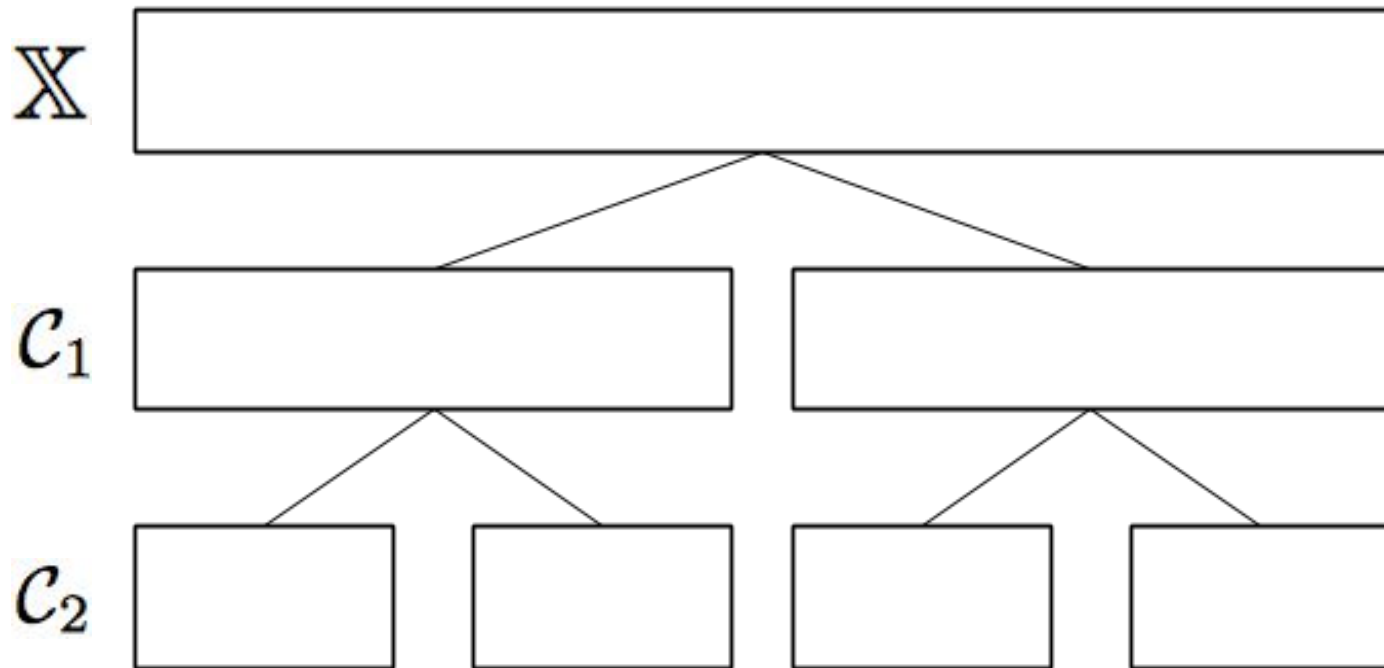
Boundary matrix of
a blowup complex



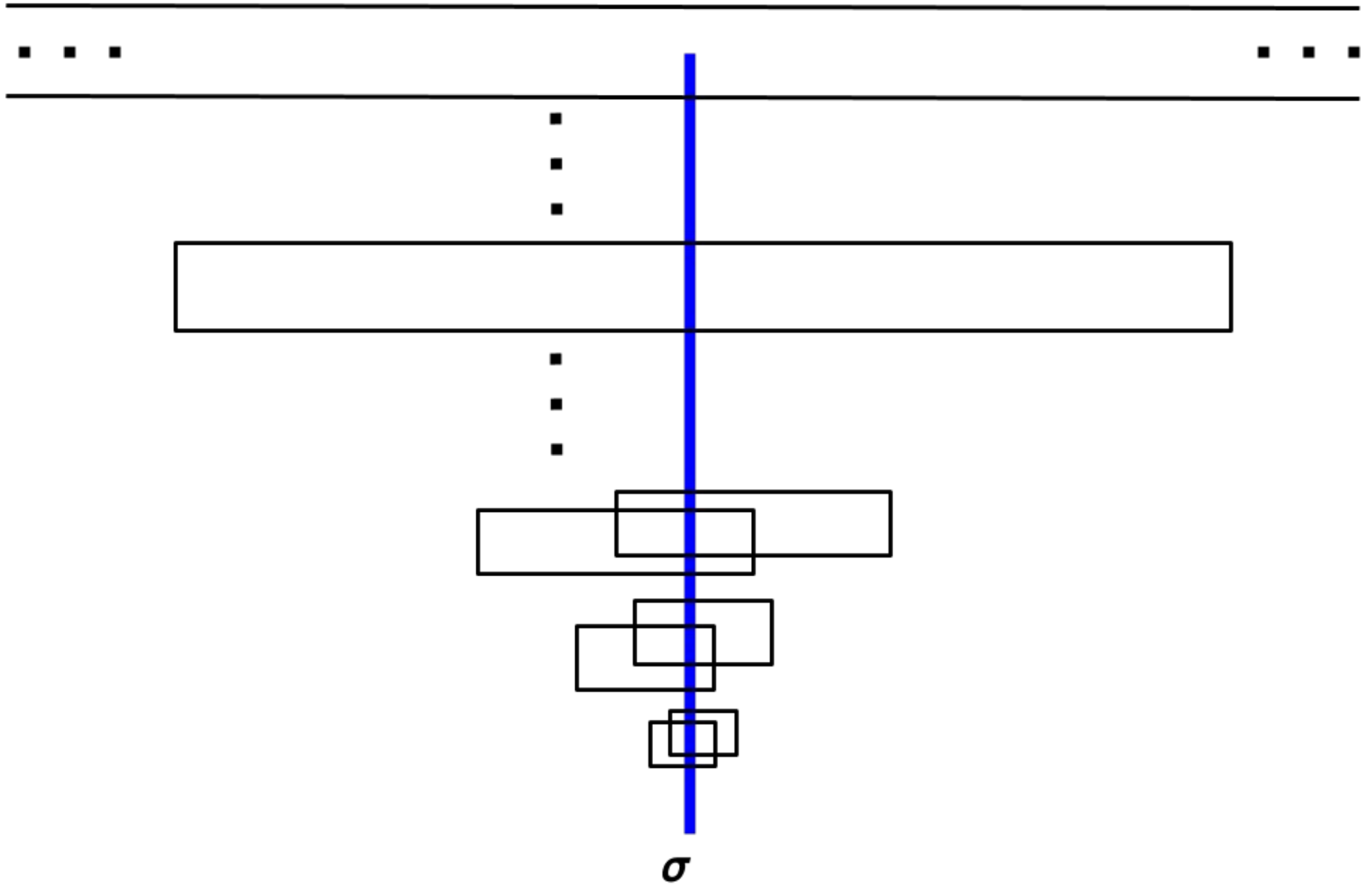
