

[E-WEB] Mathematics Story (1)

Hilbert's Problems

(1) The Origin of the Coordinates
Hilbert and Göttingen

by Ahmed Abdelkader



David Hilbert (~1885)

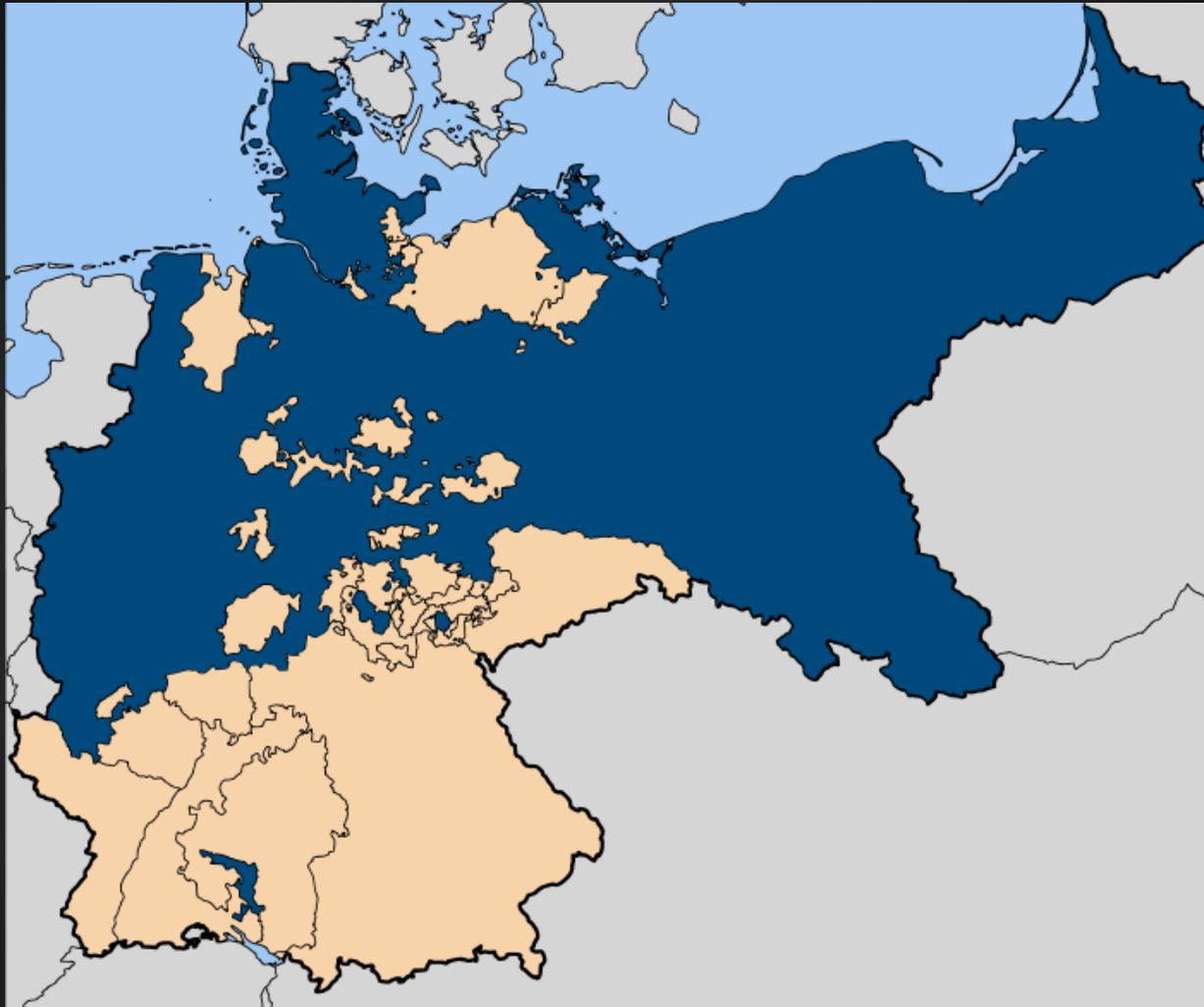
Il est encore plus facile
de juger de l'esprit d'un homme
par ses questions que par ses réponses

(Judge a man by his questions rather than by his answers)

Pierre Marc Gaston de Lévis, Duke of Lévis

1862: Birth

- Königsberg, East Prussia
- Otto and Maria Hilbert
- Father was a judge
- Grandfather too (*Geheimrat*)
- Flourishing high middle-class culture



Prussia (blue), at its peak, the leading state of the German Empire



8320 P. Z - KÖNIGSBERG SCHLOSS OSTSEITE

Koenigsberg Schloss Ostseite (1900)



