Introduction to Quantum Mechanics

QCSYS 2012

IQC Institute for Quantum Computing

Outline

1. Polarization
2. Double-slit experiment
3. Photoelectric effect
4. No-cloning theorem

Polarization

Superposition

A basic feature of quantum mechanics is the *principle of superposition*:

If a quantum system can be in the state $|\psi\rangle$ or in the state $|\phi\rangle$, then it can also be in state $\alpha|\psi\rangle + \beta|\phi\rangle$ for any complex numbers α, β (subject to normalization).

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We'll explore superposition in the context of *polarization* of light.

The electromagnetic field

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electric field $\vec{E}(x, y, z, t)$

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These fields obey the *Maxwell equations*:

$$\vec{\nabla} \cdot \vec{E} = \rho \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

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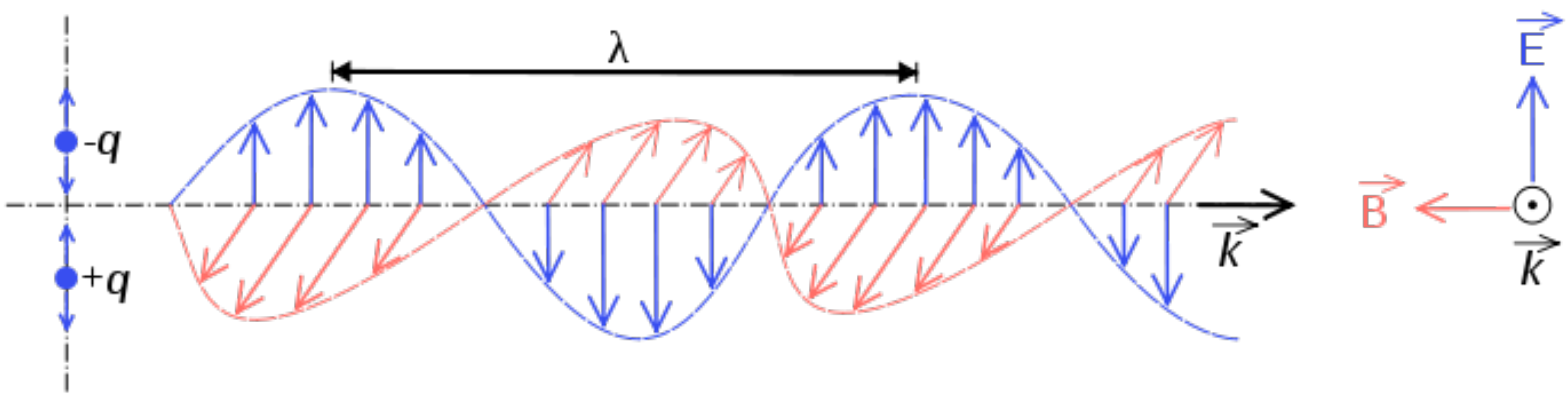
Example: Plane wave propagating in the $\hat{z}$ direction

$$\vec{E}(x,y,z,t) = \mathrm{Re}\left(\begin{pmatrix}\alpha_x\\ \alpha_y\\ 0\end{pmatrix} e^{2\pi i(z-ct)/\lambda}\right)$$

$$\vec{B}(x,y,z,t) = \hat{z} \times \vec{E}(x,y,z,t)$$



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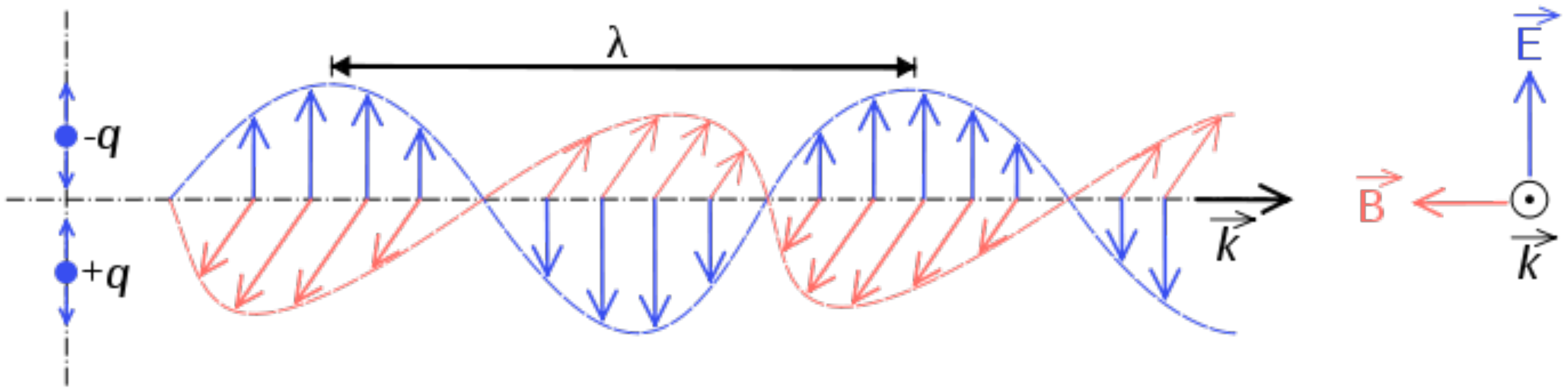

The vector $\begin{pmatrix} \alpha_x \\ \alpha_y \end{pmatrix}$ indicates the *polarization* of the wave.

Superposition of polarization states

Horizontal polarization: $|{\rightarrow}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Vertical polarization: $|{\uparrow}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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The 45° states also form an orthonormal basis:

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}|\nearrow\rangle + \frac{1}{\sqrt{2}}|\searrow\rangle$$

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Analogous to quantum measurement:

- The fraction of light that passes through the filter is given by the inner product squared with the filter direction.
- The outgoing light is entirely in the same direction as the filter.

Polarizing filter examples

Incident polarization: $|\nearrow\rangle$

Polarizing filter orientation: $\rightarrow$

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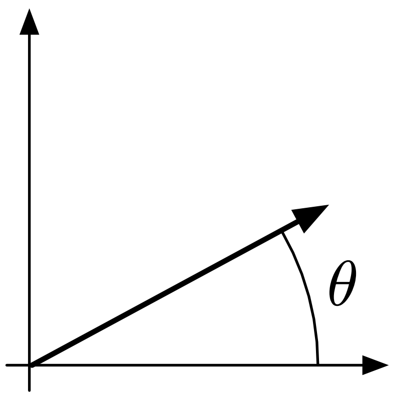

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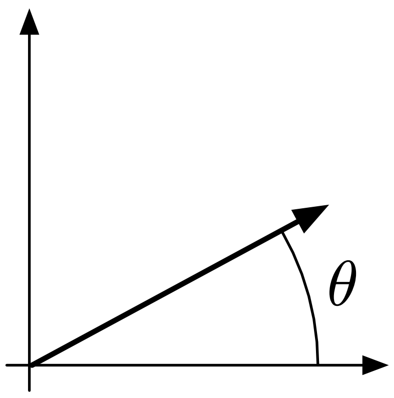

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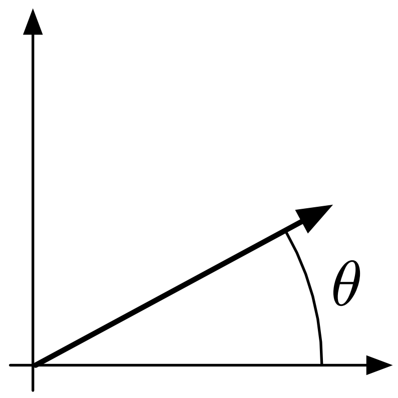

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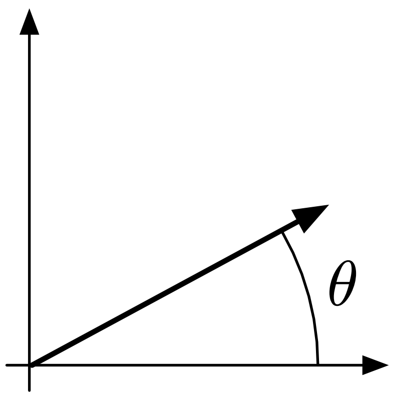

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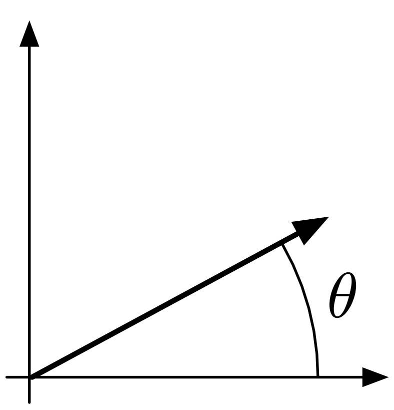

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Polarizing filter demo

What happens to the incident light if we orient polarizers as follows?

- crossed polarizers (0 and 90 degrees)
- diagonal polarizers (0 and 45 degrees)
- polarizers at 0, 45, 90 degrees

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Circular polarization

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Examples:

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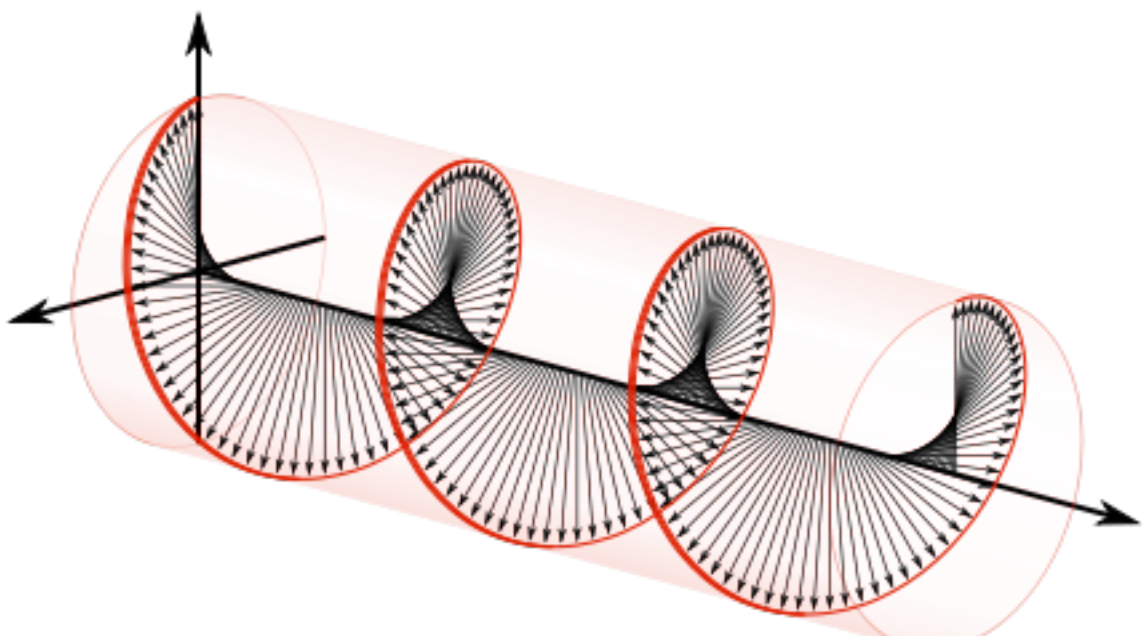
The direction of the electric field moves in a circle as the wave propagates, so this is called *circular polarization*.



Qubits

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Single photons also have polarization.

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Many other systems can be used to store qubits, and the components of the state vector need not represent directions in space:

- Atom: $\alpha|\mathrm{ground}\rangle + \beta|\mathrm{excited}\rangle$
- Electron spin: $\alpha|\mathrm{up}\rangle + \beta|\mathrm{down}\rangle$
- Photon number: $\alpha|0\rangle + \beta|1\rangle$
- Superconducting flux: $\alpha|\mathrm{cw}\rangle + \beta|\mathrm{ccw}\rangle$
- ...

Exercise: Stacked polarizers

Suppose we stack n polarizers so that the angle between the polarization direction of each filter and the next is π/n. What fraction of the light passing the first polarizer passes the last polarizer?

a. Compute exact values for $n = 2, 3, 4$.

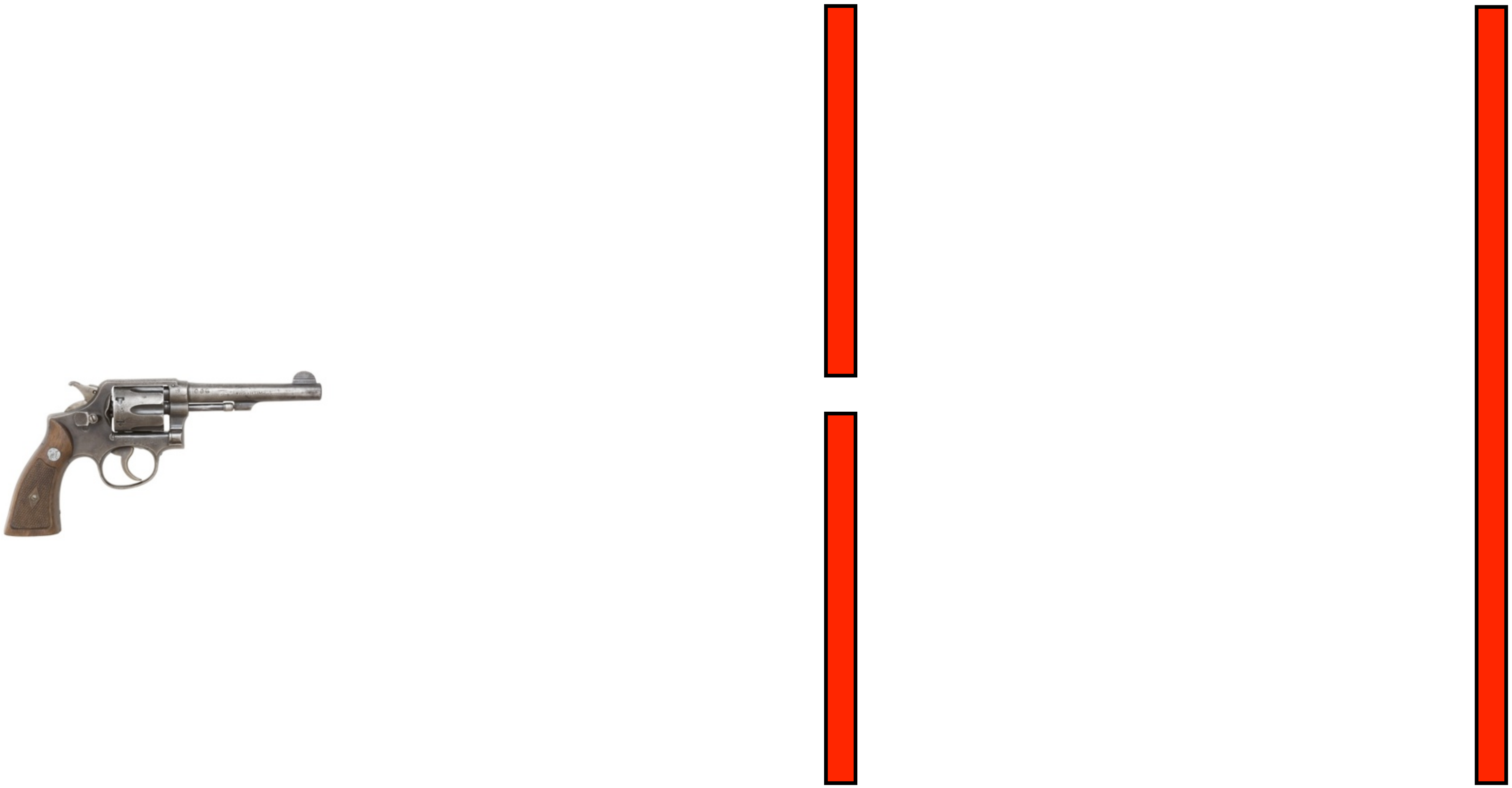
b. Give a symbolic expression for general n.

c. Using a computer, plot the values for $n = 2$ through 50.

d. What happens in the limit as $n \to \infty$?

Double-slit experiment

Firing bullets at a slit



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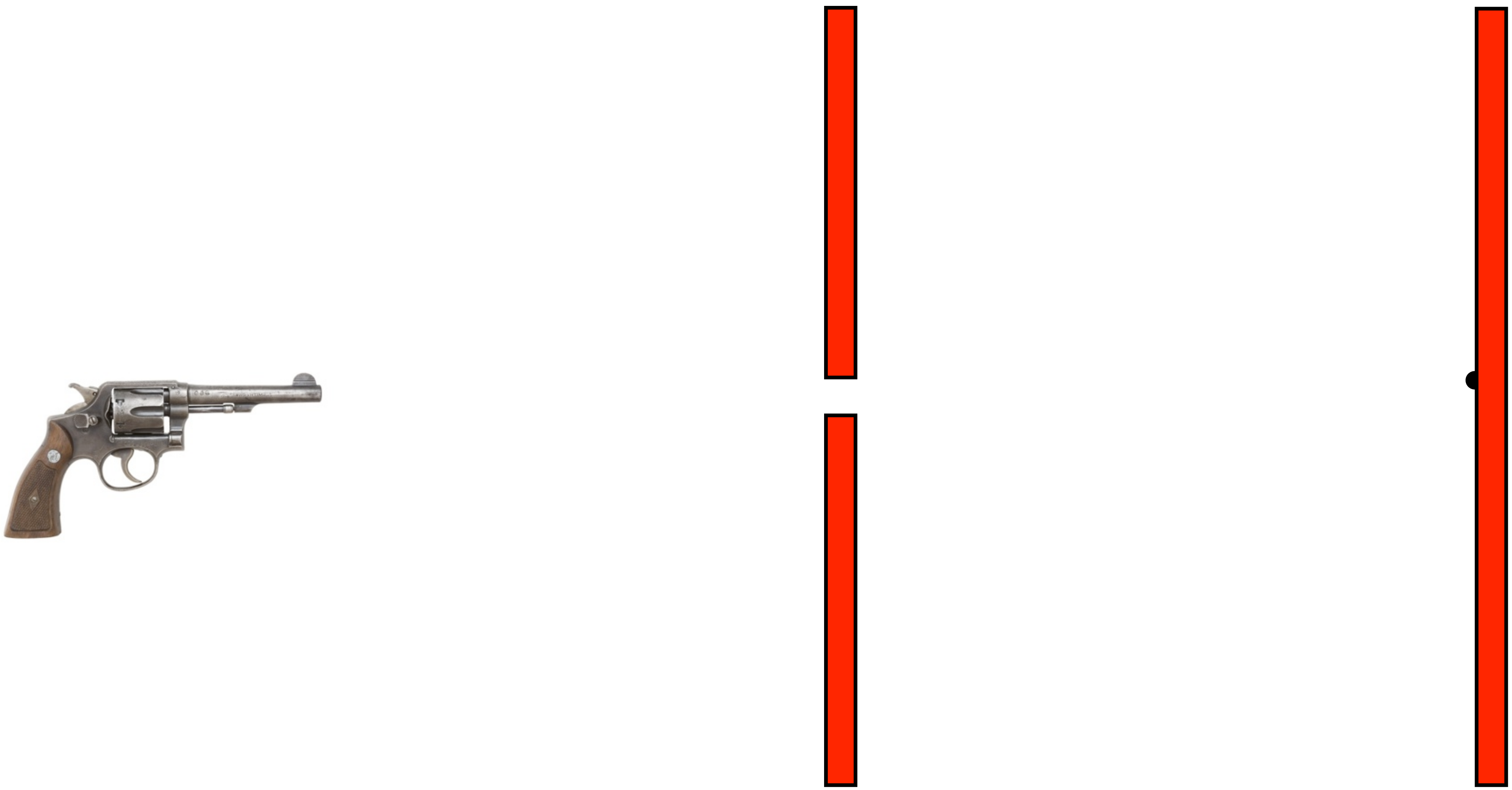

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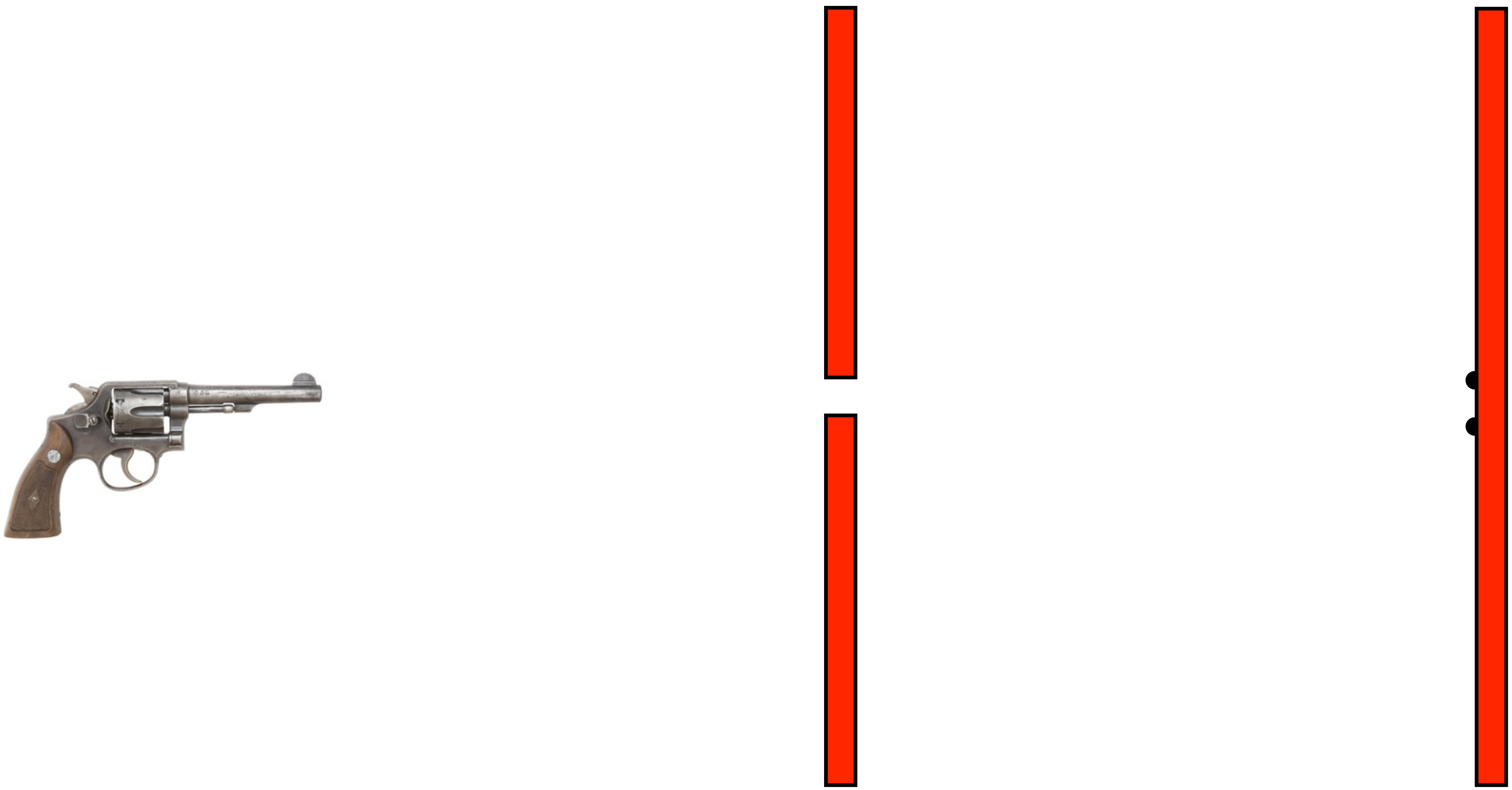