.. for 3 consecutive levels

Hierarchical Covers

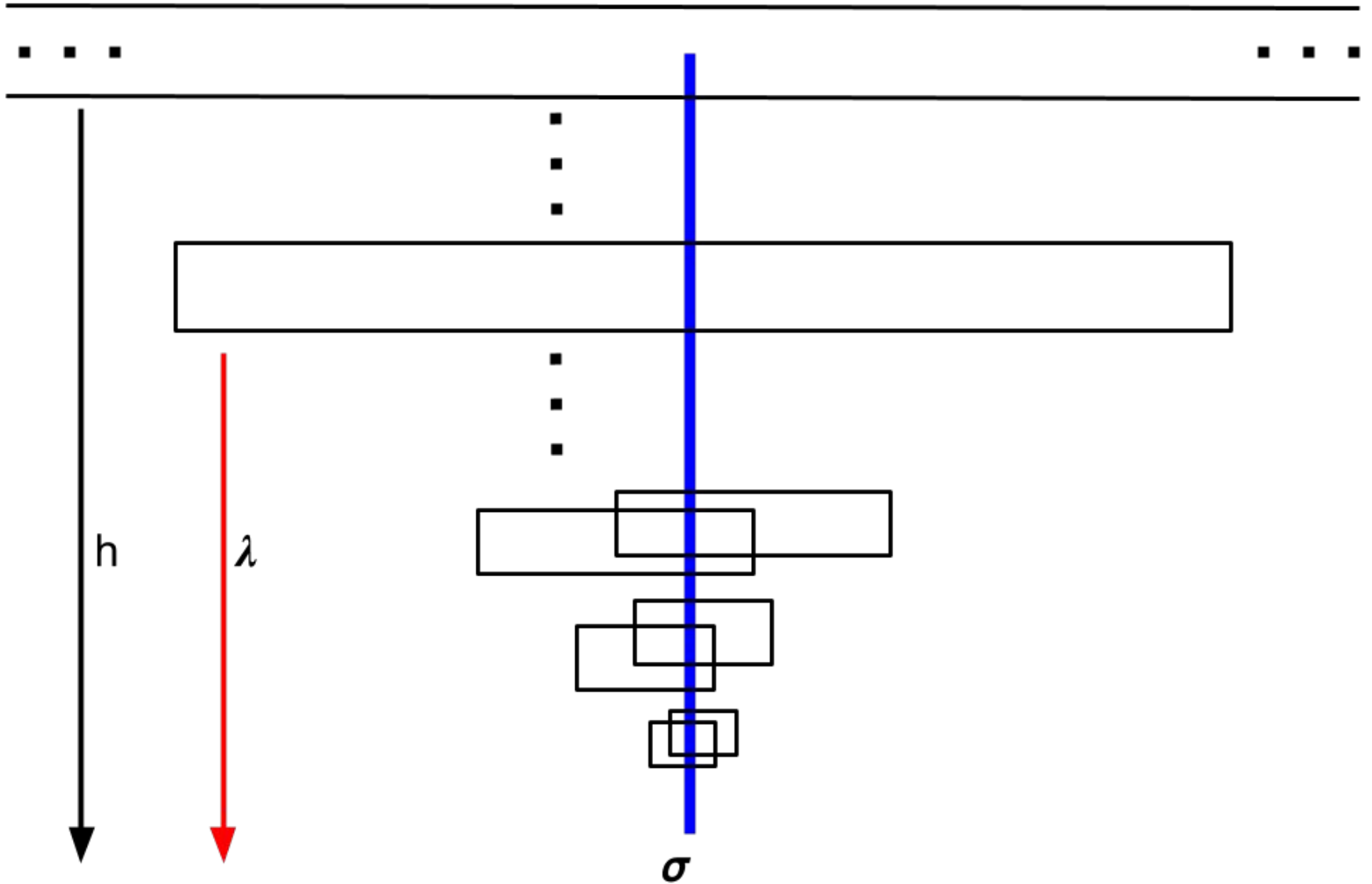
(e.g., quadtrees, net-trees, recursive separators, ...)