Königsberg Castle before World War I

Königsberg's Mathematical Tradition

- Friedrich Wilhelm Bessel (1784 - 1846)
- Carl Jacobi (1804 - 1851)
- Friedrich Richolet (1808 - 1875)

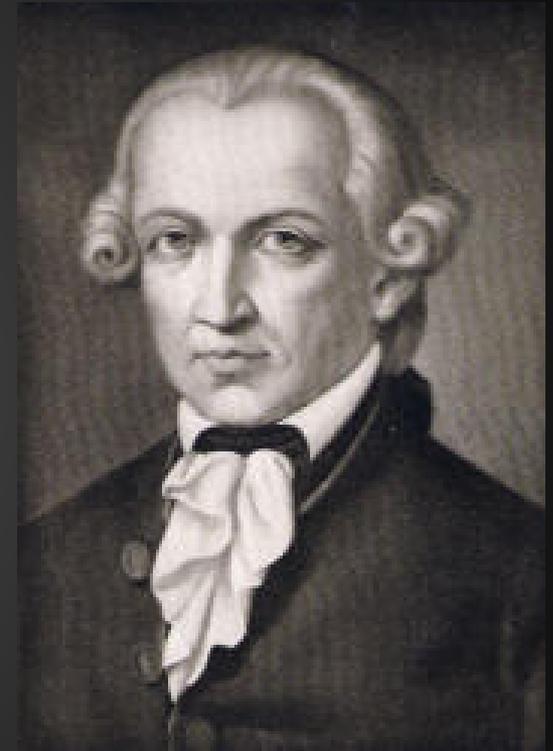
Karl Theodor Wilhelm Weierstrass

- (1815 - 1897)
- Effort to introduce "rigor" into mathematical analysis
- Calculus
 - Isaac Newton (-1727)
 - Gottfried Wilhelm Leibniz (-1716)
 - Intuitive and pragmatic
- Augustin-Louis Cauchy (-1857)
- Rigorously defined "limit" and "continuous"
- Introduced ϵ 's and δ 's



Immanuel Kant

- (1724 - 1804)
- How we know things?
- *Arithmetic and geometry concepts are a priori, not learned*
- Works
 - Critique of Pure Reason (81, 87)
 - Critique of Practical Reason (88)
 - Critique of Judgement (90)
- German idealism
 - Romanticism
 - The Enlightenment





Statue of Immanuel Kant in Kaliningrad (Königsberg), Russia
Replica donated by a German entity in the early 1990's

Intuition no longer enough

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Every assumption had to be reconsidered

Hermann Minkowski

- (1864 – 1909)
- Geometry of numbers
- Minkowski spacetime



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Einstein first viewed Minkowski's treatment as a mere mathematical trick, before eventually realizing that a geometrical view of space-time would be necessary in order to complete his own later work in general relativity

Hermann Minkowski (**Best Friend**)

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 - Algebra, number theory, and analysis
 - Riemann surface (\sim Dedekind)



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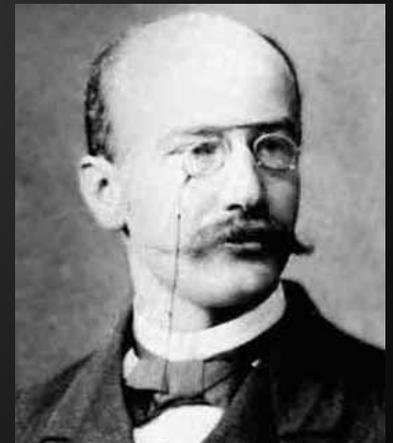
- Ferdinand von Lindemann
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 - First to prove π is transcendental
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Neither inspired Hilbert

Lazarus Immanuel Fuchs (Heidelberg)

- (1833 – 1902)
- Linear differential equations
- Fuchs's theorem

$$p(x)y'' + q(x)y' + r(x)y = 0$$

$$y = \sum_{n=0}^{\infty} a_n(x - x_0)^{n+\sigma}, \quad a_0 \neq 0$$



- Ferdinand Georg Frobenius (1849 - 1917)

Adolf Hurwitz (Königsberg - 1884)

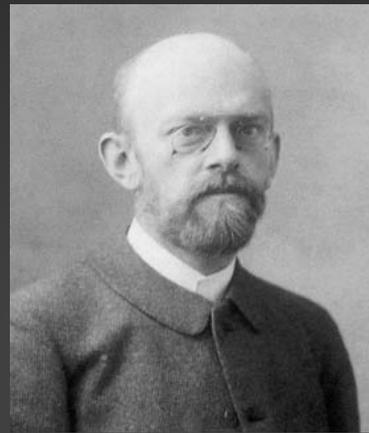
- (1859 - 1919)
- Taught by Hermann Schubert
- Riemann surface theory
- Algebraic curves
- Number theory



Hilbert, Minkowski and Hurwitz

Met daily at "precisely five"
to walk "to the apple tree"
and discuss mathematics

Friendship for Life



"On unending walks we engrossed ourselves in the actual problems of the mathematics of the time; exchanged our newly acquired understandings, our thoughts and scientific plans; and formed a friendship for life."