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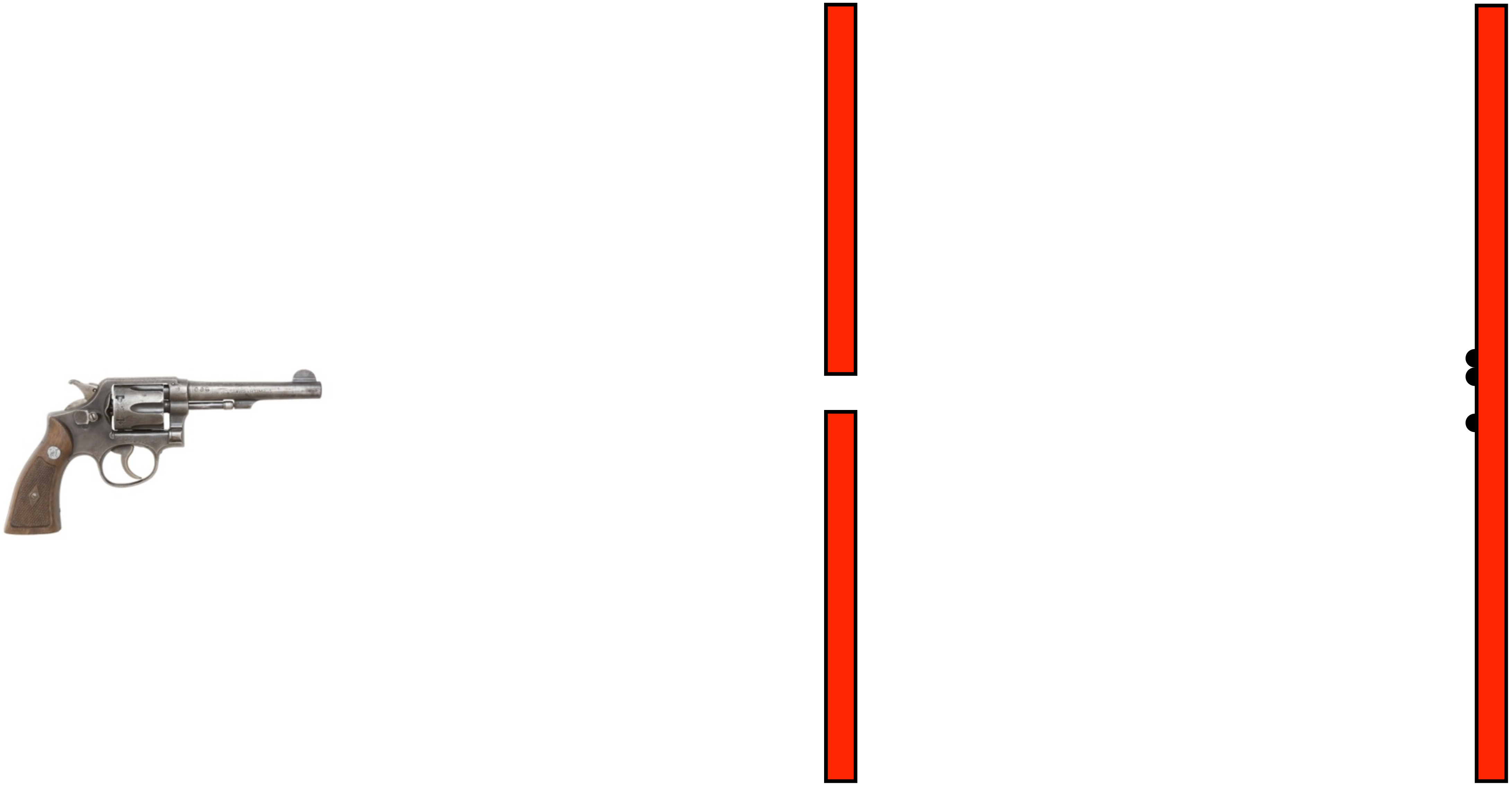

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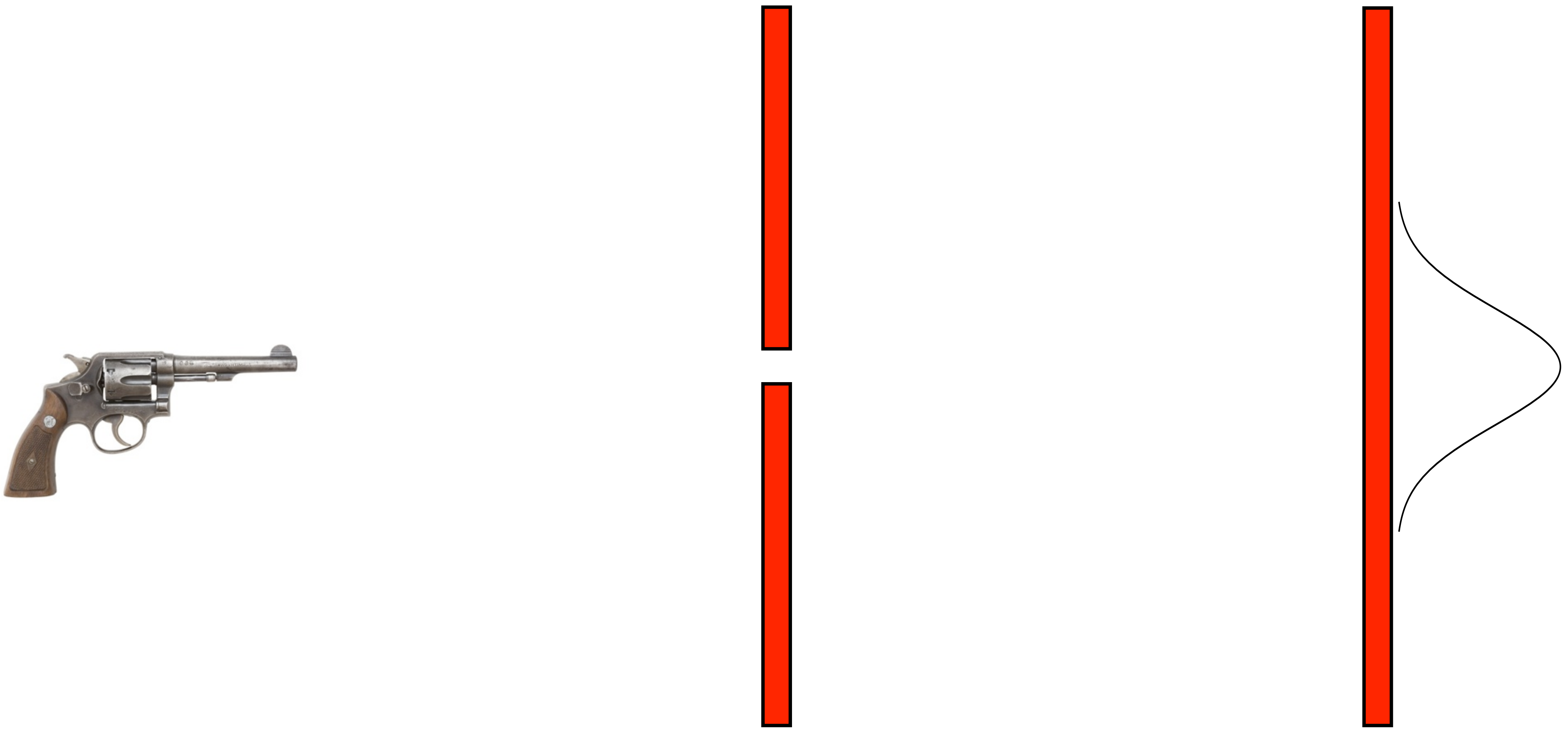

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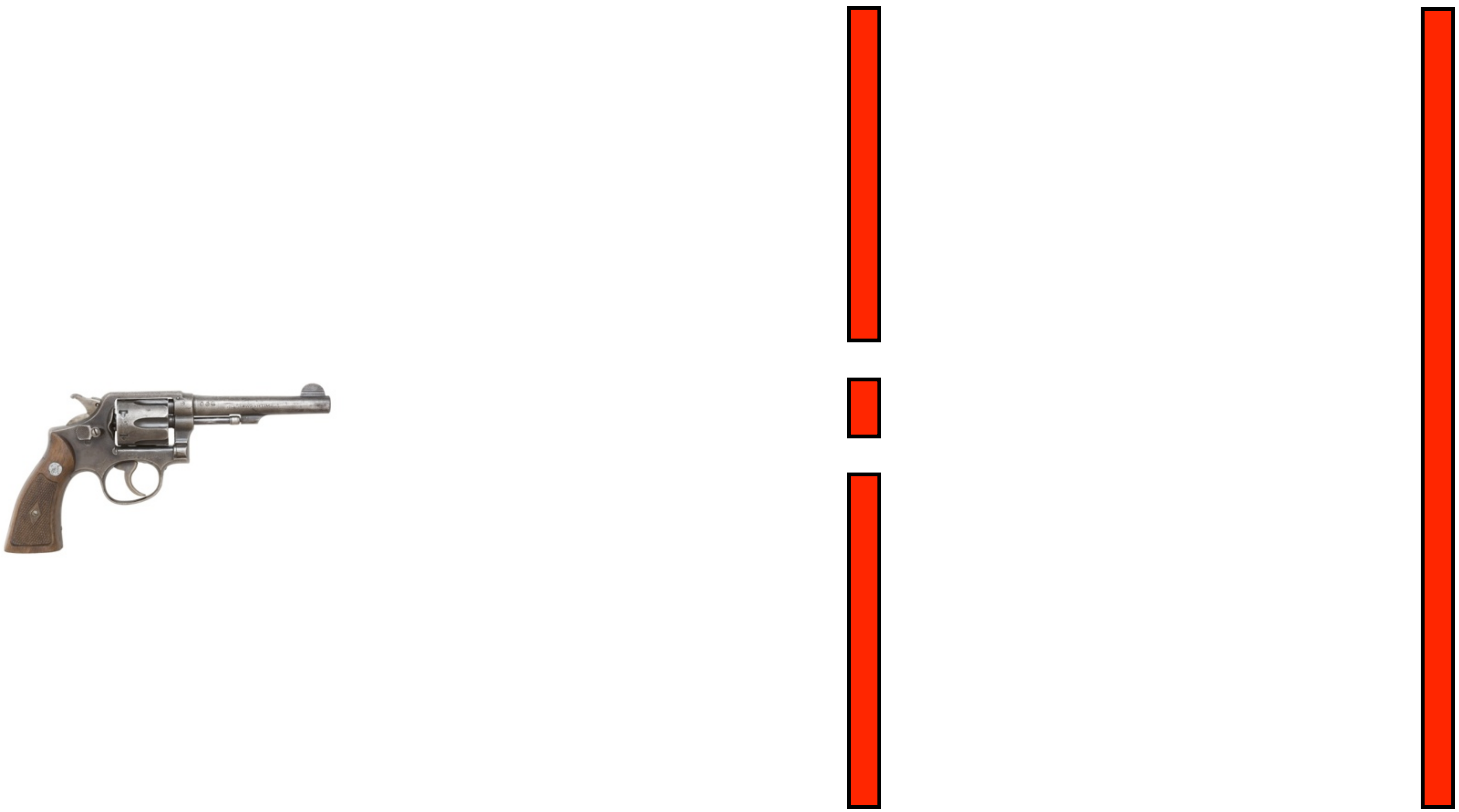

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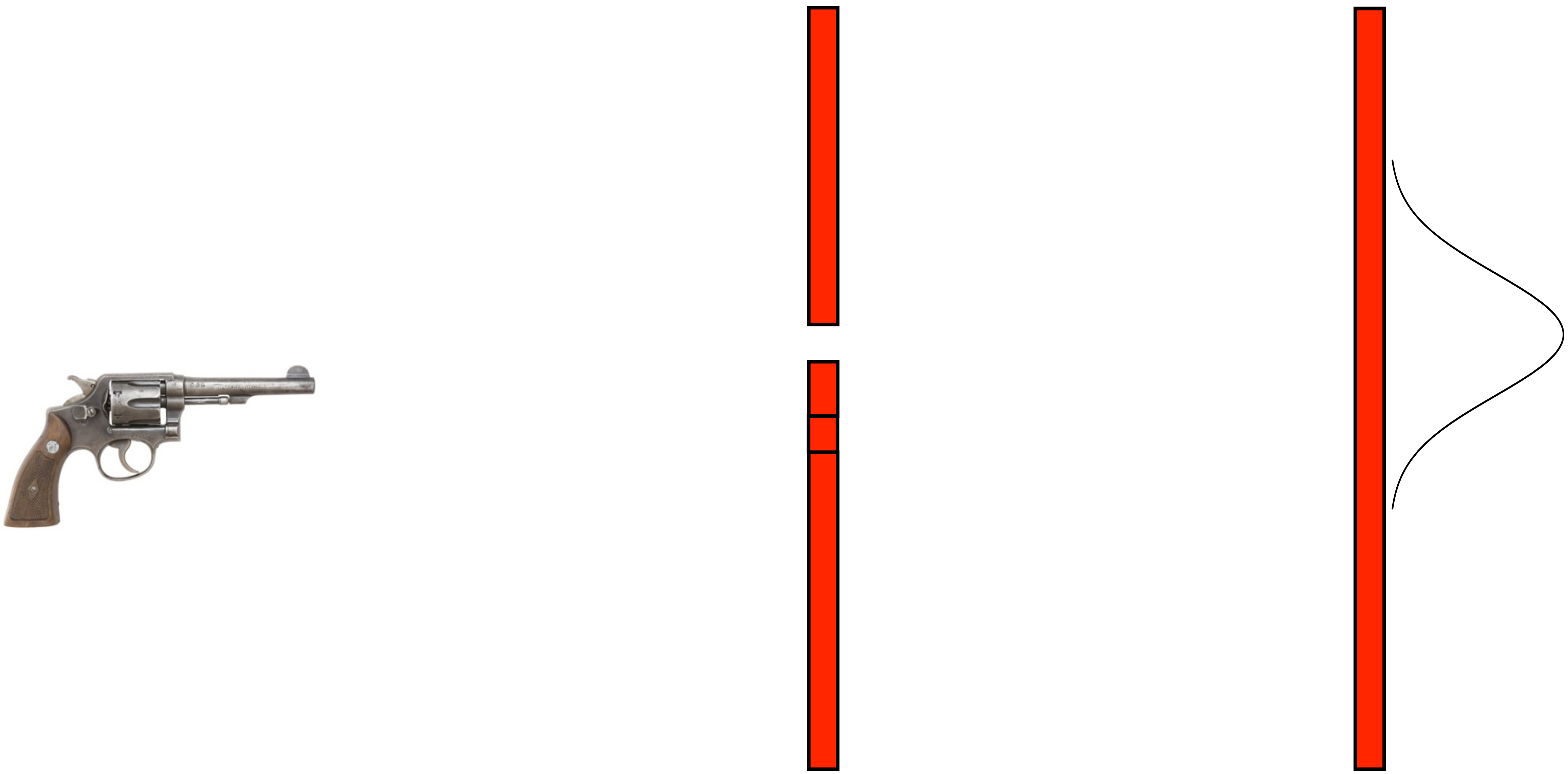

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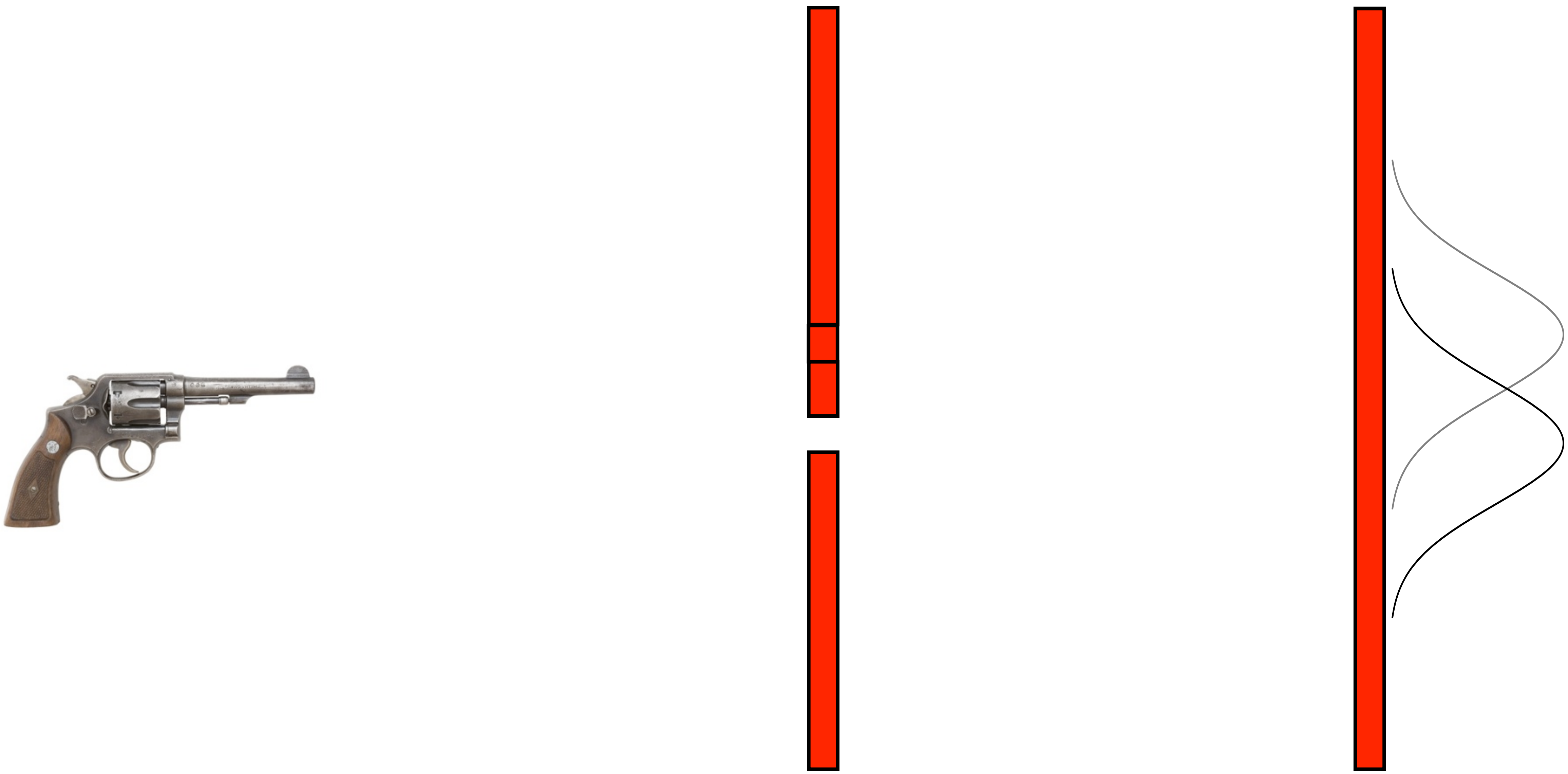

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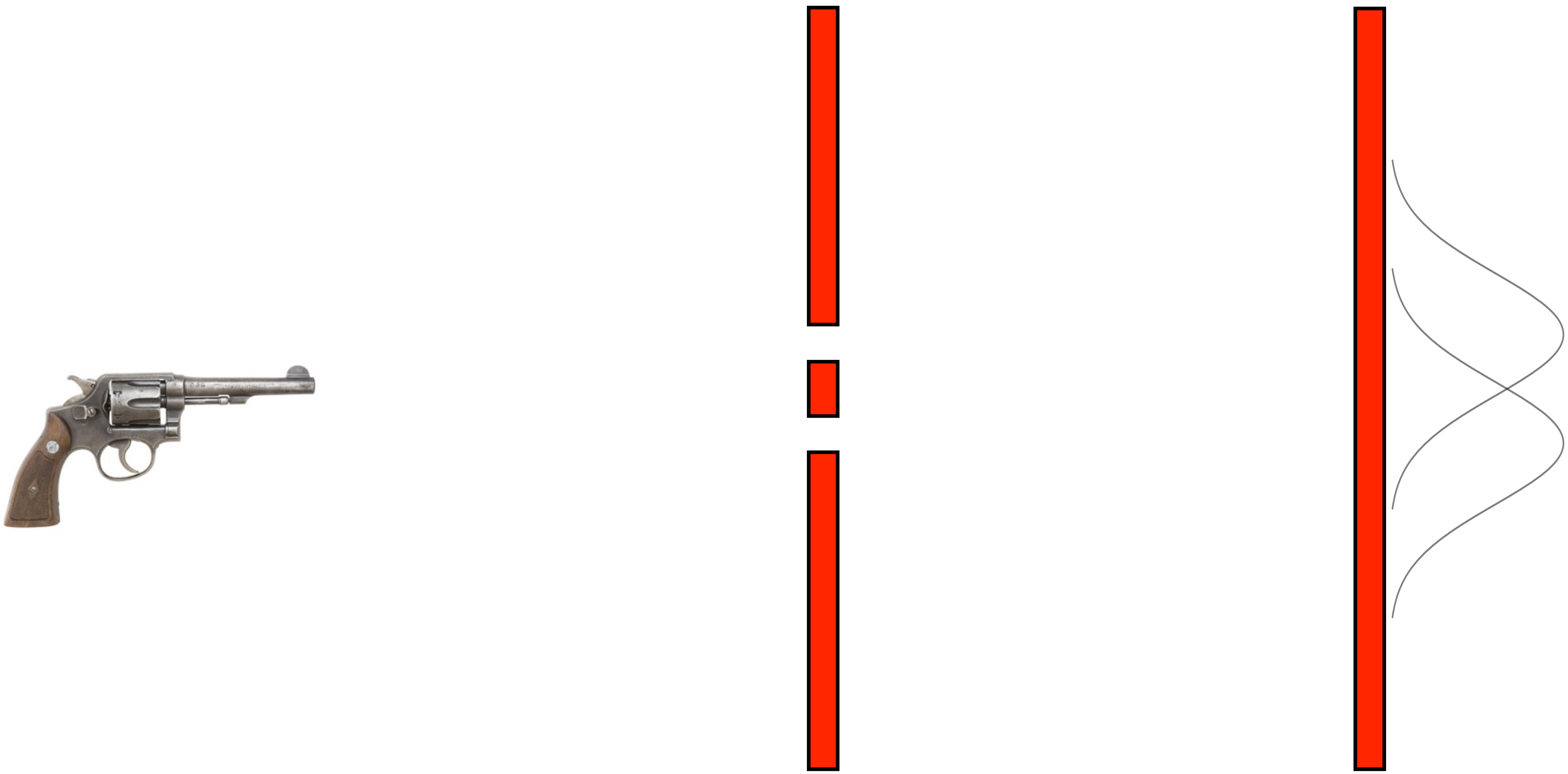