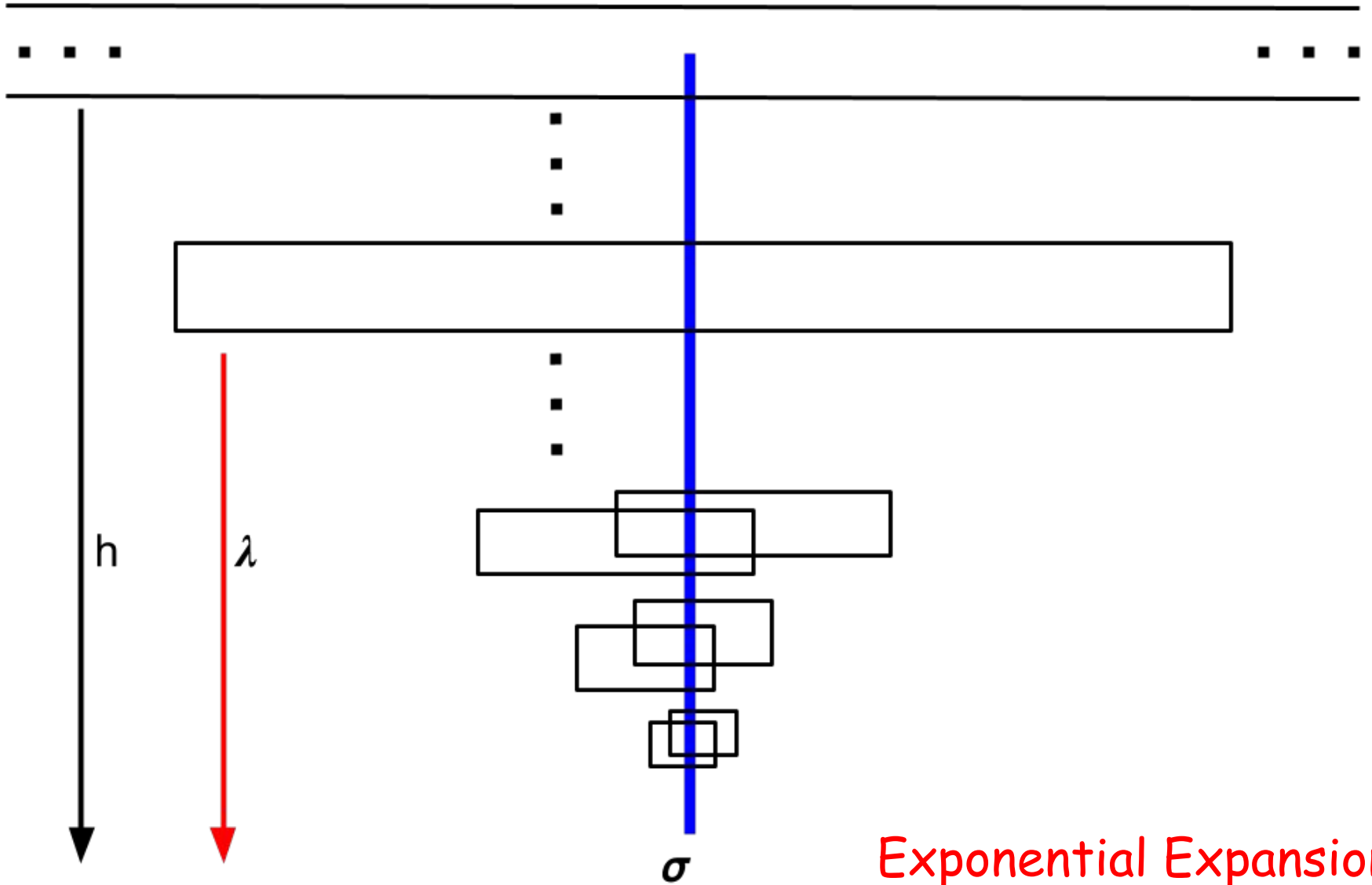
Overlaps



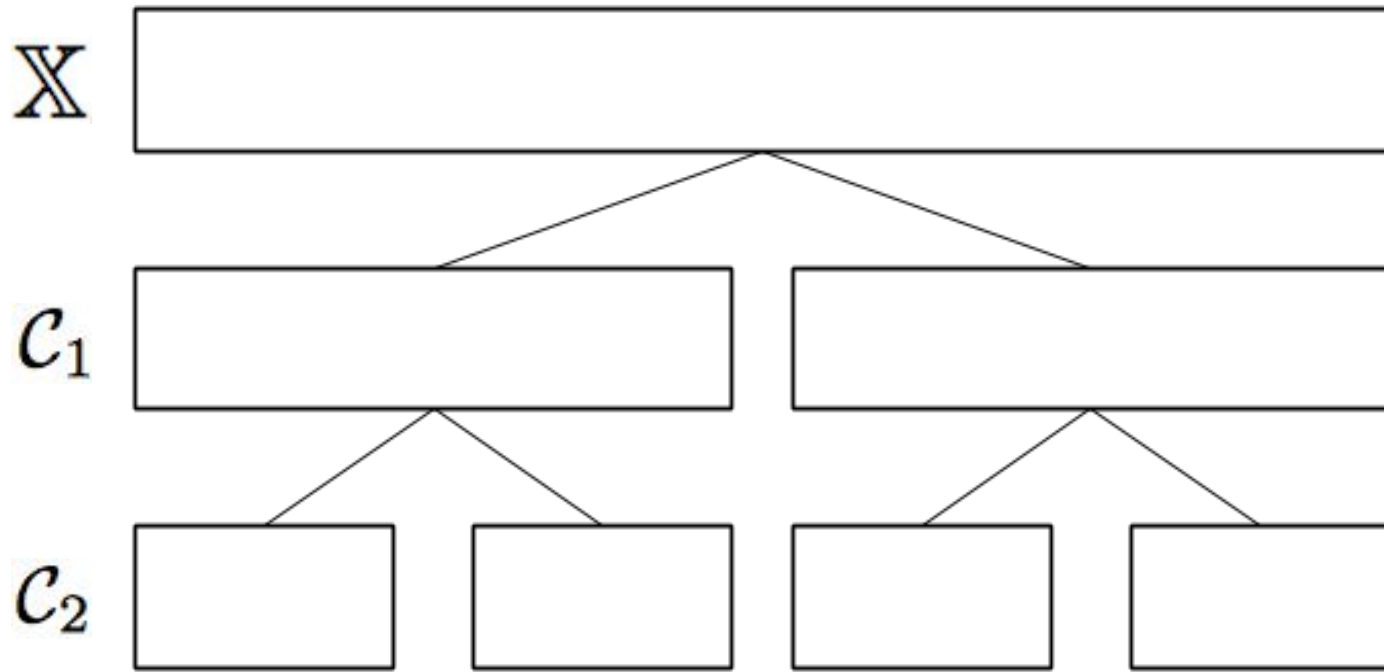
Overlaps



Overlaps



Alternate Definition



$$\mathcal{K}^{\mathcal{H}_1} = \mathcal{K}^{C_1}$$

$$\mathcal{K}^{\mathcal{H}_{i+1}} = (\mathcal{K}^{\mathcal{H}_i})^{\hat{C}_{i+1}}$$

Lifted Covers

$$\hat{c}_\alpha^{i+1} = \{(\sigma, J_1, \dots, J_i) \in \mathcal{K}^{\mathcal{H}_i} \mid \sigma \in c_\alpha^{i+1} \wedge p_i(\alpha) \in J_i\}$$

Putting it all together

$$\hat{c}_\alpha^{i+1} = \{(\sigma, J_1, \dots, J_i) \in \mathcal{K}^{\mathcal{H}_i} \mid \sigma \in c_\alpha^{i+1} \wedge p_i(\alpha) \in J_i\}$$