Breadth of Career

- Wrote a thesis of some originality on algebraic invariants
- Chose to defend two topics on his promotion
 - Electromagnetism
 - "The objections to Kant's theory of the a priori nature of arithmetical judgements are unfounded"

Christian Felix Klein (Leipzig)

- (1849 – 1925)
- Group theory
- Complex analysis
- Non-Euclidean geometry
- Chair at U. of Göttingen
 - re-establish Göttingen as the world's leading mathematics research center
- Notable students:
 - Max Planck
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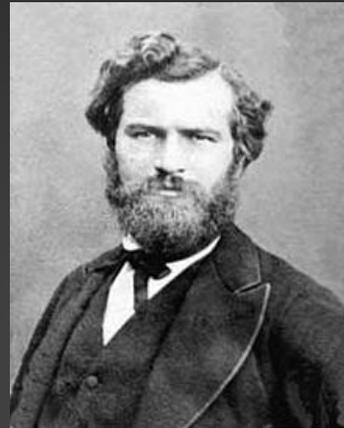


Married Anne Hegel, granddaughter of the philosopher Georg W. F. Hegel

French Mathematicians



Poincaré
(1854 - 1912)
The Last
Universalist



Jordan
(1838 - 1922)
Group Theory



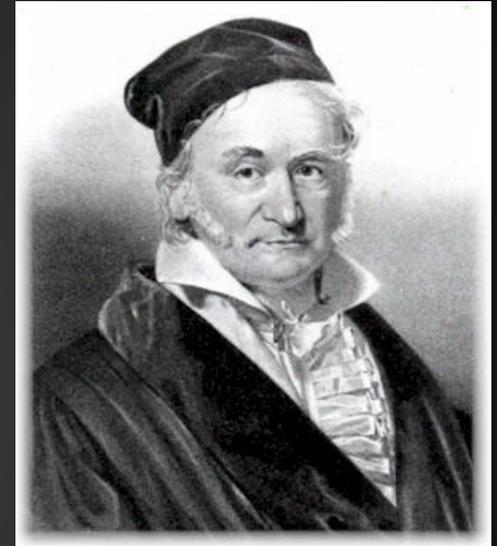
Hermite
(1822 - 1901)
Proved e is
transcendental

Back to Königsberg

- "Content and full of joy"
- Almost 25
- "A systematic exploration" of mathematics with Hurwitz during their daily walk
- Year and a half of lectures
- Another round of trips to 21 mathematicians

Invariant Theory

- Carl Friedrich Gauss (-1855)
 - $ax^2 + bx + c$
 - The discriminant $b^2 - 4ac$
- Arthur Cayley (-1895)
- James Sylvester (-1897)
- Paul Gordan (-1912)
- Alfred Clebsch (-1872)
 - (Supported Klein in becoming a professor at Erlangen)



Weird Proof of Gordan's Conjecture

- Paul Gordan (1837 – 1912)
- *Finite basis of n -ary forms*
- Brute-force demonstration for $n = 2$



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"Das ist nicht Mathematik. Das ist Theology" ~ Gordan

Marriage and the Move to Göttingen

- 1892
- Son: Franz
- *"She was a full human being in her own right, strong and clear, and always stood on the same footing with her husband, kindly and forthright, always original."*
- Minkowski expected "another great discovery"
 - New simplified proof of the transcendence of both e and π
- German Mathematical Society
 - Assigned them to report on number theory

Hilbert's Work on Geometry

"The Greeks had conceived of geometry as a deductive science which proceeds by purely logical processes once the few axioms have been established. Both Euclid and Hilbert carry this program. However, Euclid's list of axioms was still far from being complete; Hilbert's list is complete and there are no gaps in the deduction." ~ Hermann Weyl (1885 - 1955)

Turn of the Century

- Second International Congress of Mathematicians
- Algebra, number theory, geometry and analysis
- Foundational issues: models, relative consistency
- After some thought and discussion with Minkowski and Hurwitz, decided to lecture on the significance of individual problems and to give a list of problems that he thought would be the most fruitful for mathematics in the new century
- Had only 10 when he delivered the talk
- 23 in the published version
- 24th rediscovered in 2000

Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?