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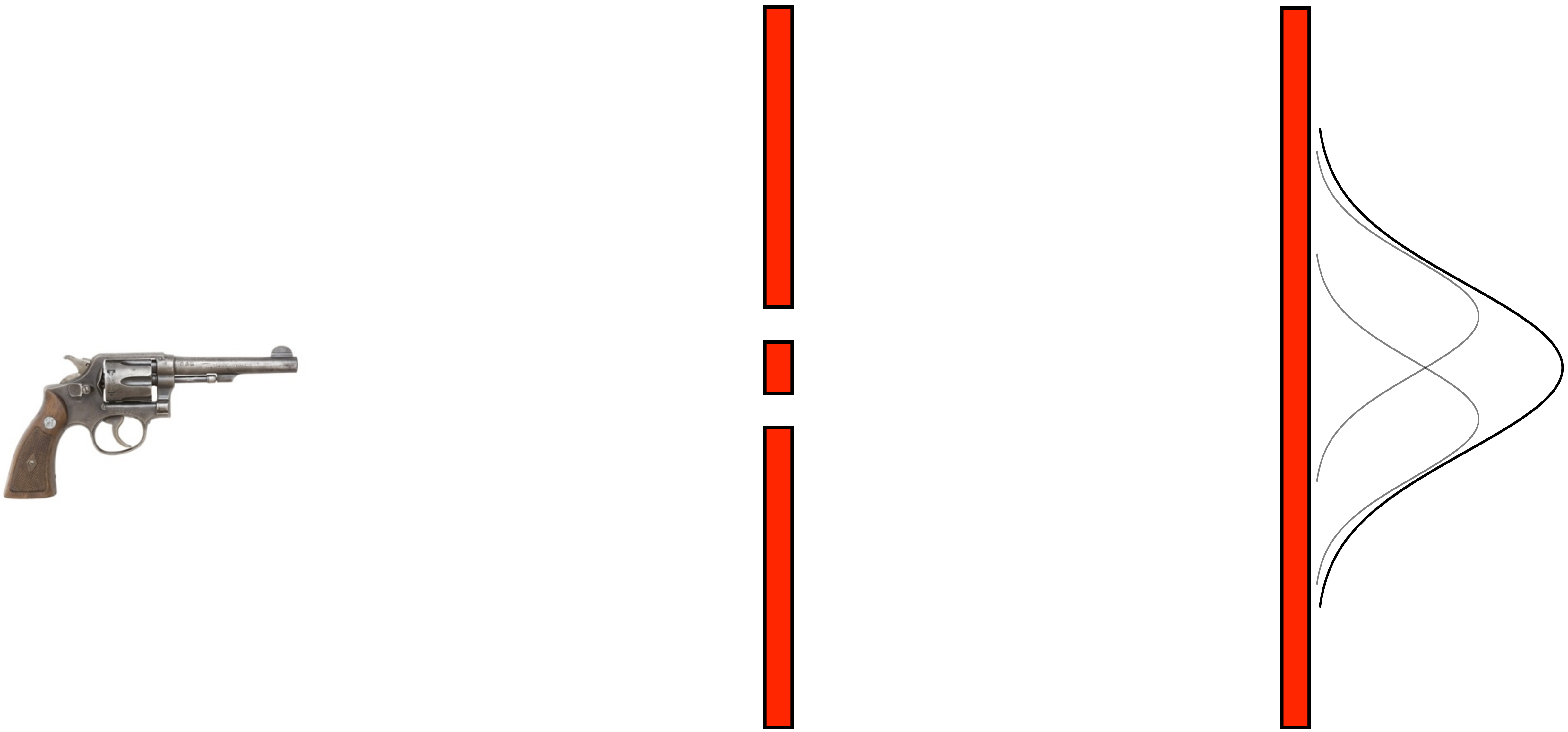

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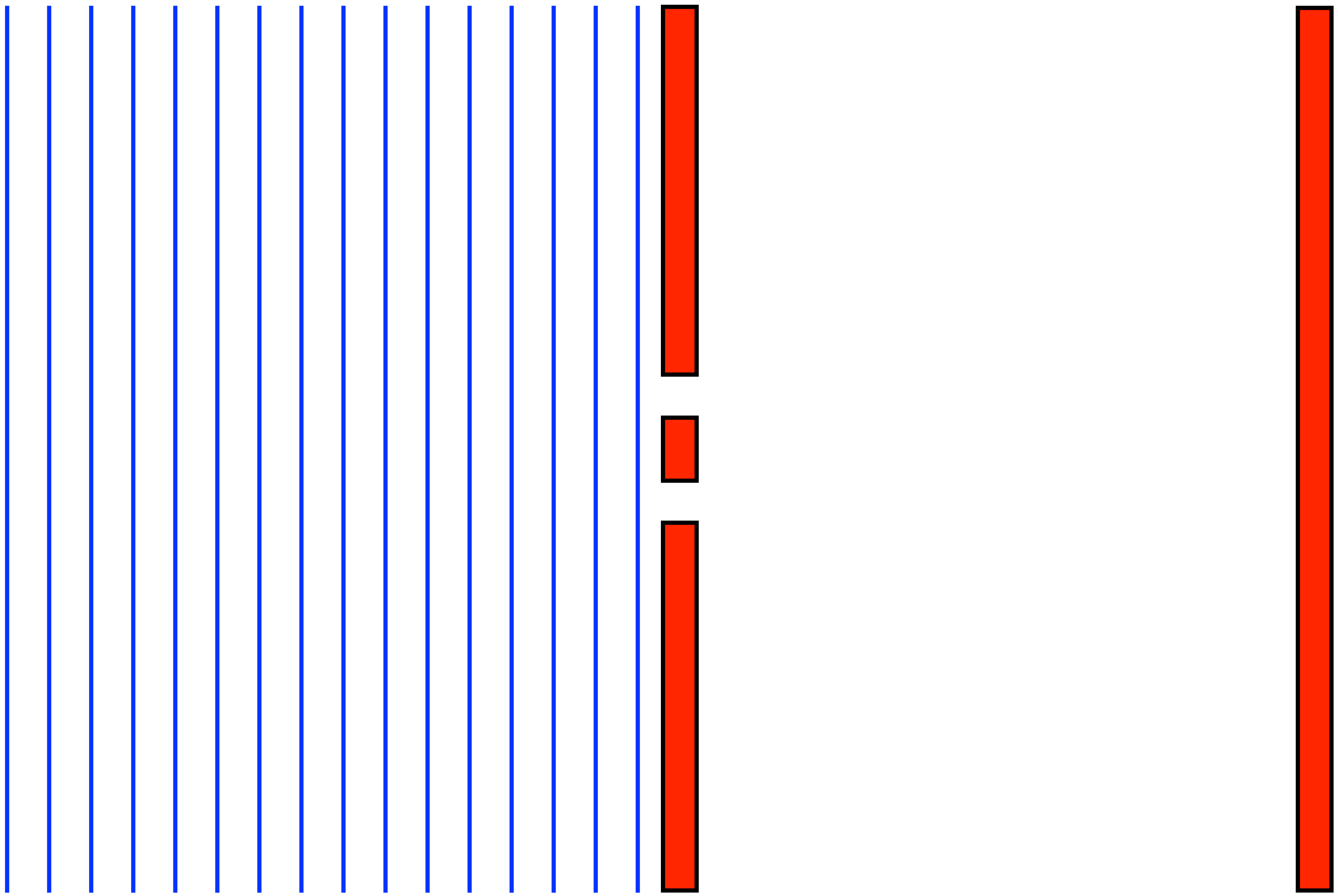

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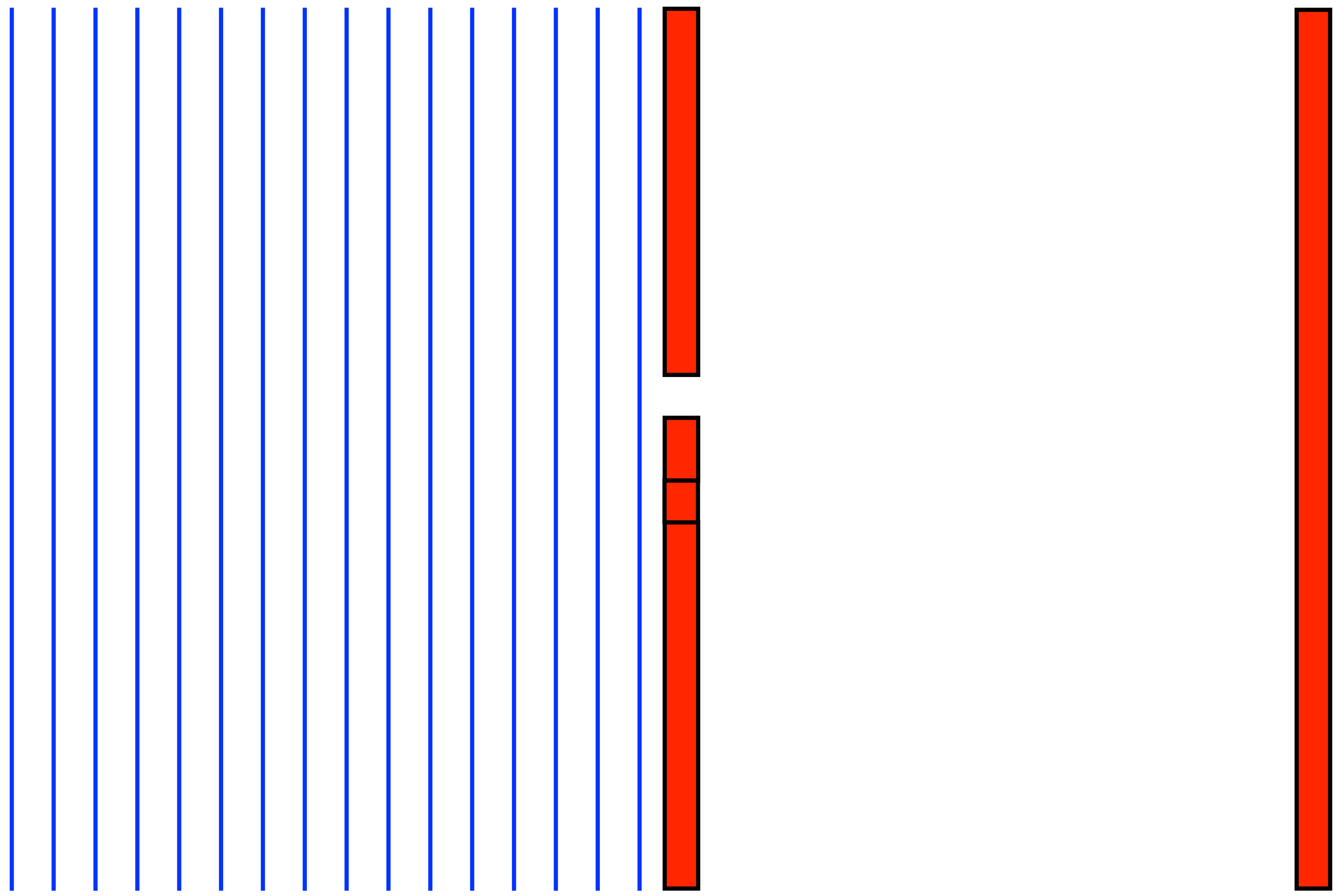

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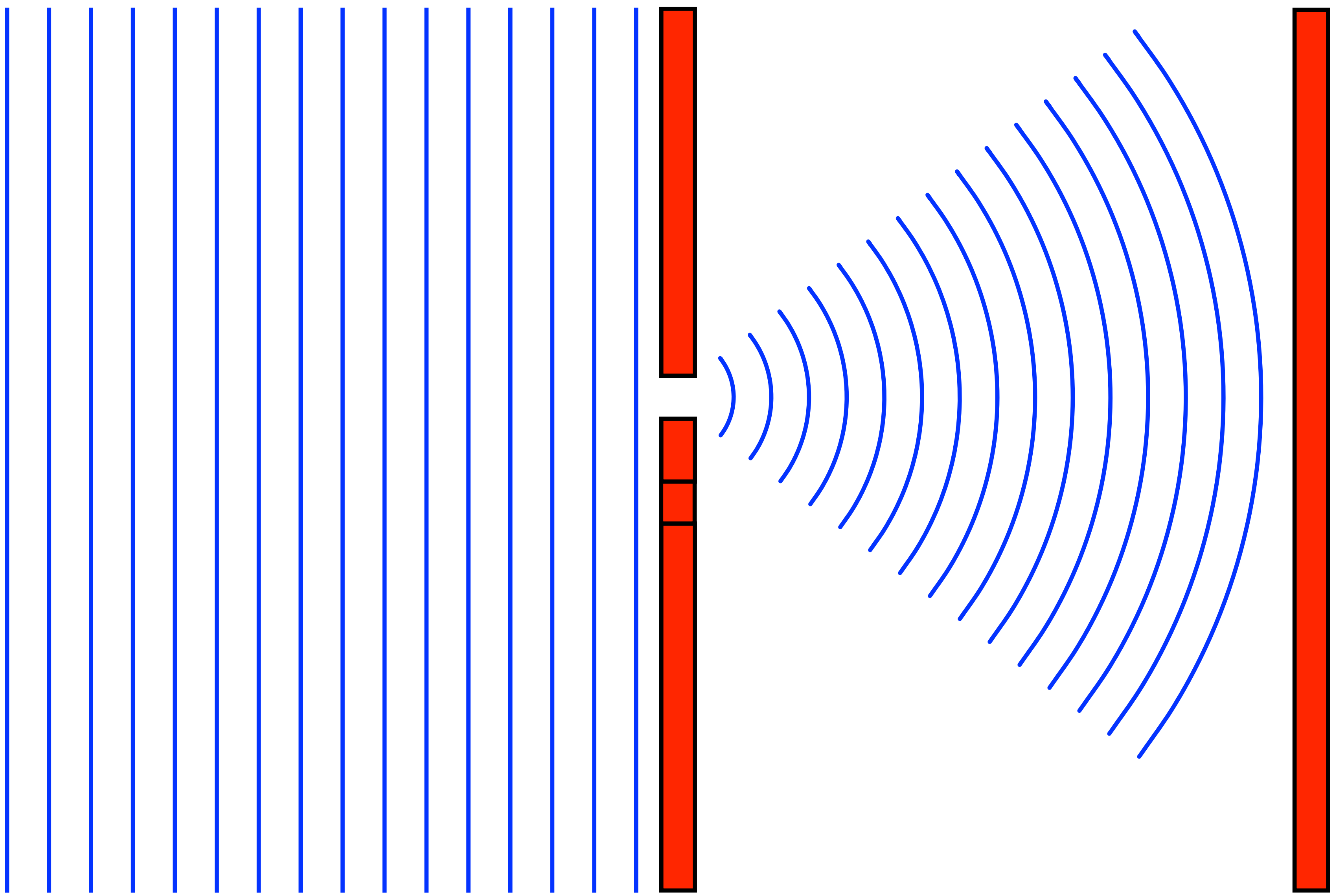

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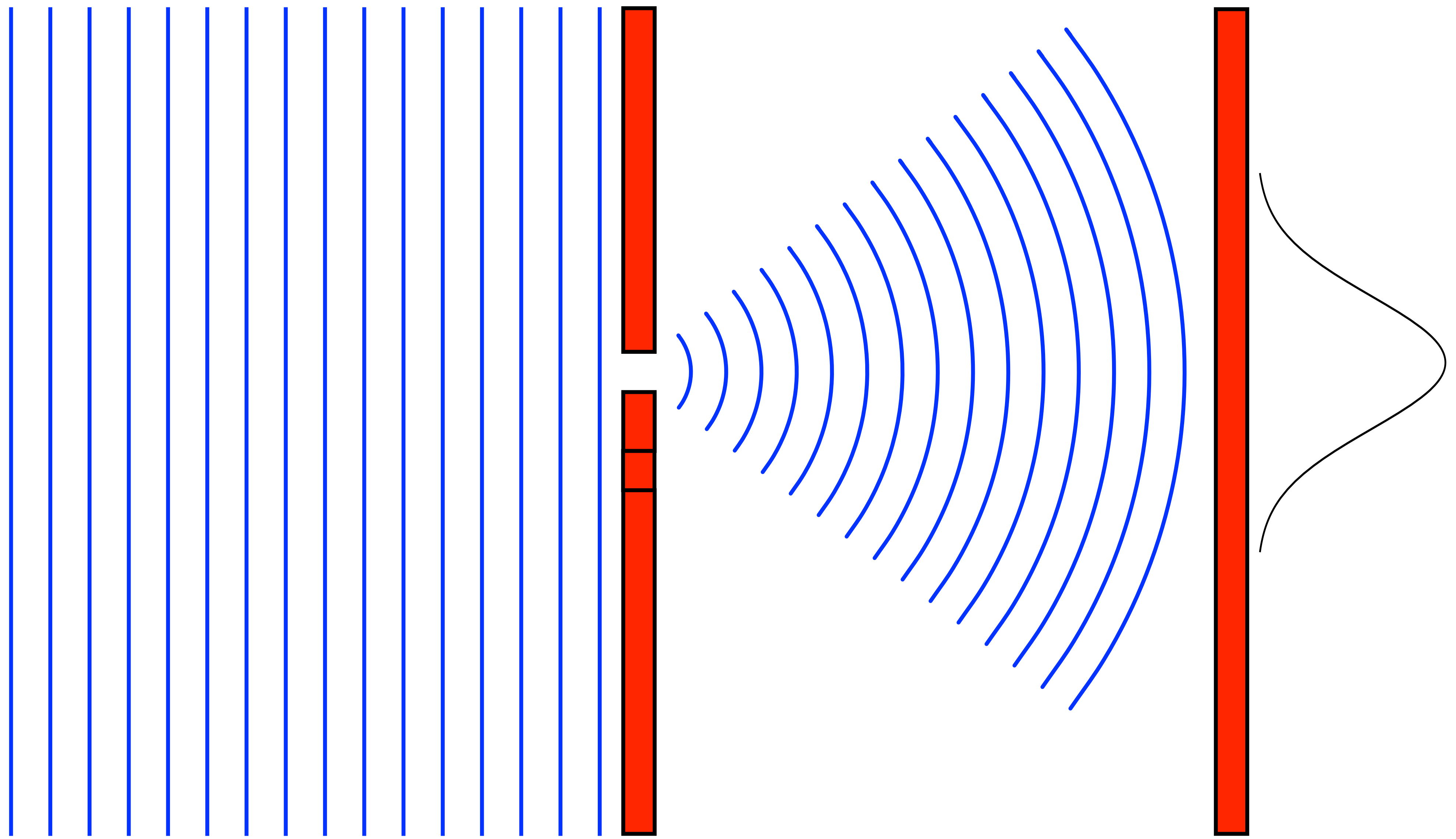

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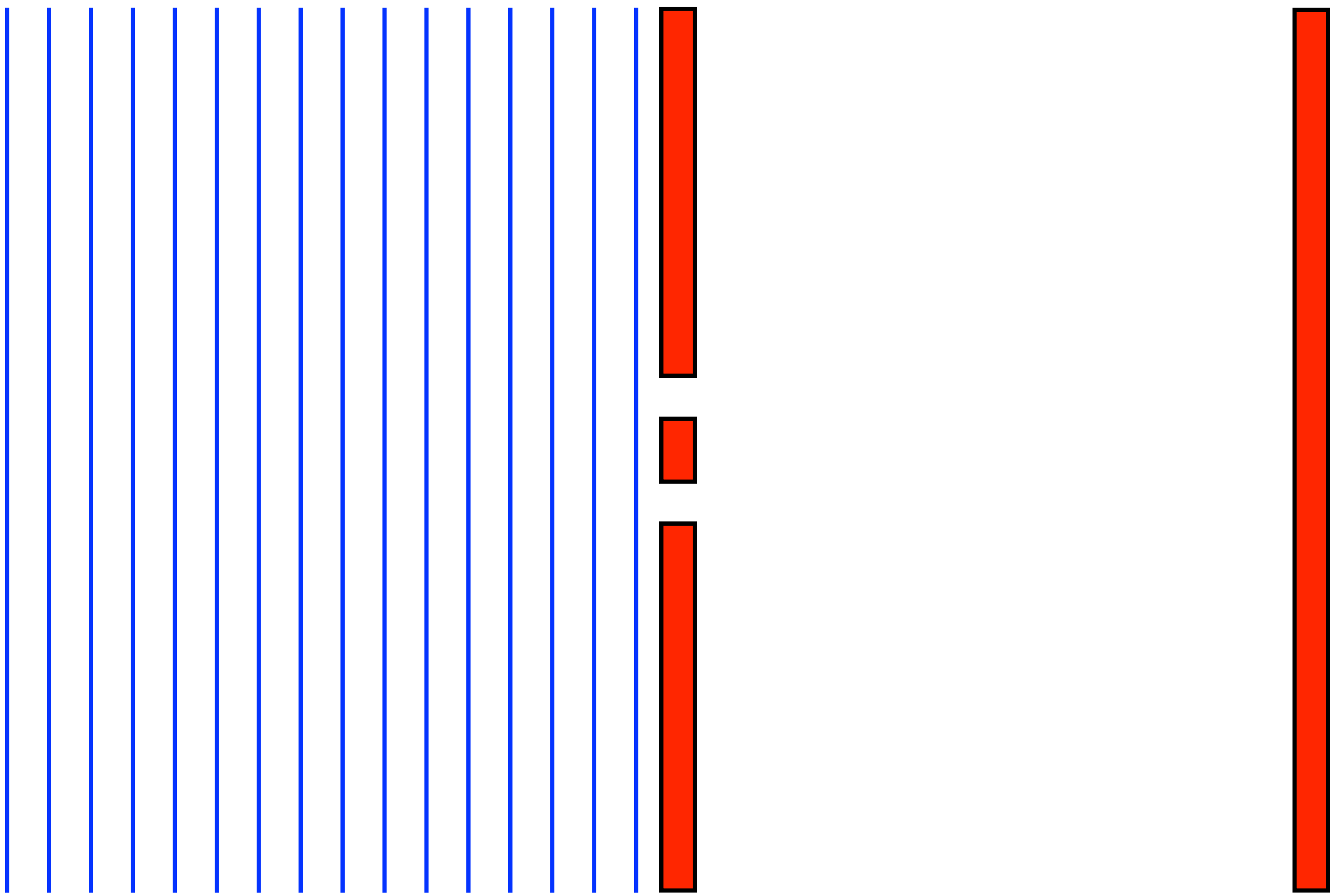