$$\mathcal{K}^{\mathcal{H}_1} = \mathcal{K}^{\mathcal{C}_1}$$

$$\mathcal{K}^{\mathcal{H}_{i+1}} = (\mathcal{K}^{\mathcal{H}_i})^{\hat{c}_{i+1}}$$

Putting it all together

$$\hat{c}_\alpha^{i+1} = \{(\sigma, J_1, \dots, J_i) \in \mathcal{K}^{\mathcal{H}_i} \mid \sigma \in c_\alpha^{i+1} \wedge p_i(\alpha) \in J_i\}$$

$$\mathcal{K}^{\mathcal{H}_1} = \mathcal{K}^{\mathcal{C}_1}$$

$$\mathcal{K}^{\mathcal{H}_{i+1}} = (\mathcal{K}^{\mathcal{H}_i})^{\hat{c}_{i+1}}$$

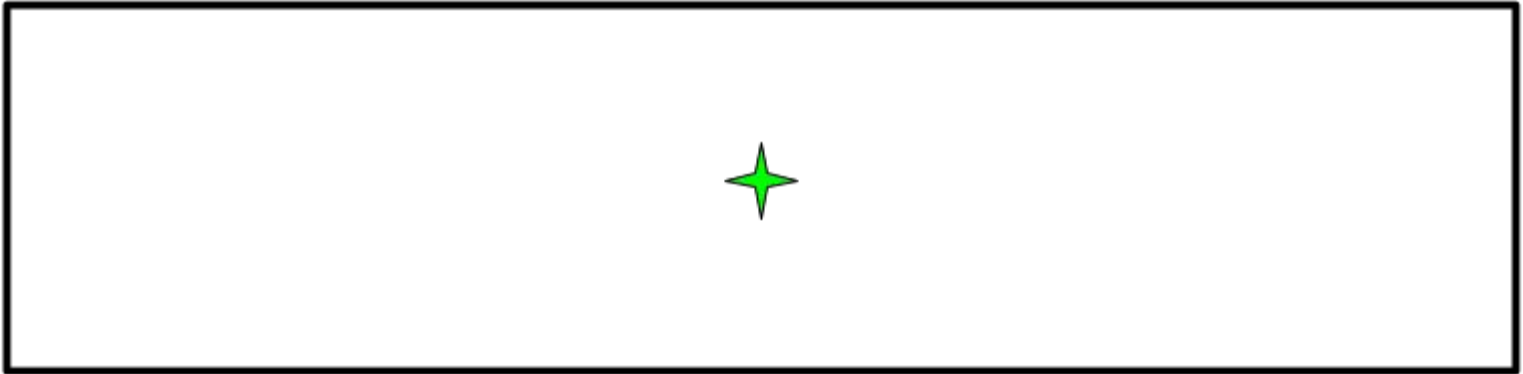
$$\pi_i : \mathcal{K}^{\mathcal{H}_{i+1}} \rightarrow \mathcal{K}^{\mathcal{H}_i}$$