David Hilbert
Opening speech

Outline of Problems

- The Foundation Problems
 - 1, 2, 10
- The Foundation of Specific Areas
 - 3, 4, 5, 6
- Number Theory
 - 7, 8, 9, 11, 12
- Algebra and Geometry: A Miscellany
 - 14, 15, 16, 17, 18
- The Analysis Problems
 - 13, 19, 20, 21, 22, 23

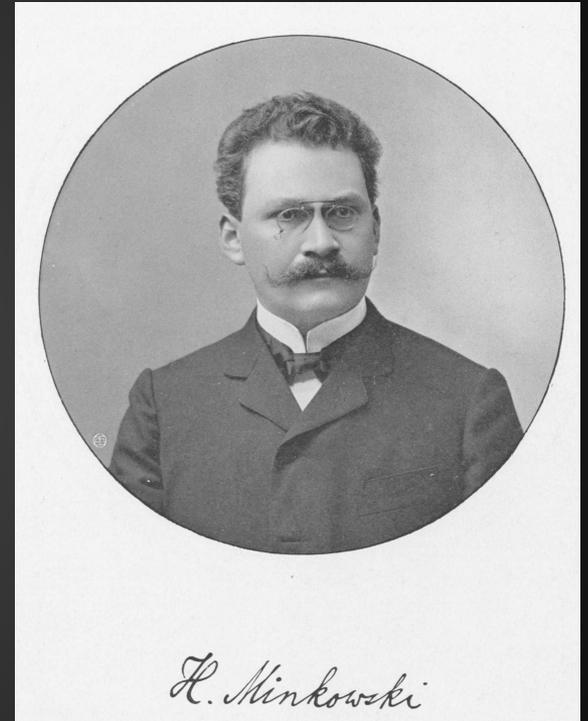
The conviction of the solvability of every mathematical problem is a powerful incentive to the worker. We hear within us the perpetual call: There is the problem. Seek its solution. You can find it by pure reason, for in mathematics there is no **ignorabimus.**

David Hilbert

Towards the end of his famous lecture

Hilbert's Obituary of Minkowski

"Since my student years Minkowski was my best, most dependable friend who supported me with all the depth and loyalty that was so characteristic of him. Our science, which we loved above all else, brought us together; it seemed to us a garden full of flowers. In it, we enjoyed looking for hidden pathways and discovered many a new perspective that appealed to our sense of beauty, and when one of us showed it to the other and we marvelled over it together, our joy was complete. He was for me a rare gift from heaven and I must be grateful to have possessed that gift for so long. Now death has suddenly torn him from our midst. However, what death cannot take away is his noble image in our hearts and the knowledge that his spirit in us continue to be active."

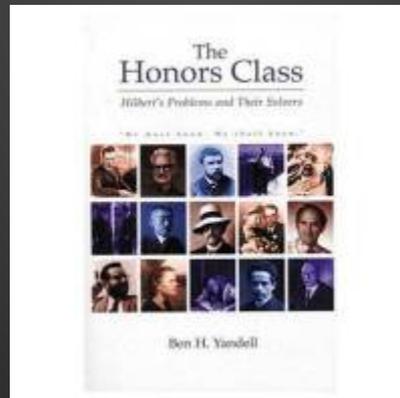


Axiomatization of Physics

- Made a contribution to general relativity
- *"Every boy in the streets of Göttingen understands more about four-dimensional geometry than Einstein. Yet, in spite of that, Einstein did the work and not the mathematicians"*

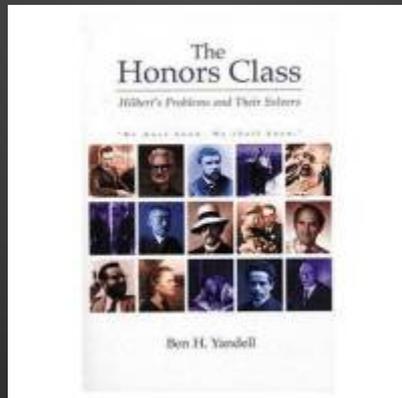
Textbook

The Honor Class Hilbert's Problems and their Solvers *by Benjamin Yandell*



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Euler Book Prize - 2008

That I have been able to accomplish anything in mathematics is really due to the fact that I have always found it so difficult. When I read, or when I am told about something, it nearly always seems so difficult, and practically impossible to understand, and then I cannot help wondering if it might not be simpler. And on several occasions it has turned out that it really was simple!

David Hilbert

Thank you!

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