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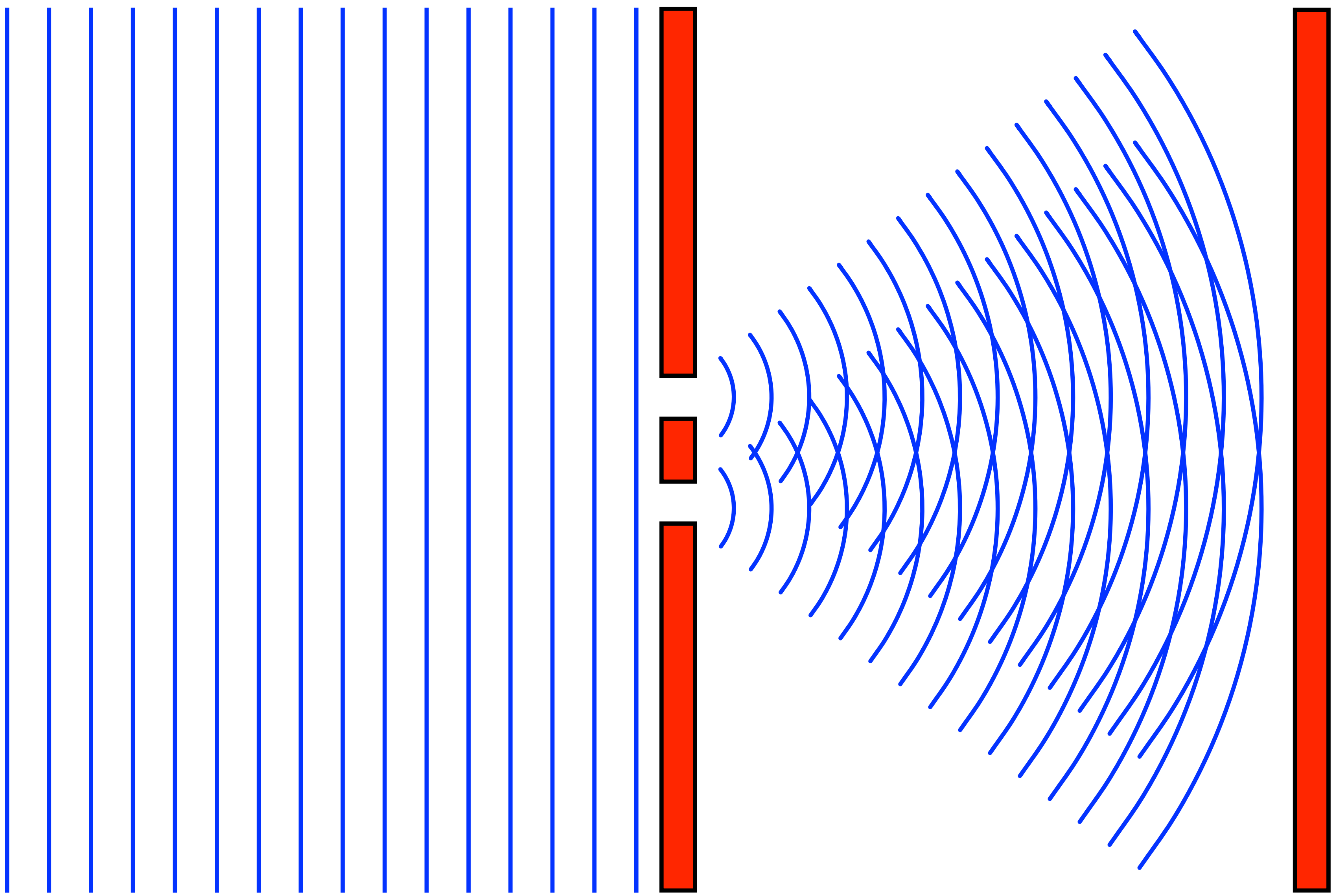

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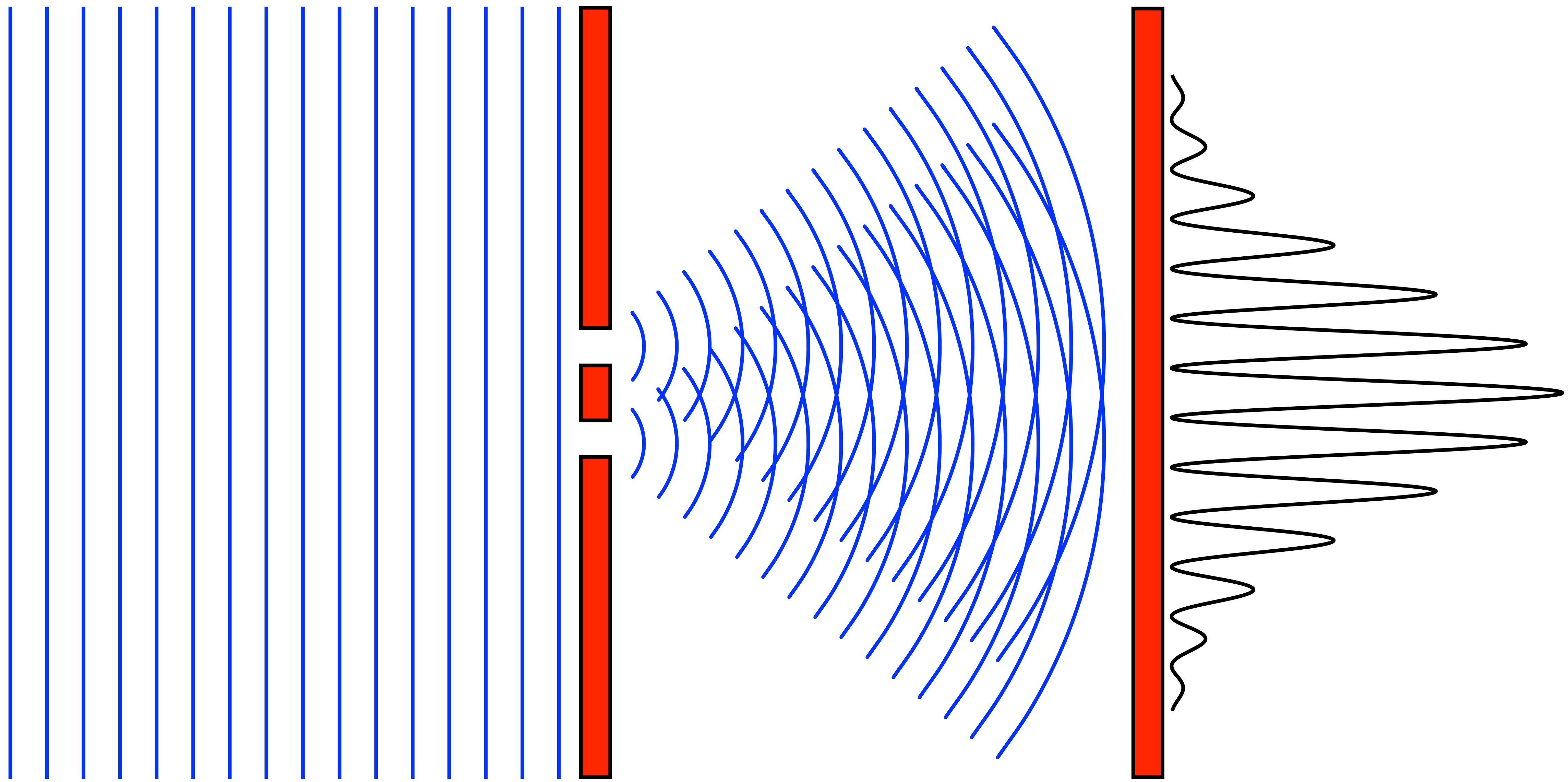
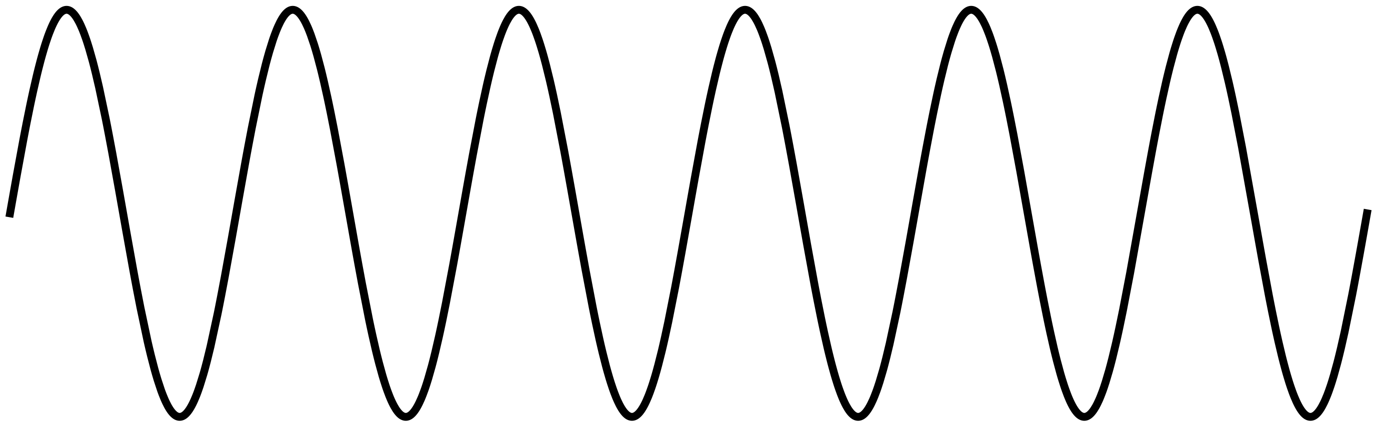


Demo

Interference



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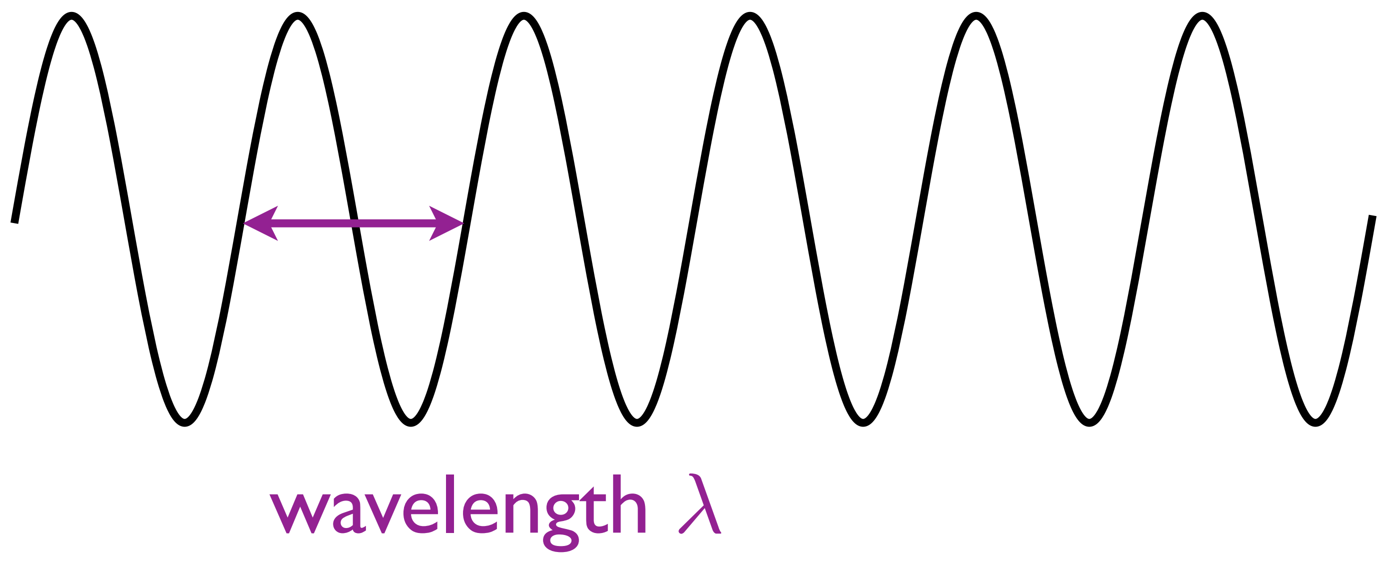

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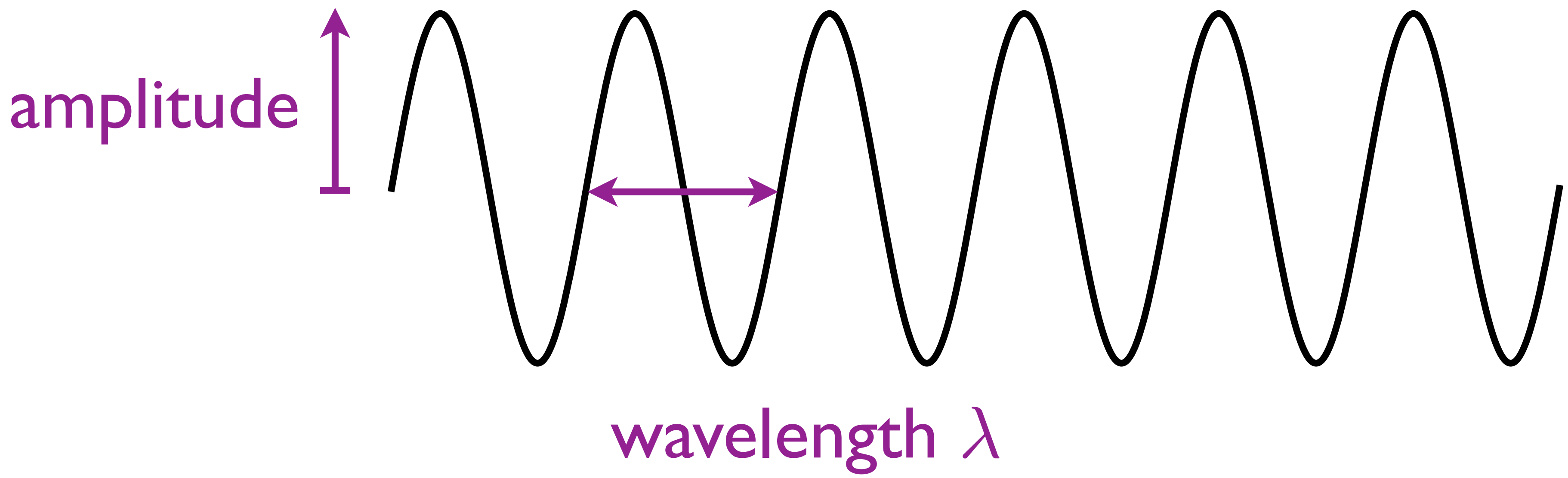

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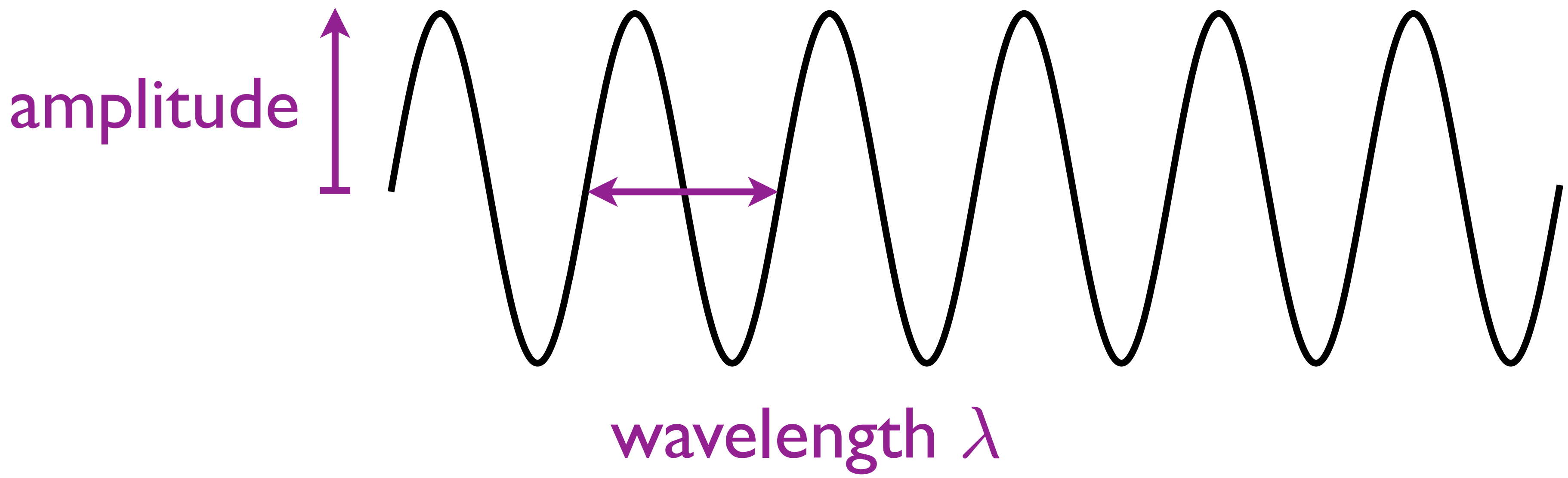

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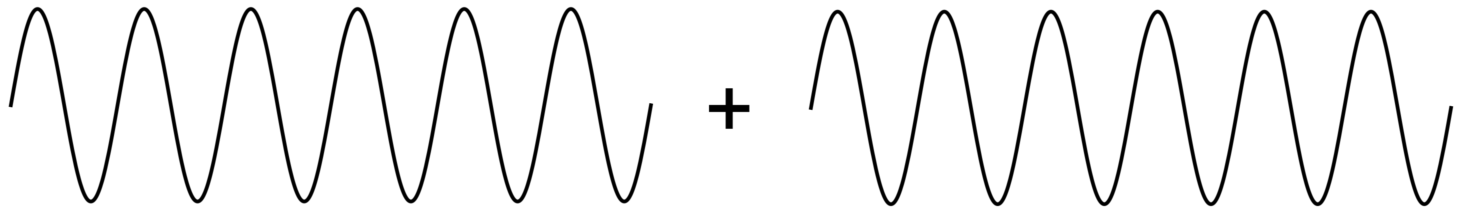

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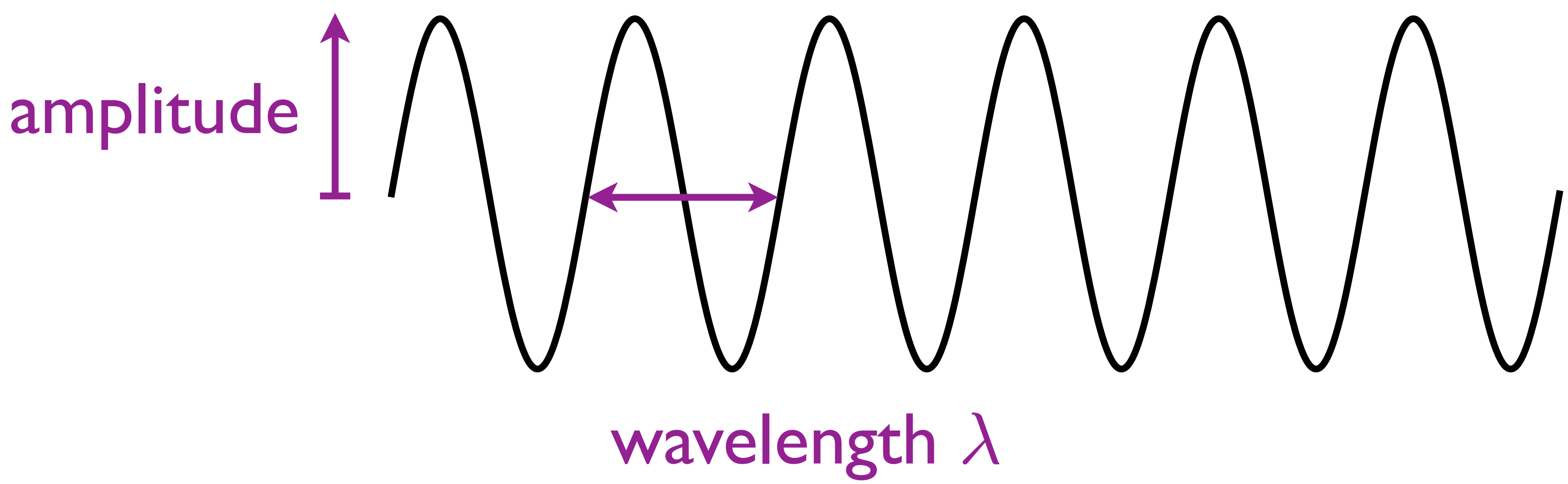