$$\pi : \mathcal{K}^{\mathcal{C}} \rightarrow \mathcal{K}$$

Putting it all together

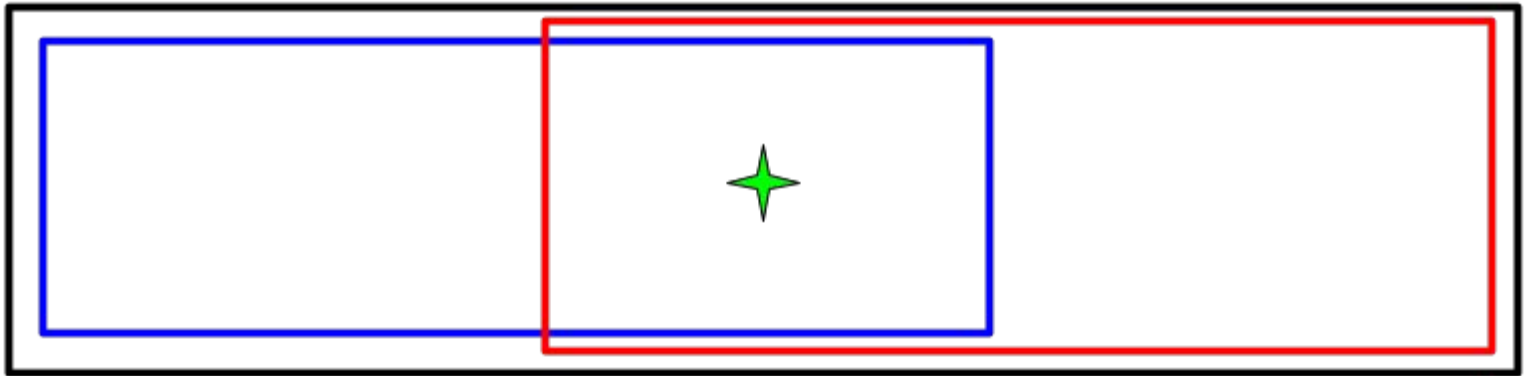
$$\begin{array}{ccccccc} H(K_1) & \longrightarrow & \dots & \longrightarrow & H(K_i) & \longrightarrow & \dots & \longrightarrow & H(K_n) \\ \uparrow \pi_{1,1}^* & & & & \uparrow \pi_{1,i}^* & & & & \uparrow \pi_{1,n}^* \\ H(K_1^{\mathcal{H}_1}) & \longrightarrow & \dots & \longrightarrow & H(K_i^{\mathcal{H}_1}) & \longrightarrow & \dots & \longrightarrow & H(K_n^{\mathcal{H}_1}) \\ \uparrow \pi_{2,1}^* & & & & \uparrow \pi_{2,i}^* & & & & \uparrow \pi_{2,n}^* \\ H(K_1^{\mathcal{H}_2}) & \longrightarrow & \dots & \longrightarrow & H(K_i^{\mathcal{H}_2}) & \longrightarrow & \dots & \longrightarrow & H(K_n^{\mathcal{H}_2}) \end{array}$$

Overlaps?



σ

Overlaps?

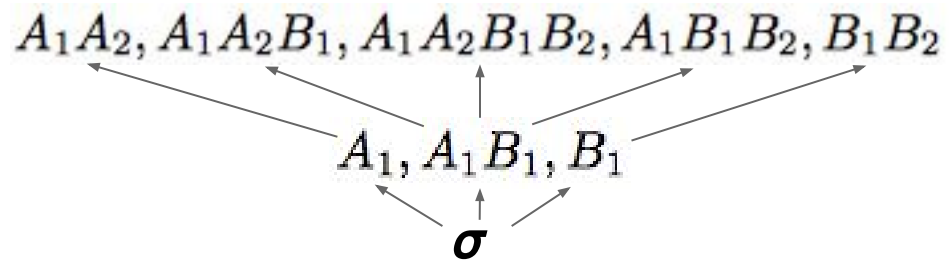
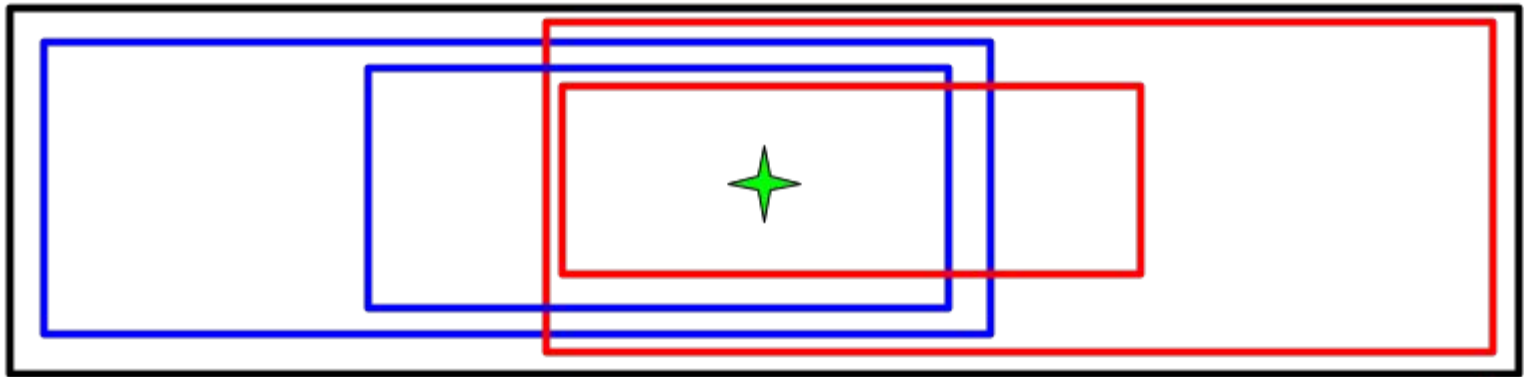