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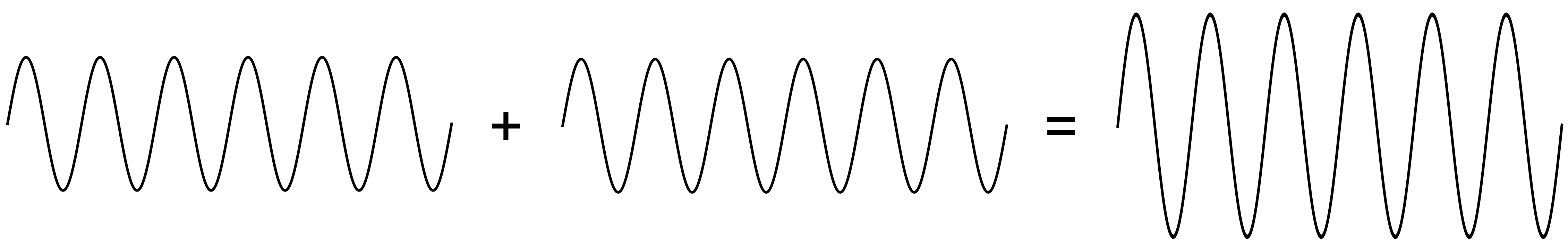

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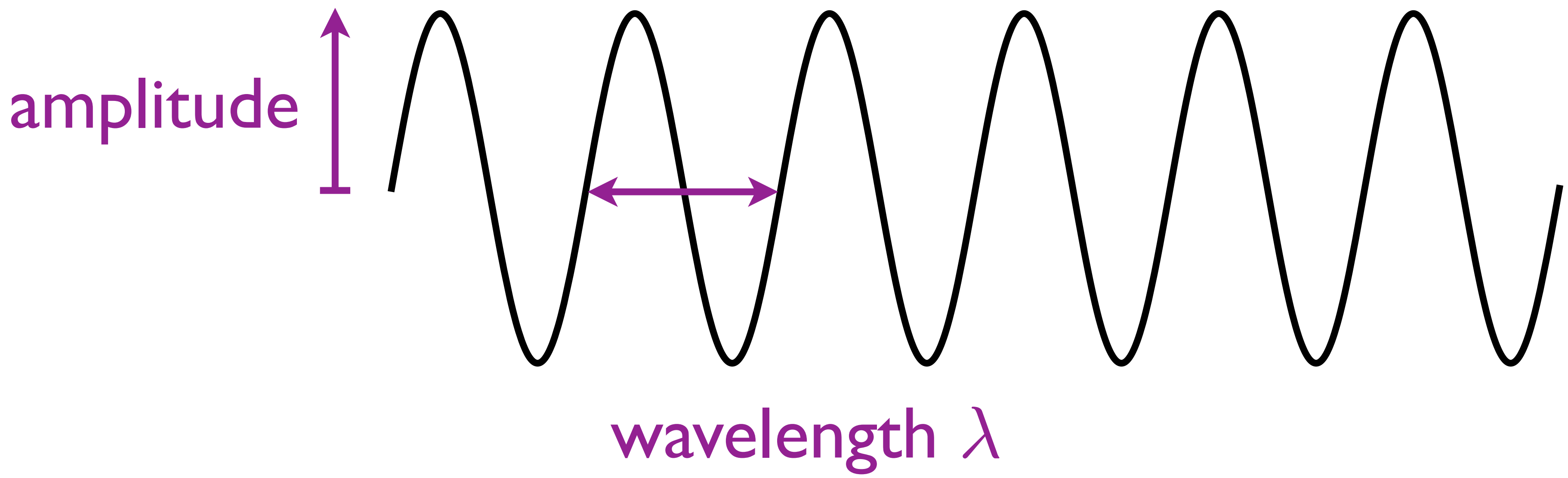

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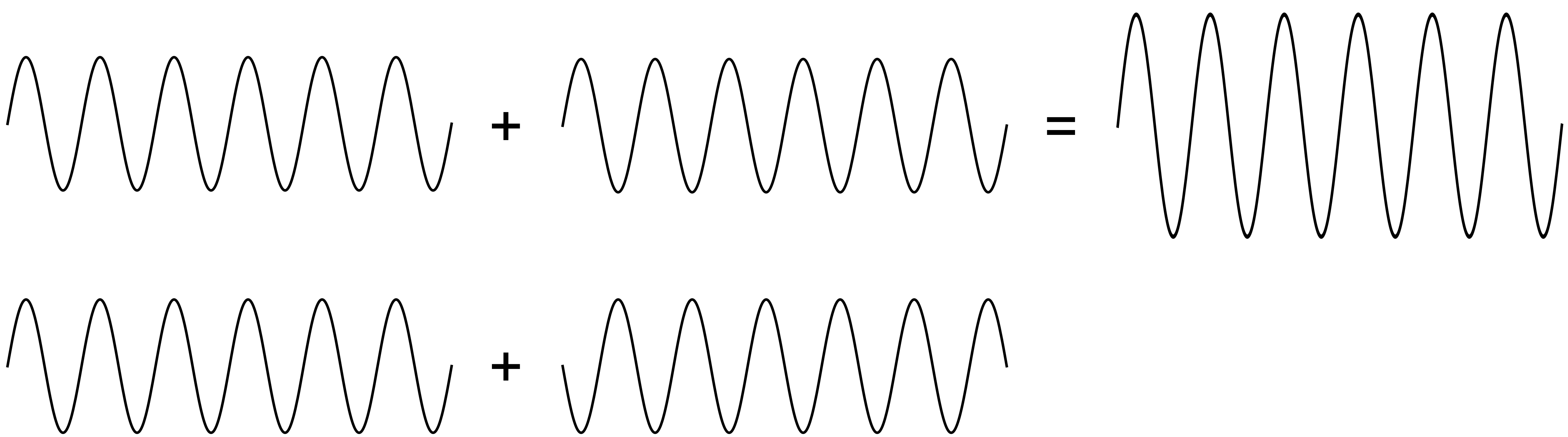

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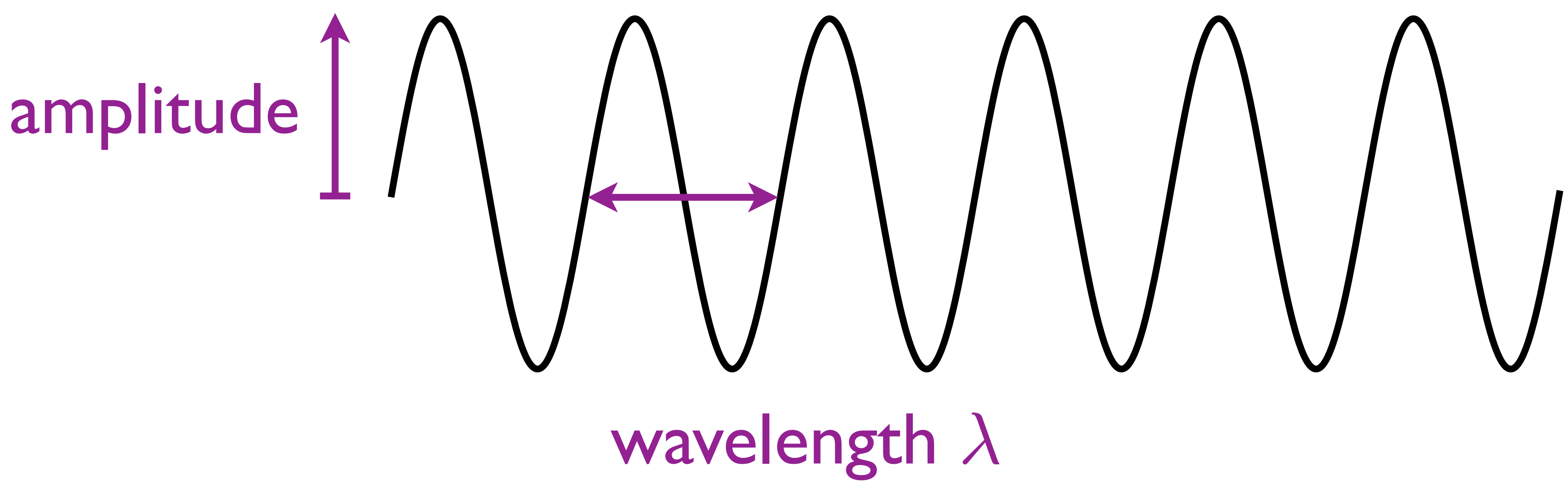
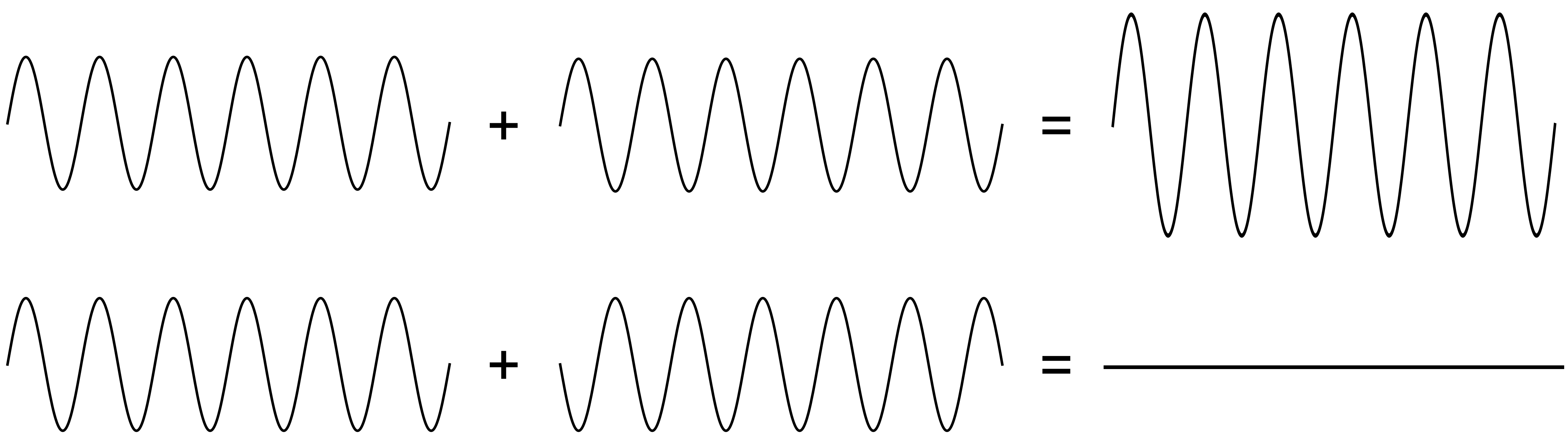
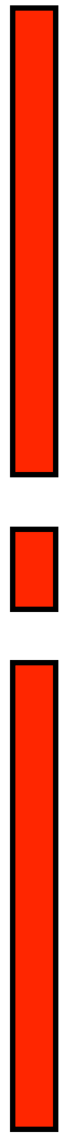


Amplitude can be positive (water is above sea level)
or negative (water is below sea level)



Form of the interference pattern



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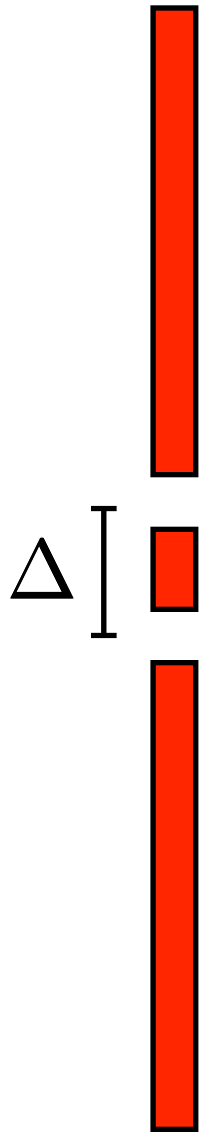

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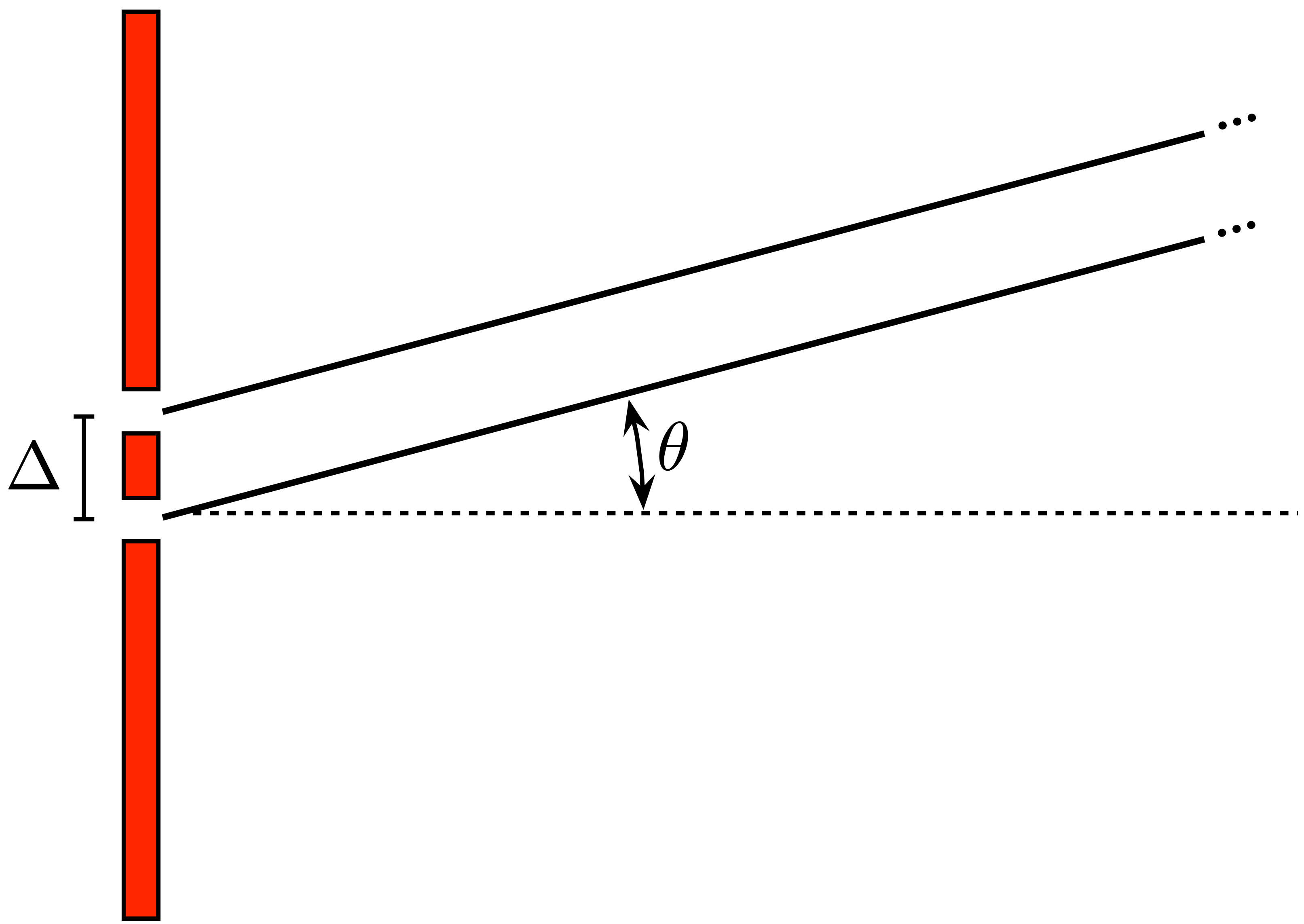

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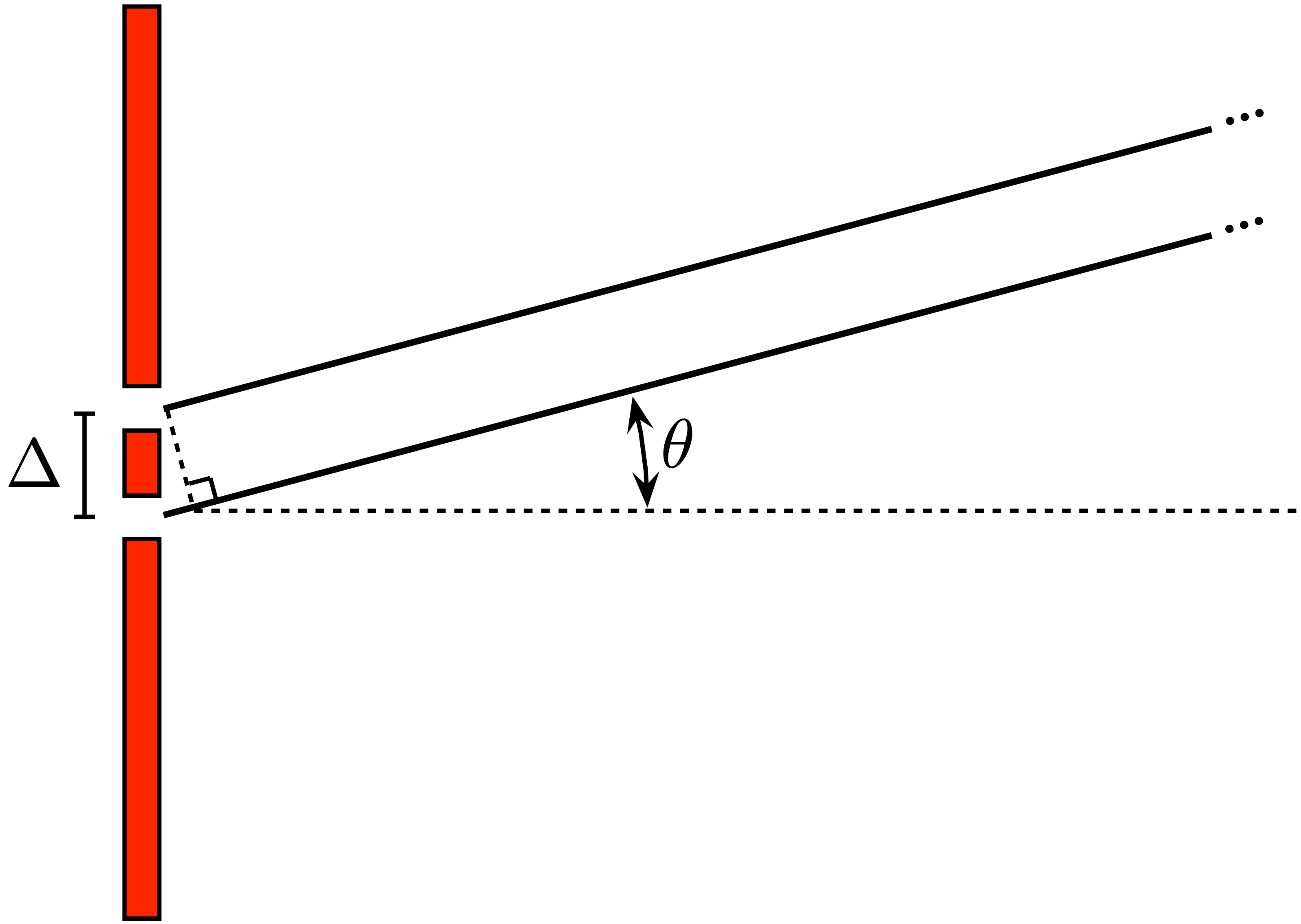

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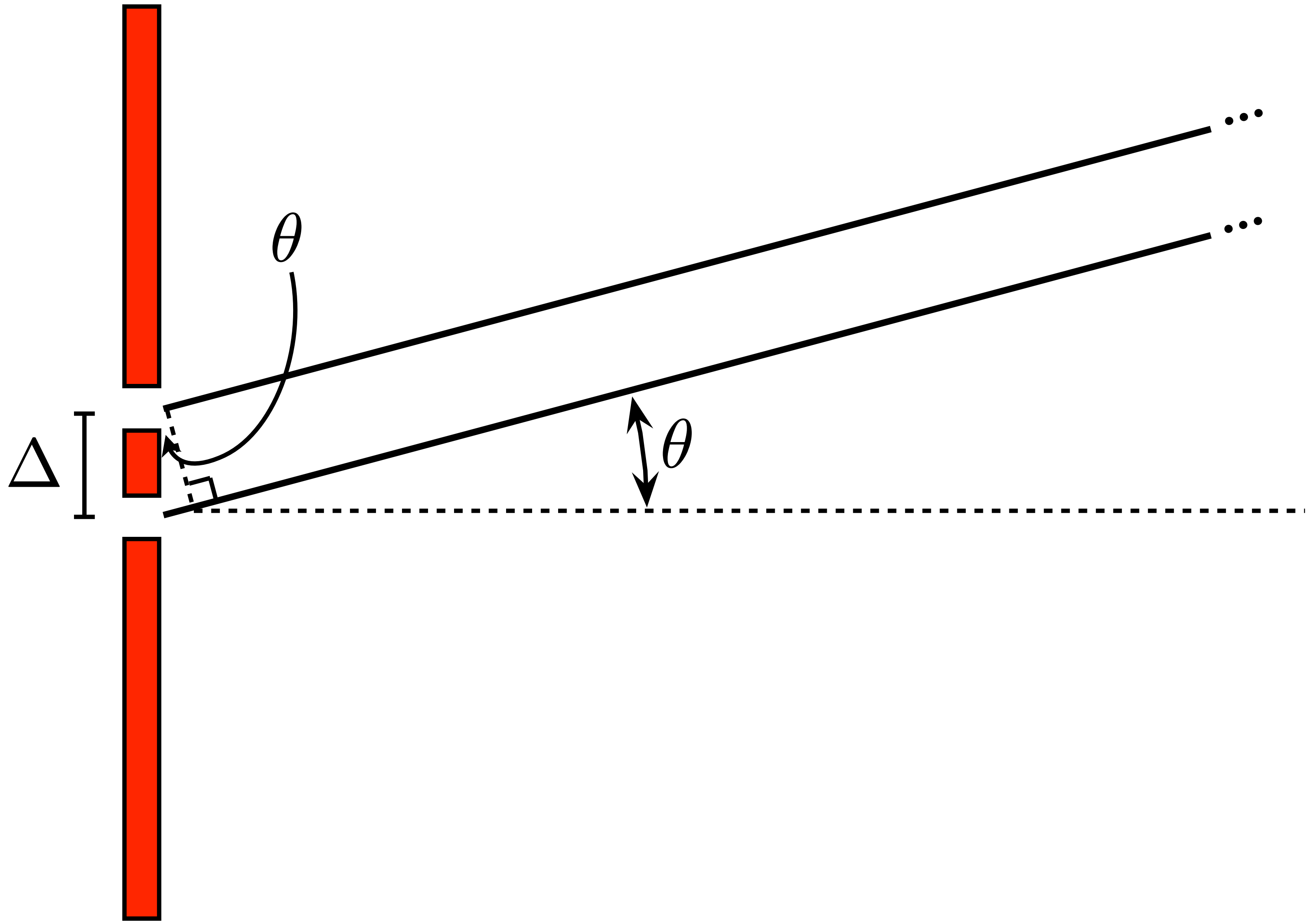