$$A_1, A_1 B_1, B_1$$

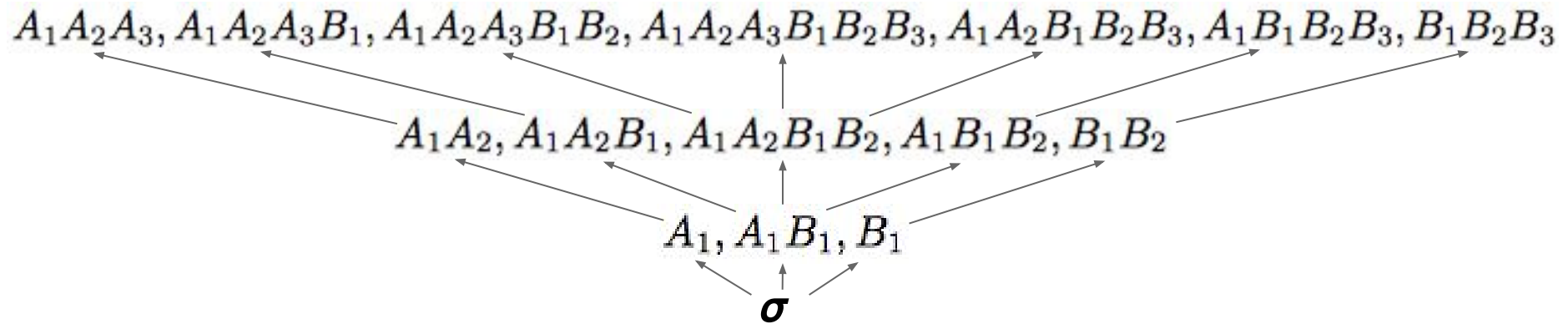
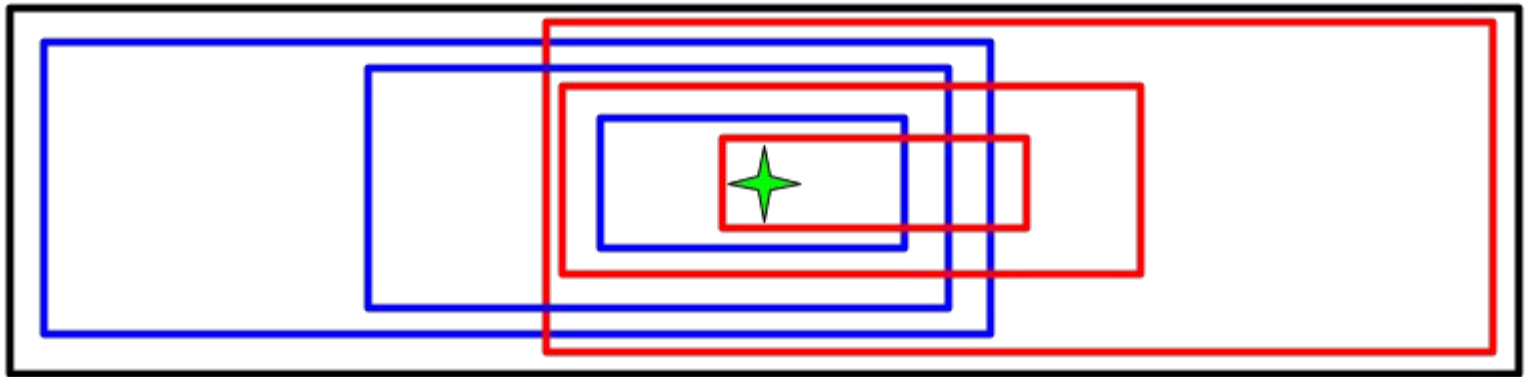
σ

The diagram shows the expression $A_1, A_1 B_1, B_1$ with arrows pointing from a central σ to each of the three terms. The arrow to A_1 points up and to the left, the arrow to $A_1 B_1$ points straight up, and the arrow to B_1 points up and to the right.

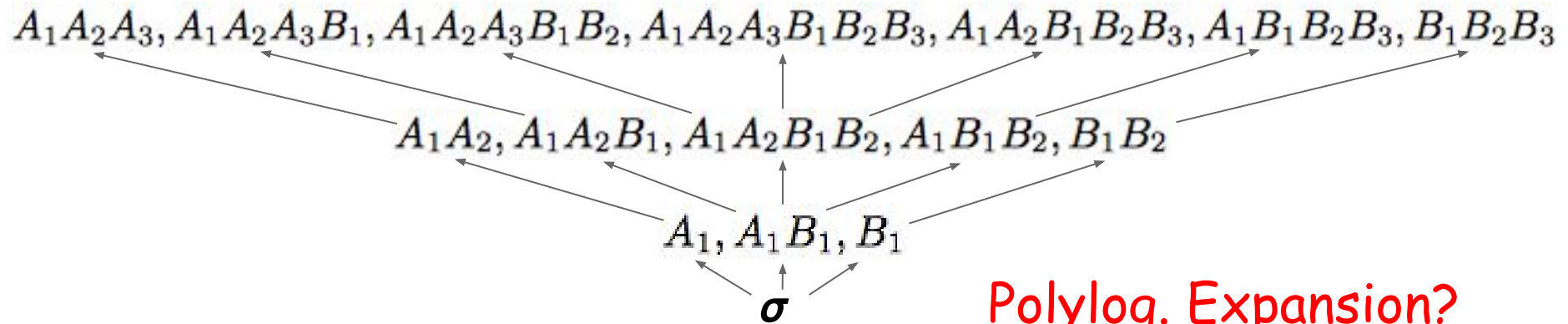
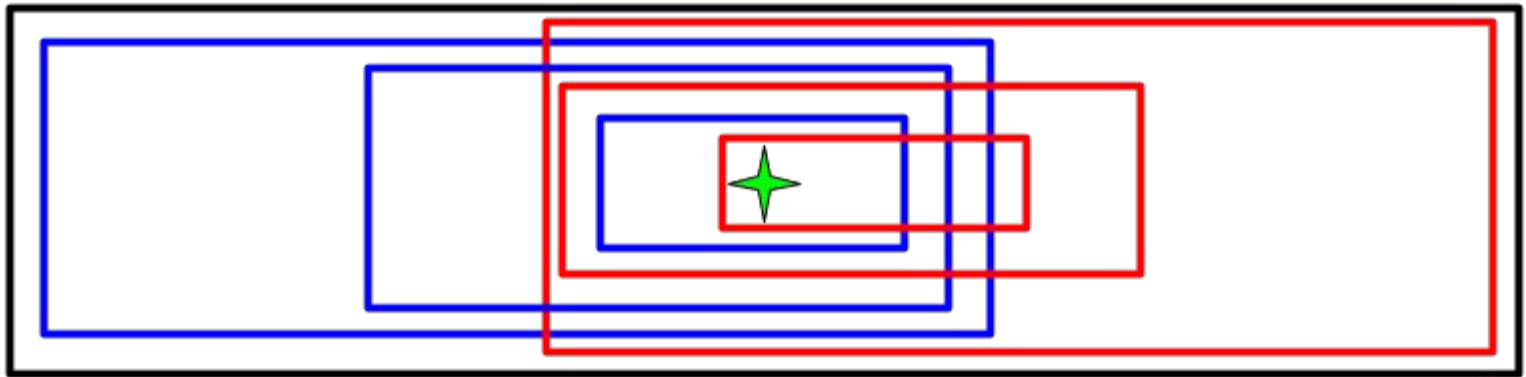
Overlaps?



Overlaps?



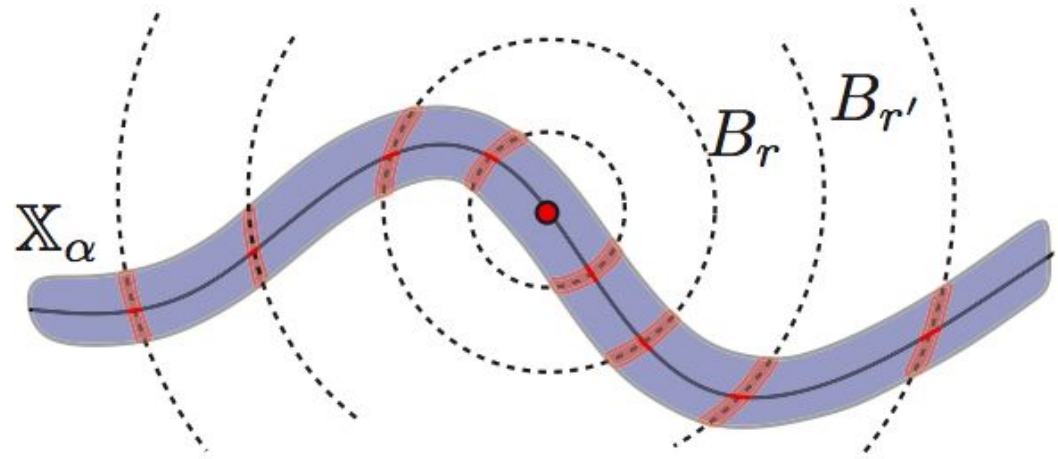
Overlaps?