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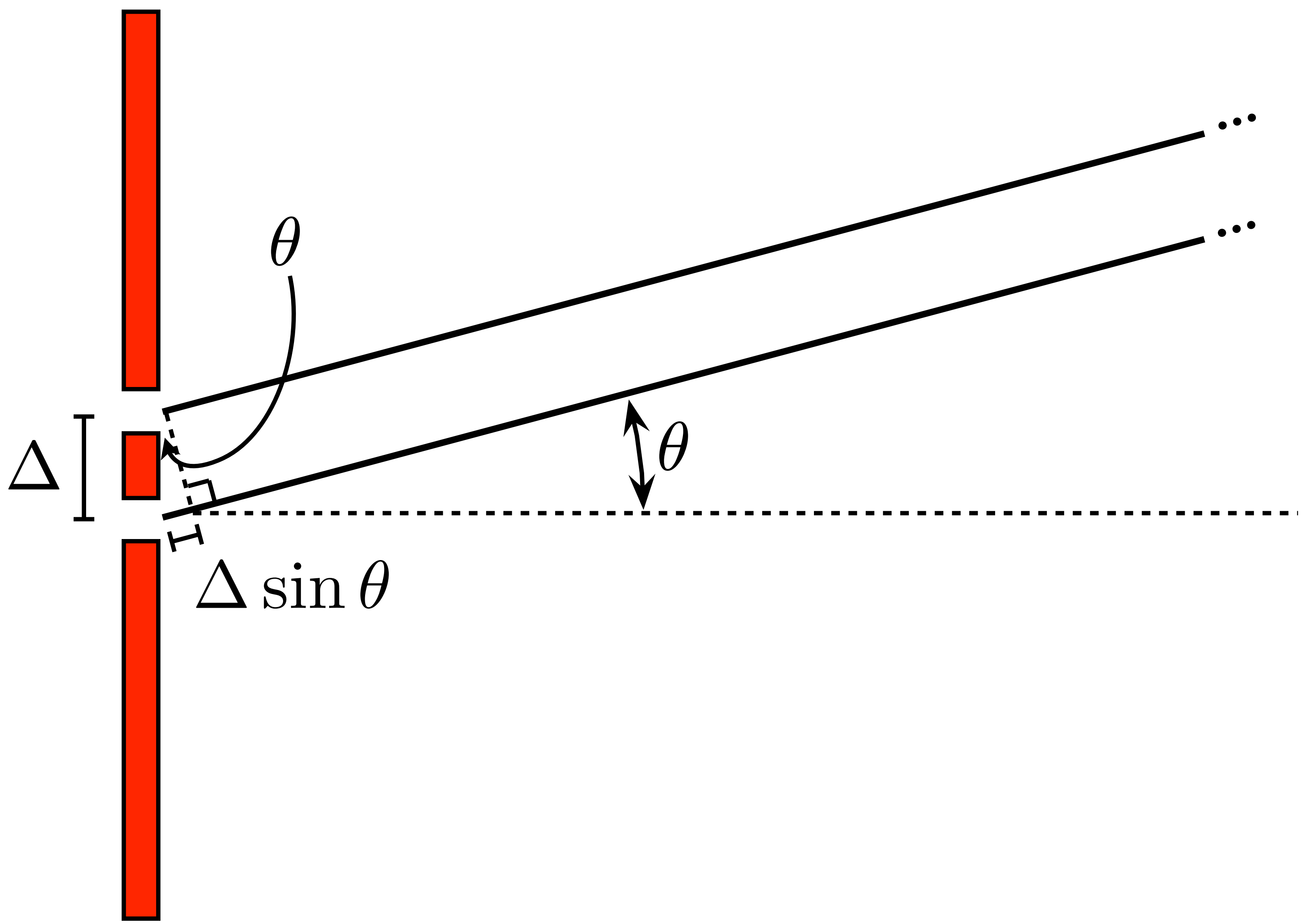

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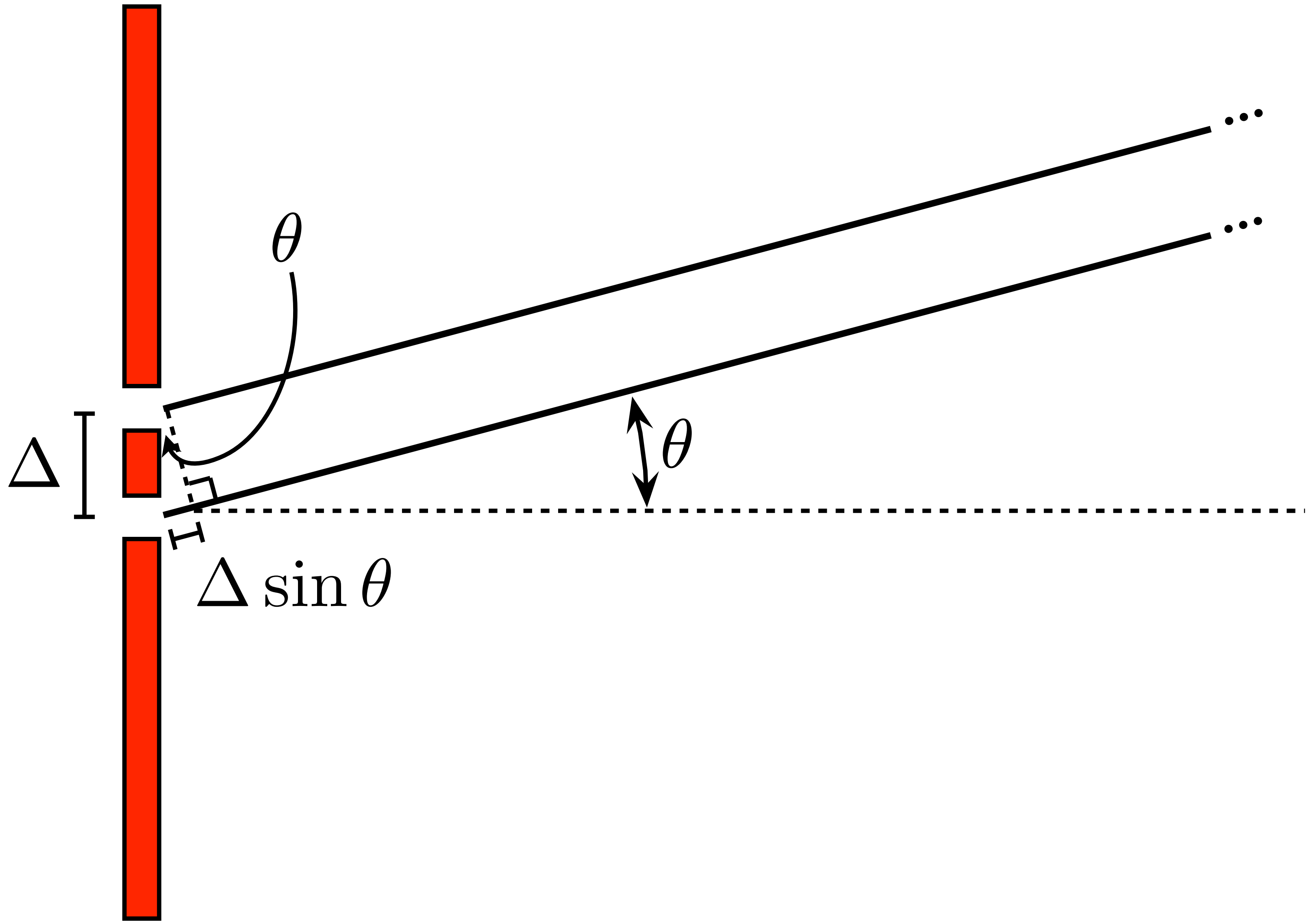

One fringe: $\Delta \sin \theta = \lambda$

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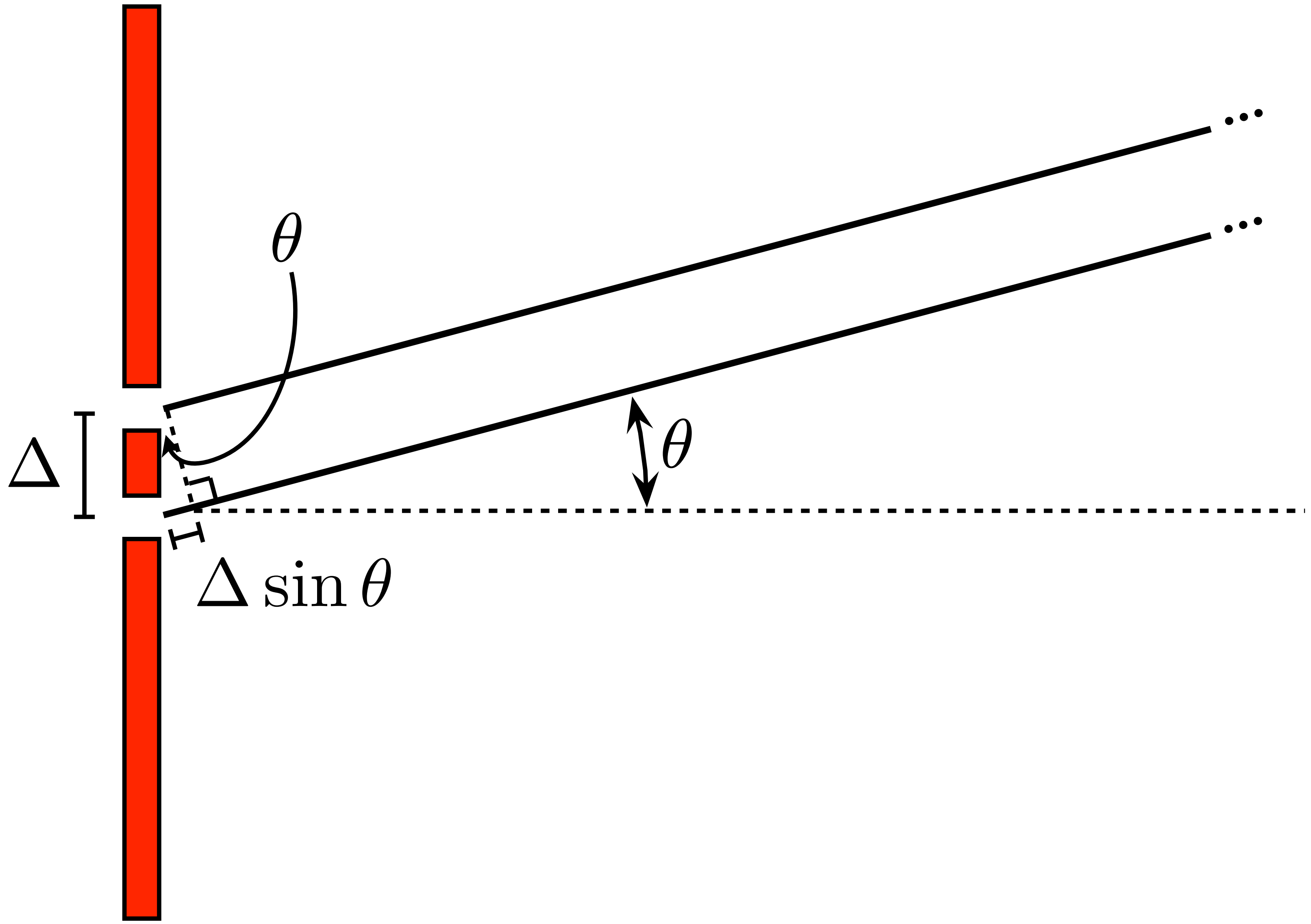

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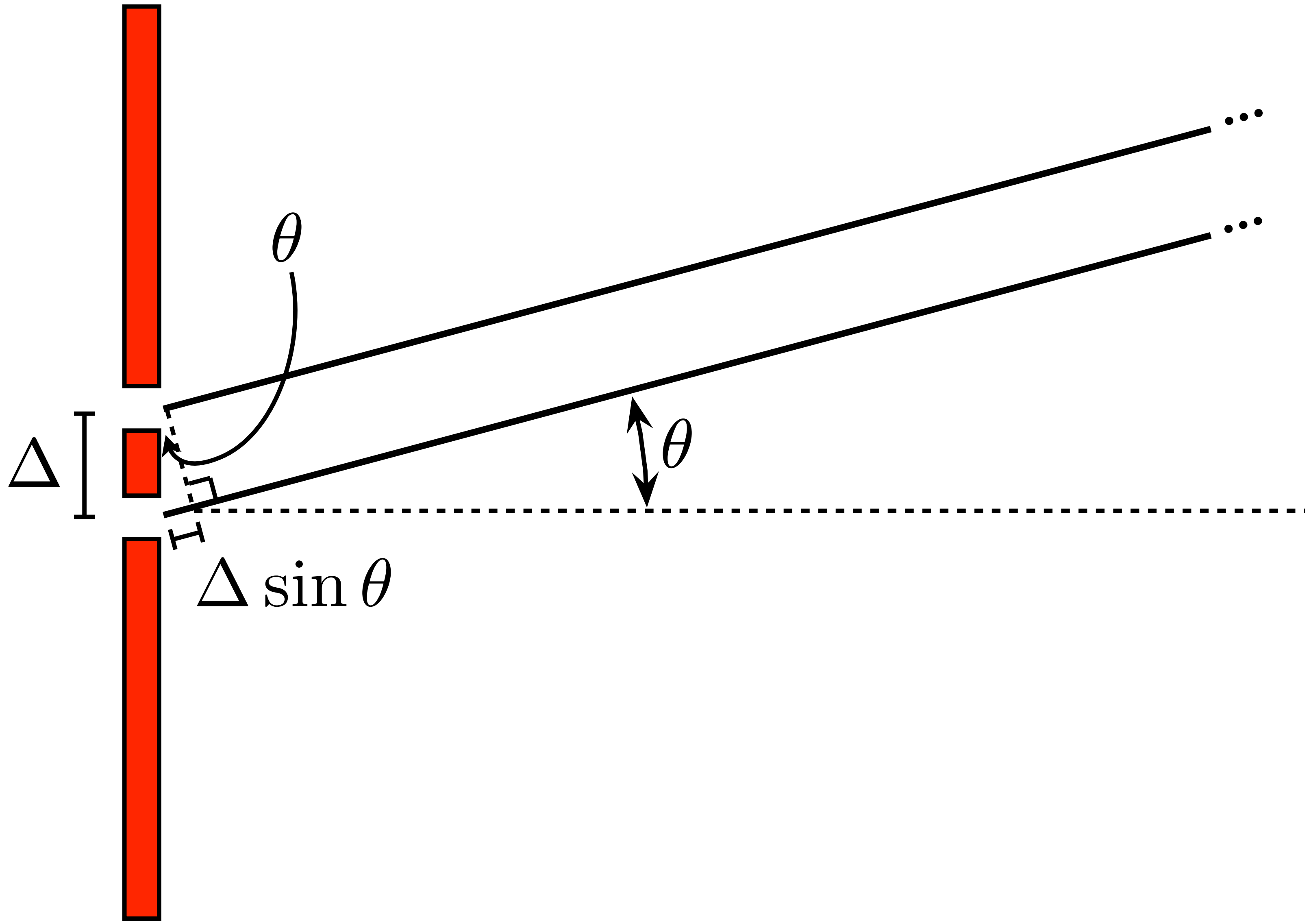

One fringe: $\Delta \sin\theta = \lambda \qquad \theta \approx \dfrac{\lambda}{\Delta}$

Screen at a distance d away: fringe spacing is approximately $d \cdot \theta \approx \dfrac{d \cdot \lambda}{\Delta}$

Double slit with electrons

How will the experiment behave if we use electrons instead of bullets or water waves?

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Same behavior with light, which is composed of individual photons.

Uncertainty principle

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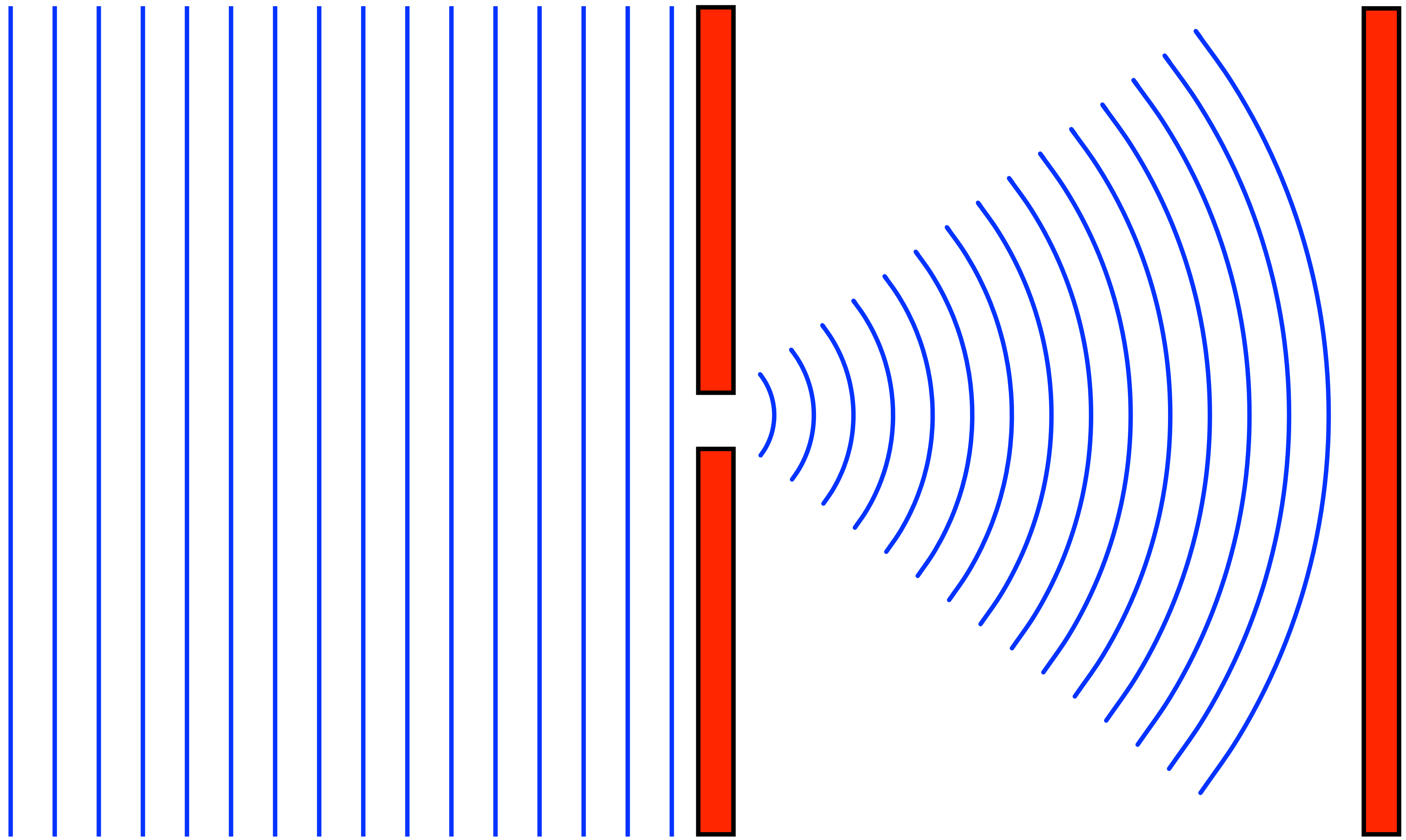
Heisenberg uncertainty principle: $\Delta x \, \Delta p \geq \frac{\hbar}{2}$ $\qquad (\hbar = \frac{h}{2\pi})$

Uncertainty principle and diffraction



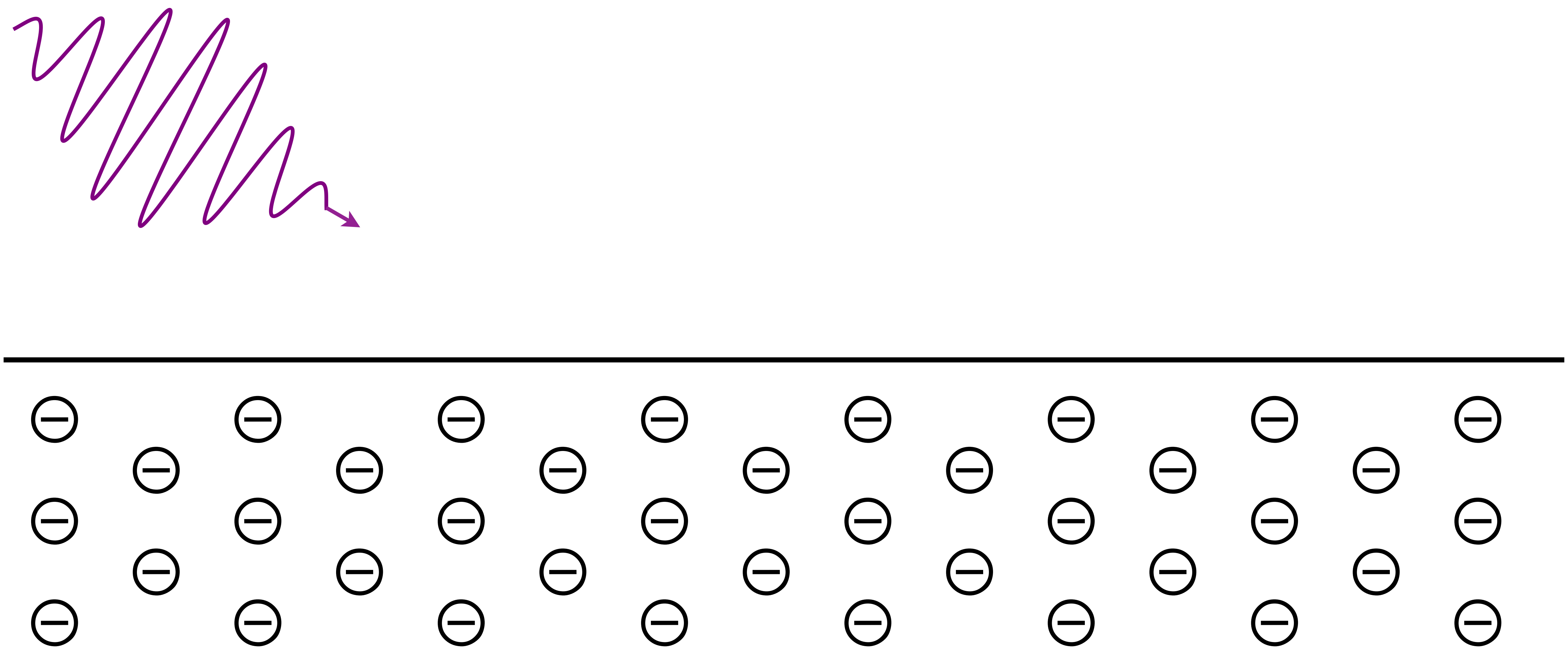
Exercise: Double slit with laser light

Suppose you perform the double slit experiment using a green laser with a wavelength of 523 nm and slits spaced by 1 mm.

a. What is the angular spacing between two adjacent fringes of the interference pattern?
b. If the pattern is projected onto a screen at a distance of 5 m, what is the distance between adjacent fringes?