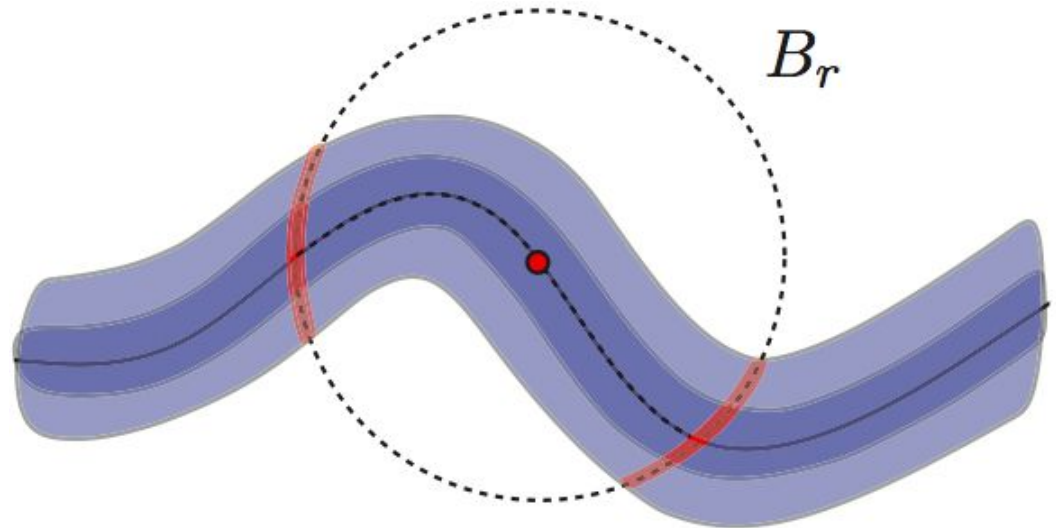
Polylog. Expansion?

Topological Range Queries?

r -filtrations



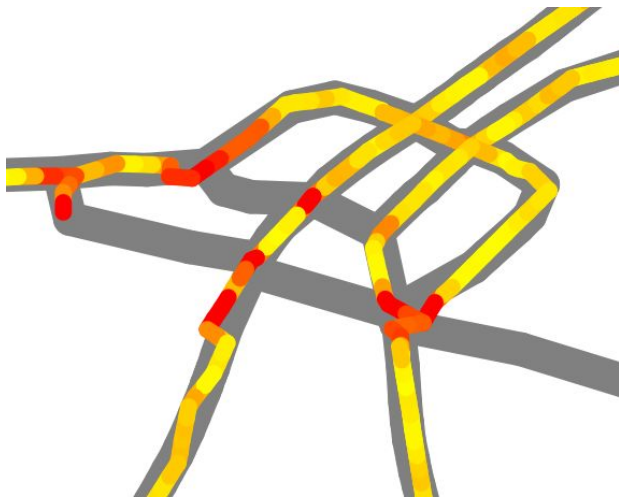
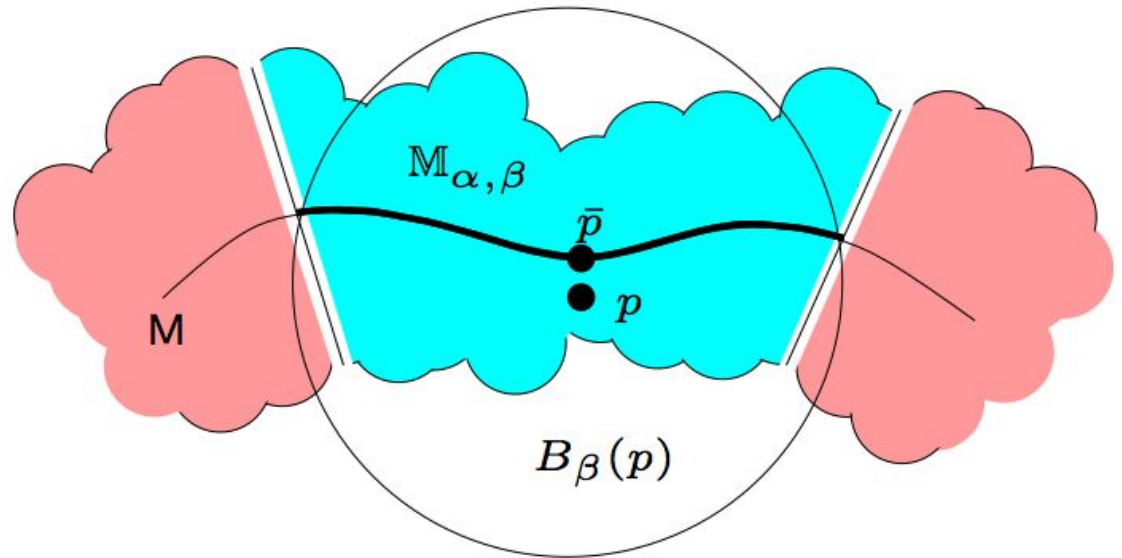
α -filtrations



Topological Range Queries?

Manifold Learning

[Dey, Fan, Wang, CCCG'14]



Map Reconstruction

[M. Ahmed, Fasy, Wenk, SIGSPATIAL'14]

Questions?

akader@cs.umd.edu