Photoelectric effect

The photoelectric effect



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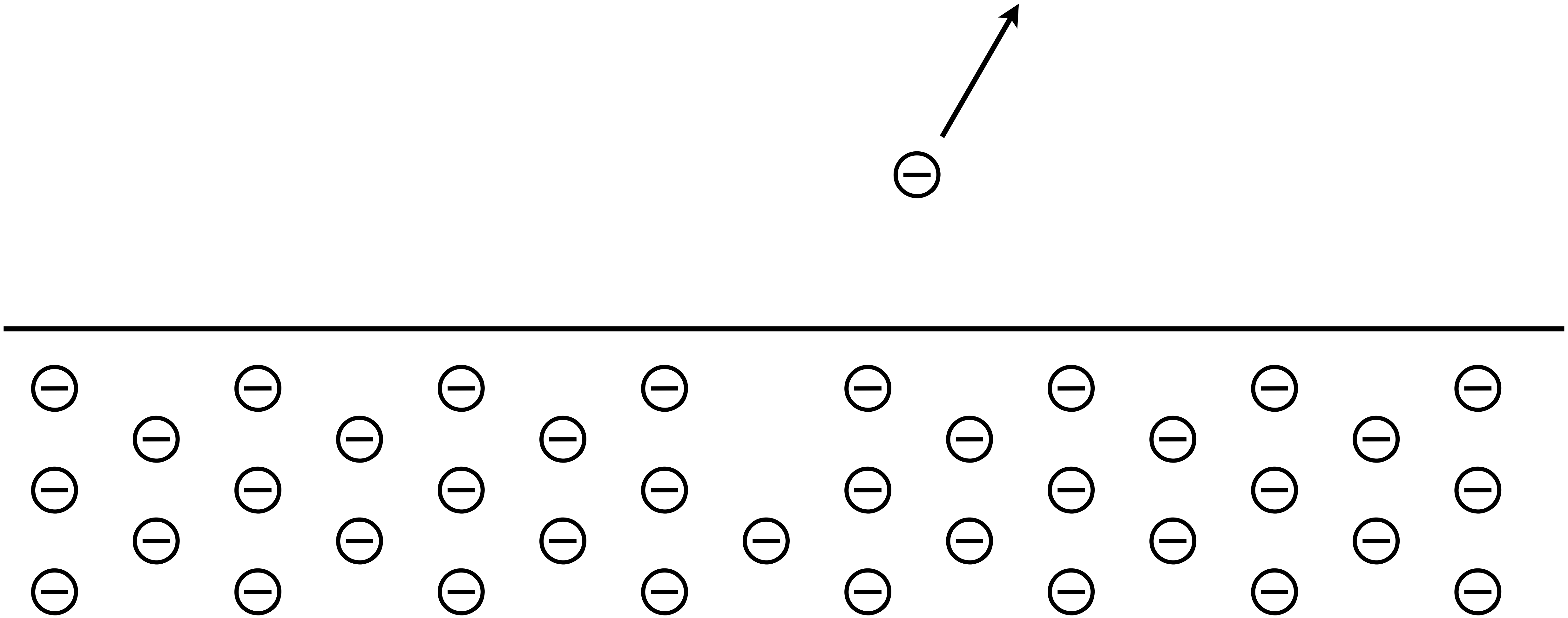

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Electrons are massive particles with negative electric charge.

$$E = \frac{1}{2}mv^2 + \phi$$

$$m = 9.11 \times 10^{-31}\,\mathrm{kg}$$
$$e = 1.60 \times 10^{-19}\,\mathrm{C}$$

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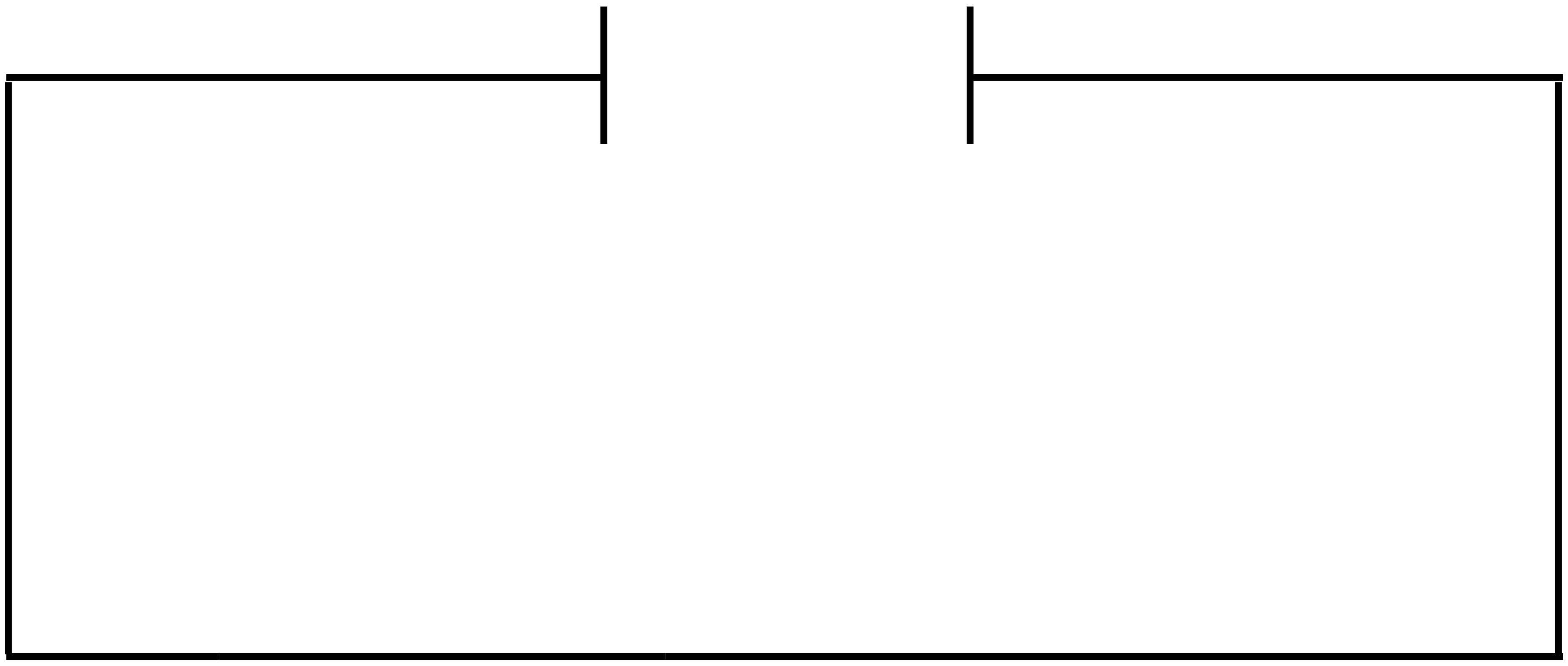
Whether emission occurs depends on the frequency of the light, not on its intensity (the number of photons arriving per unit time).

Photoelectric effect experiment



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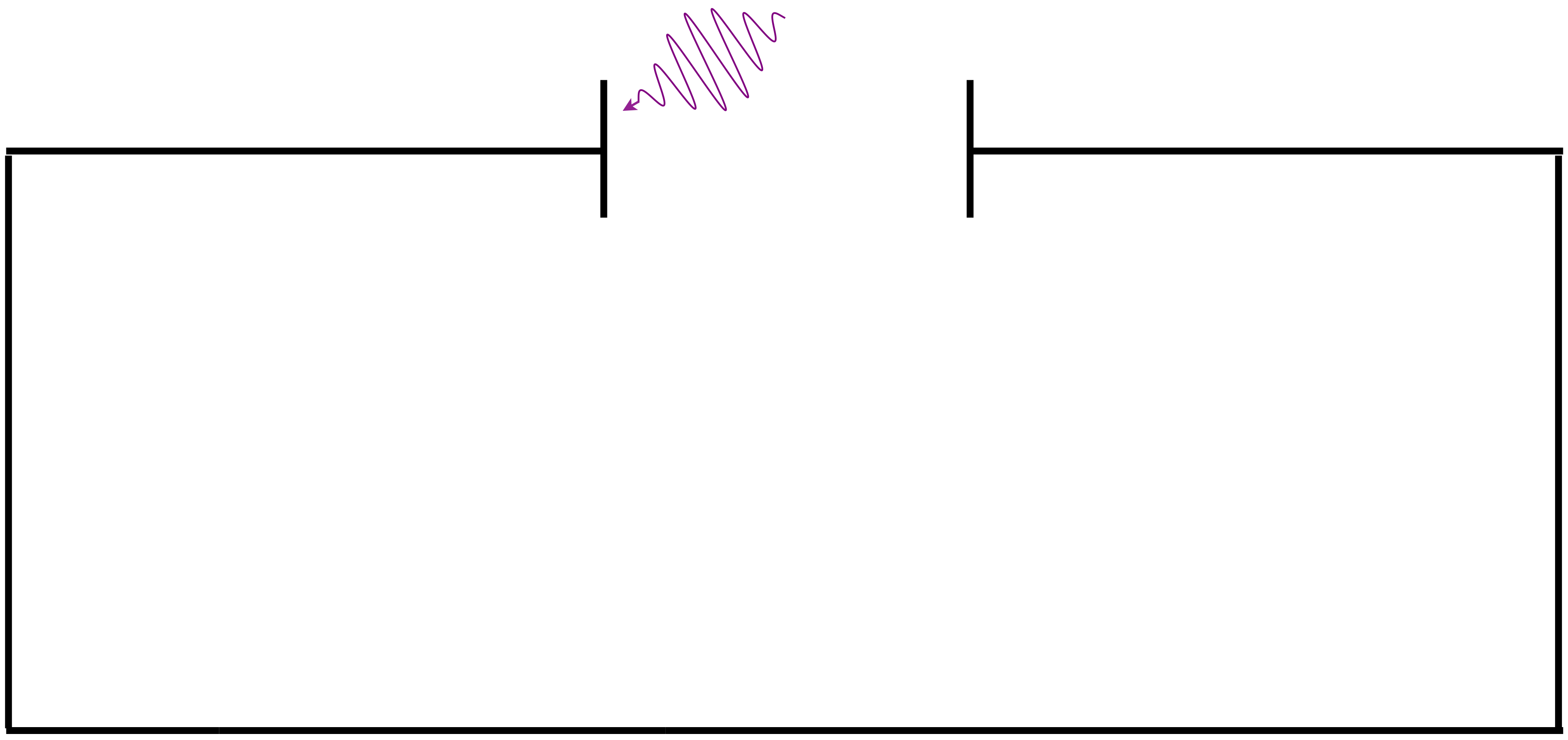

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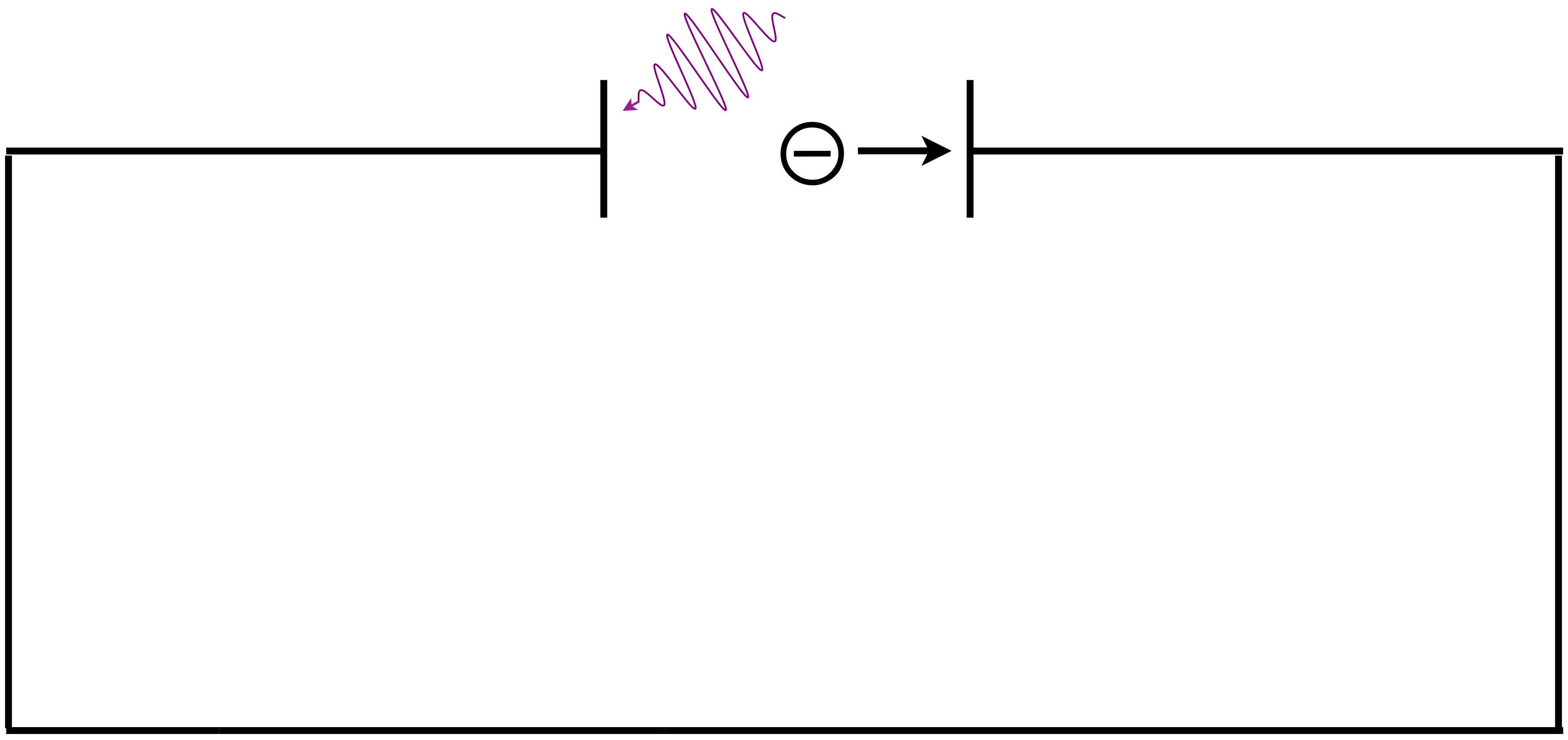

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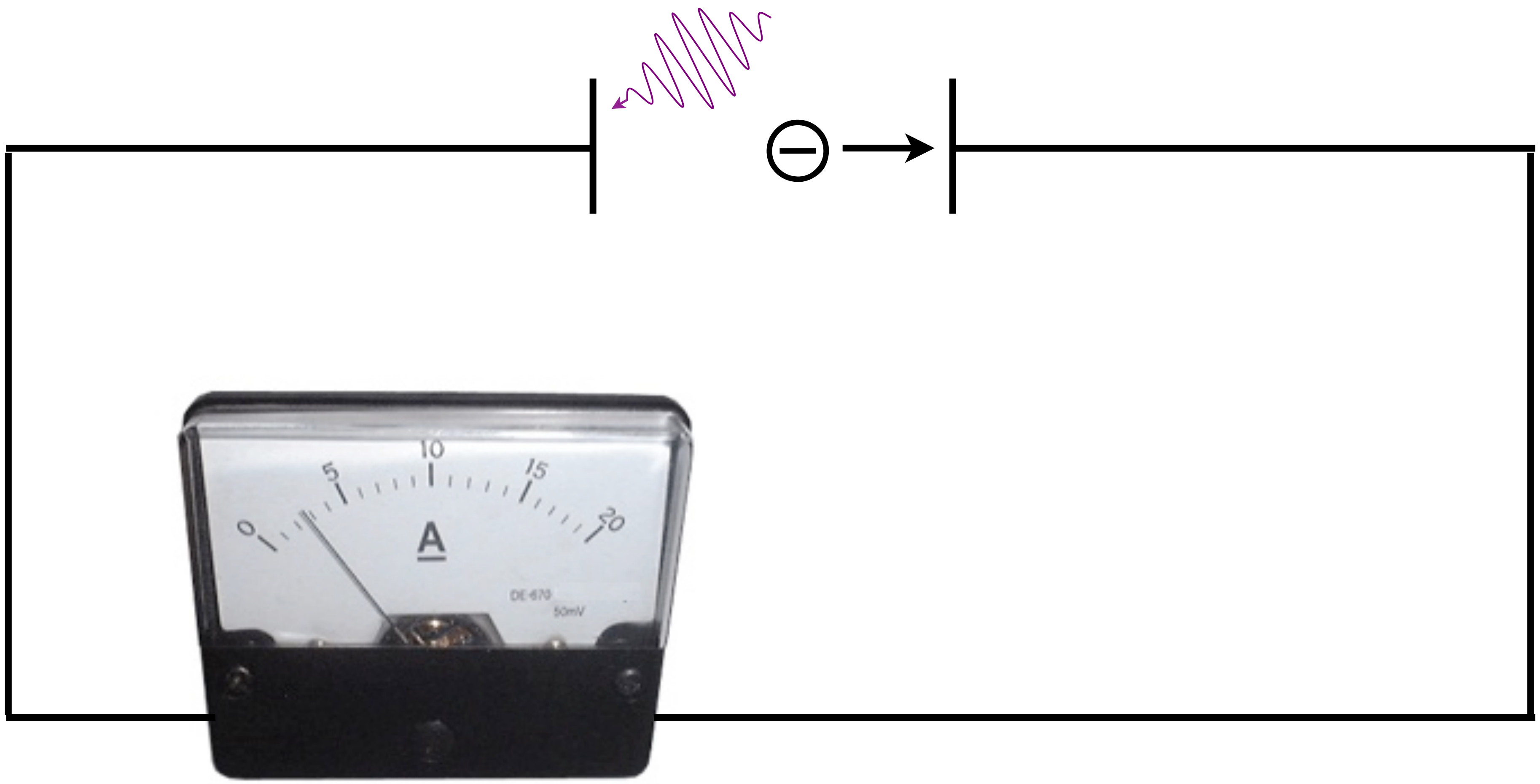

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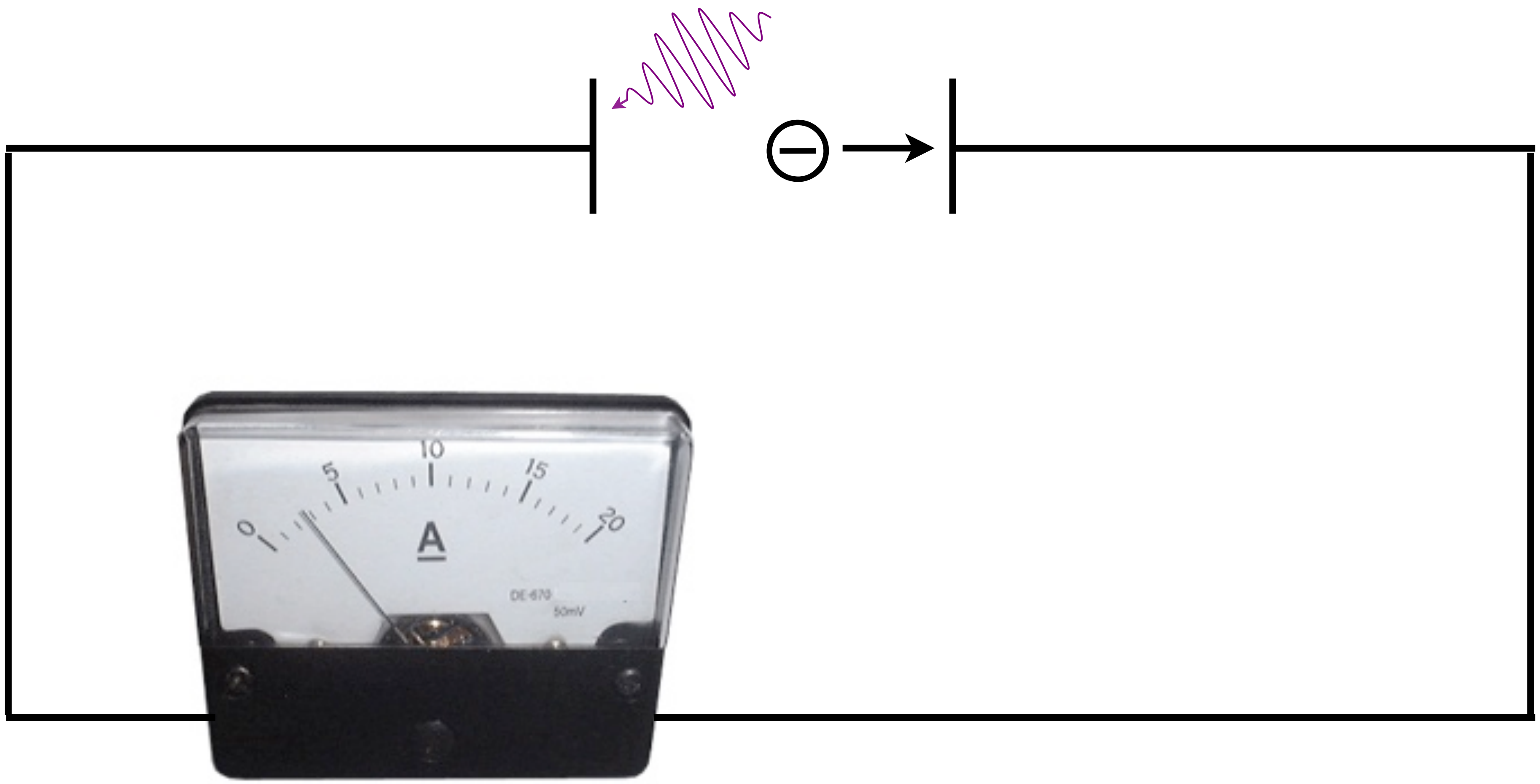

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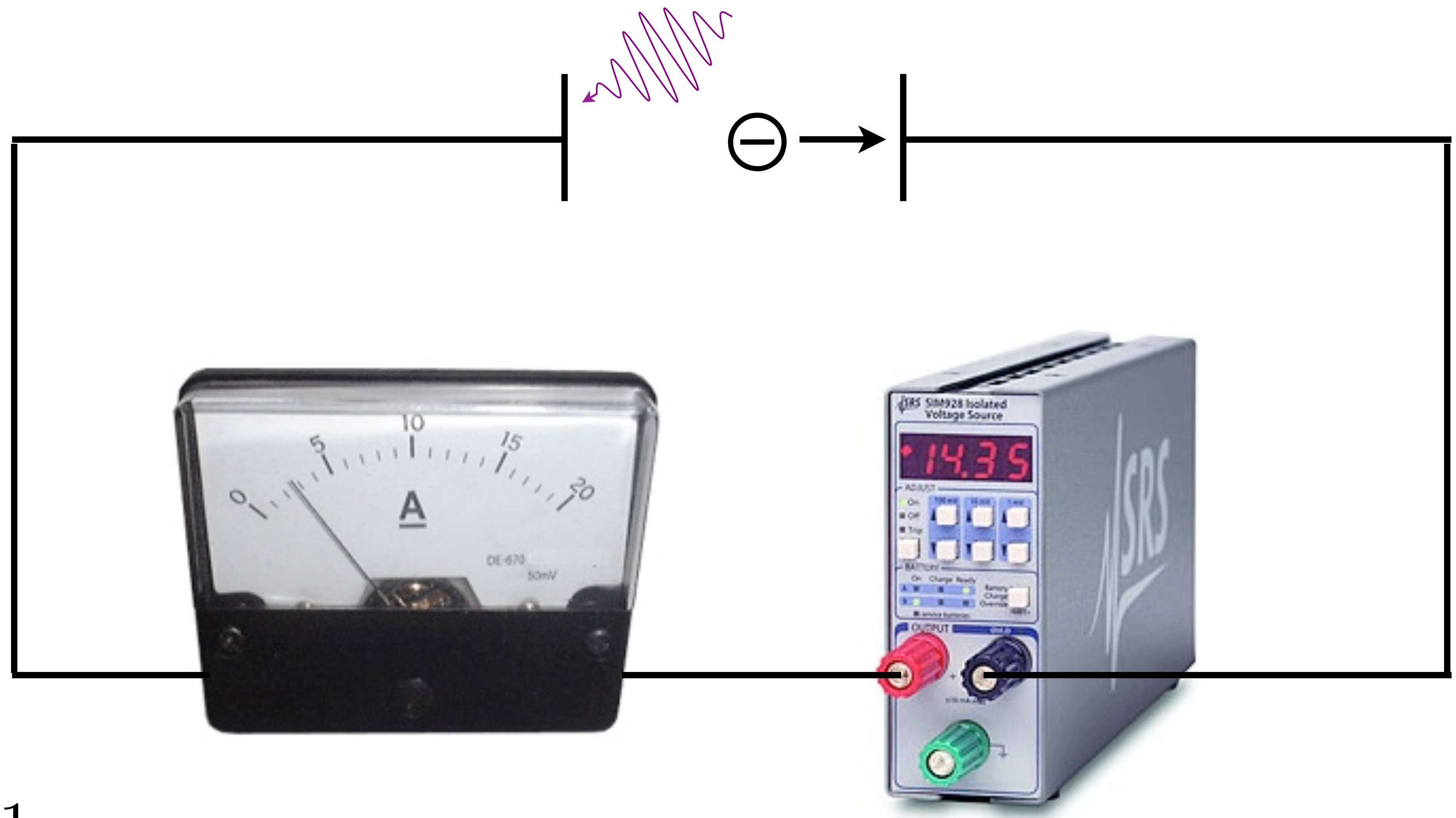

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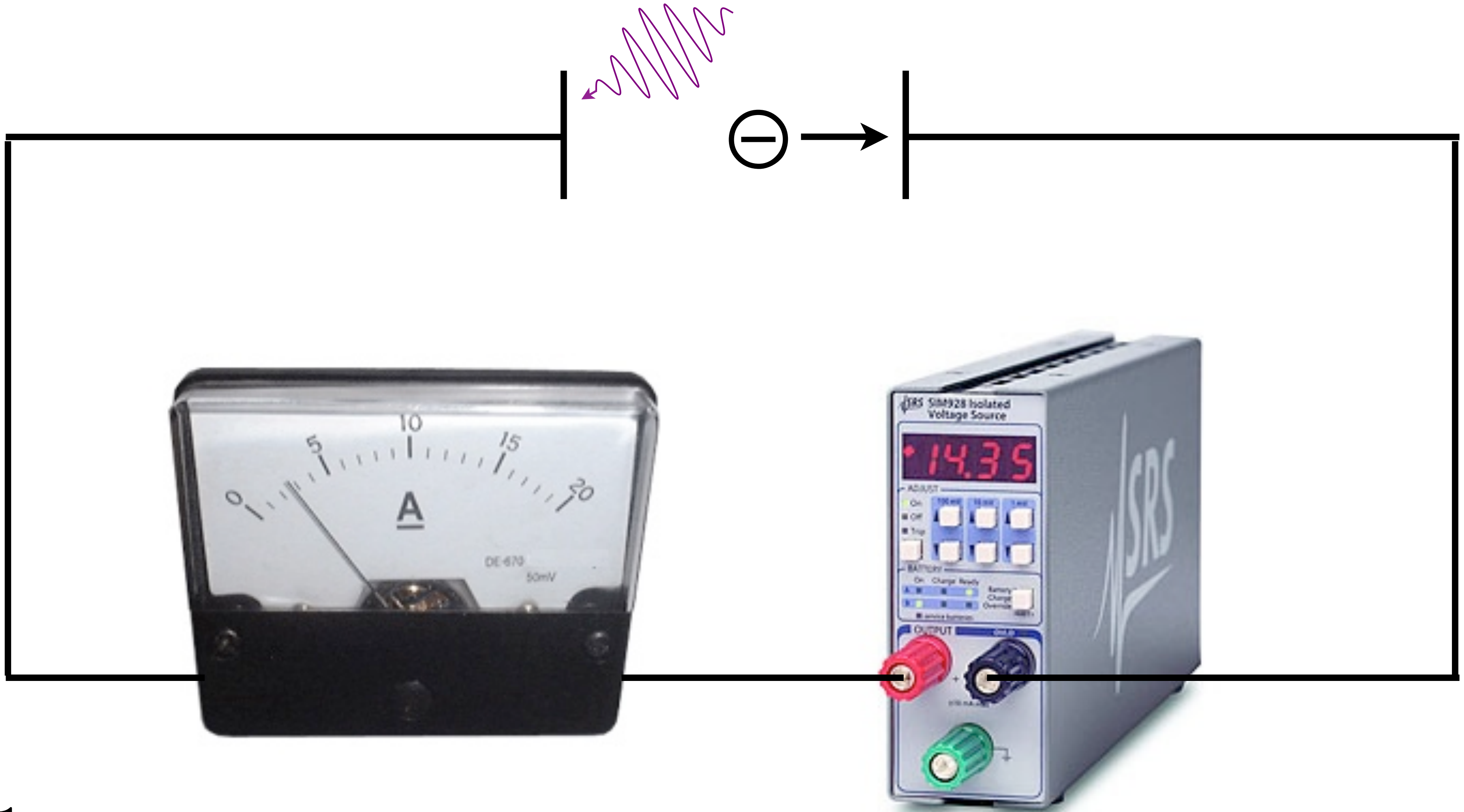

$$h\nu = \frac{1}{2}mv^2 + \phi$$

$$\frac{1}{2}mv^2 = eV_0$$ where V_0 is the *stopping potential*, the voltage that must be applied to stop the current from flowing

Photoelectric effect experiment: results

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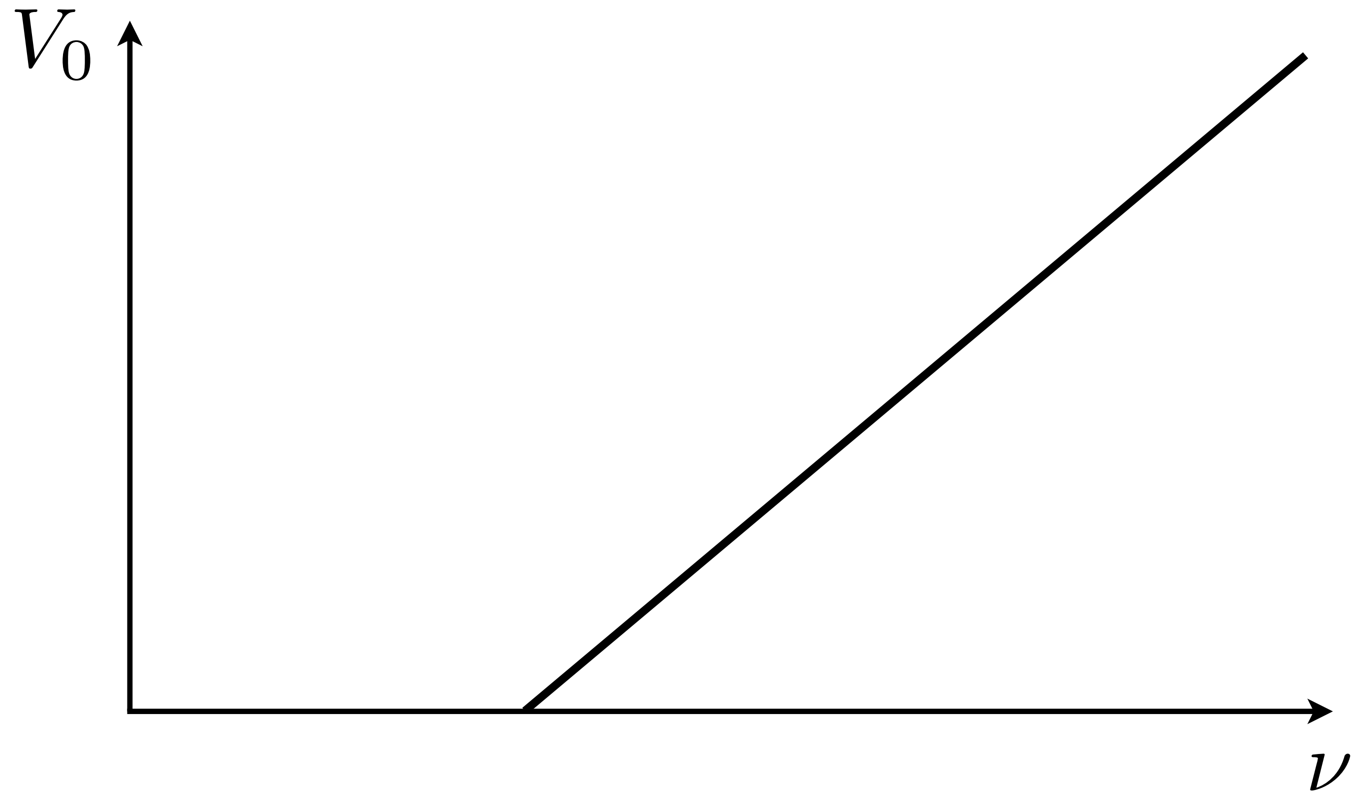

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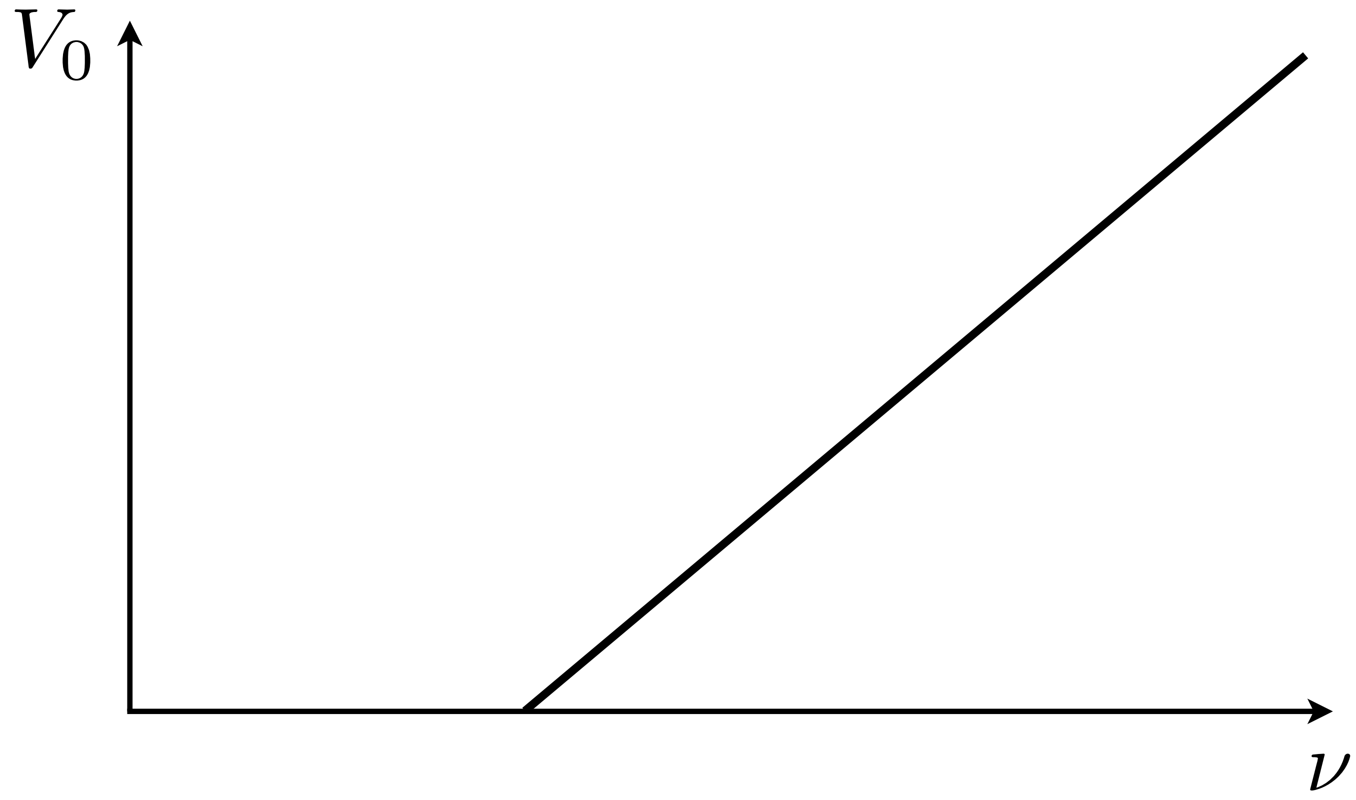

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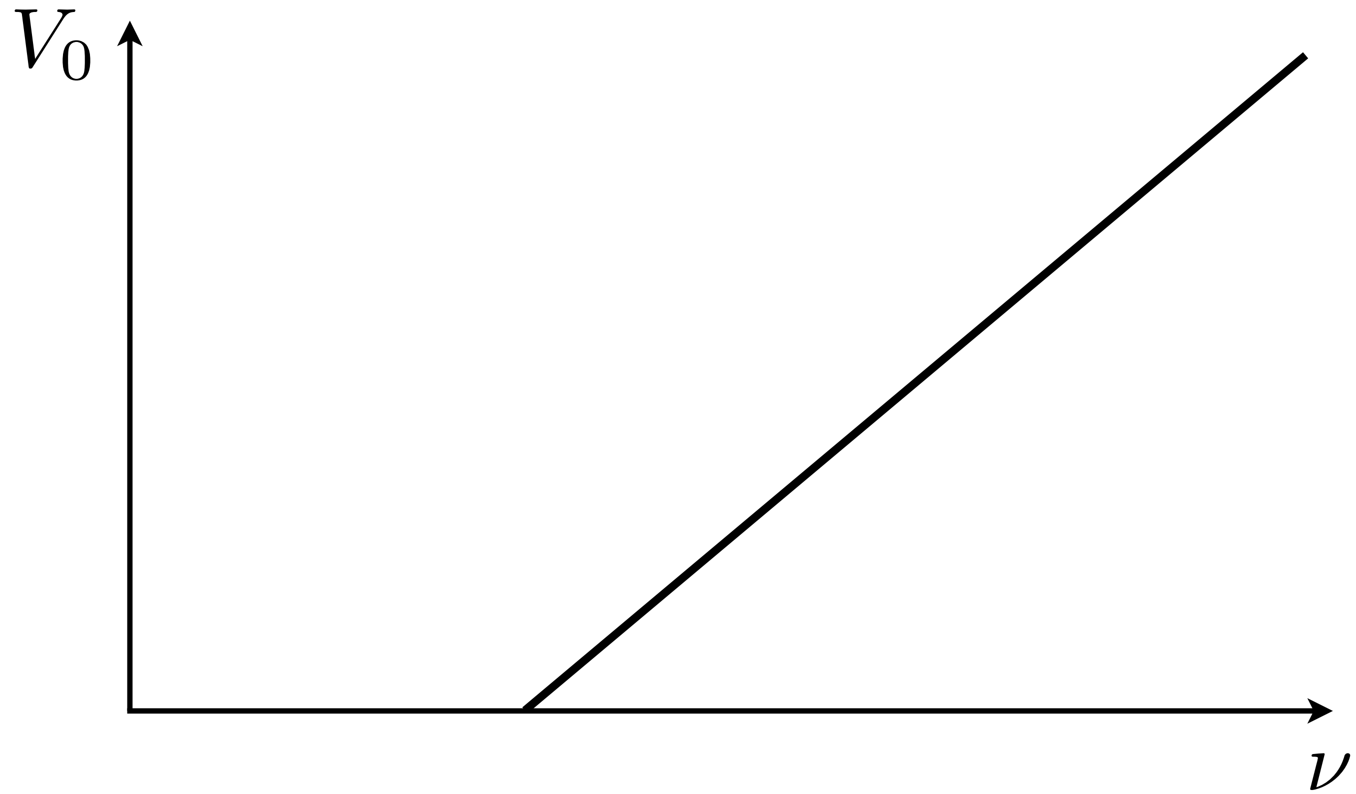

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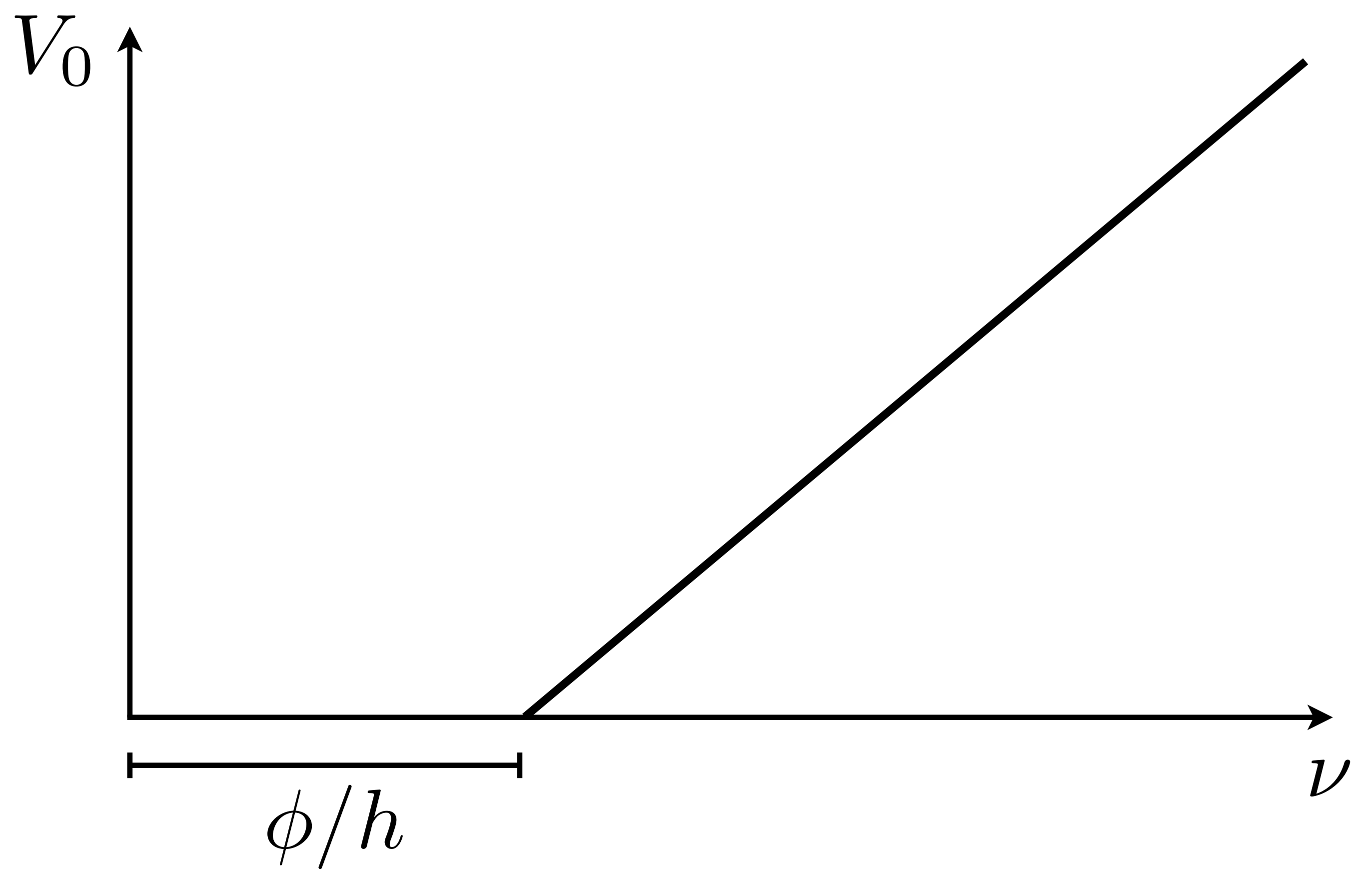

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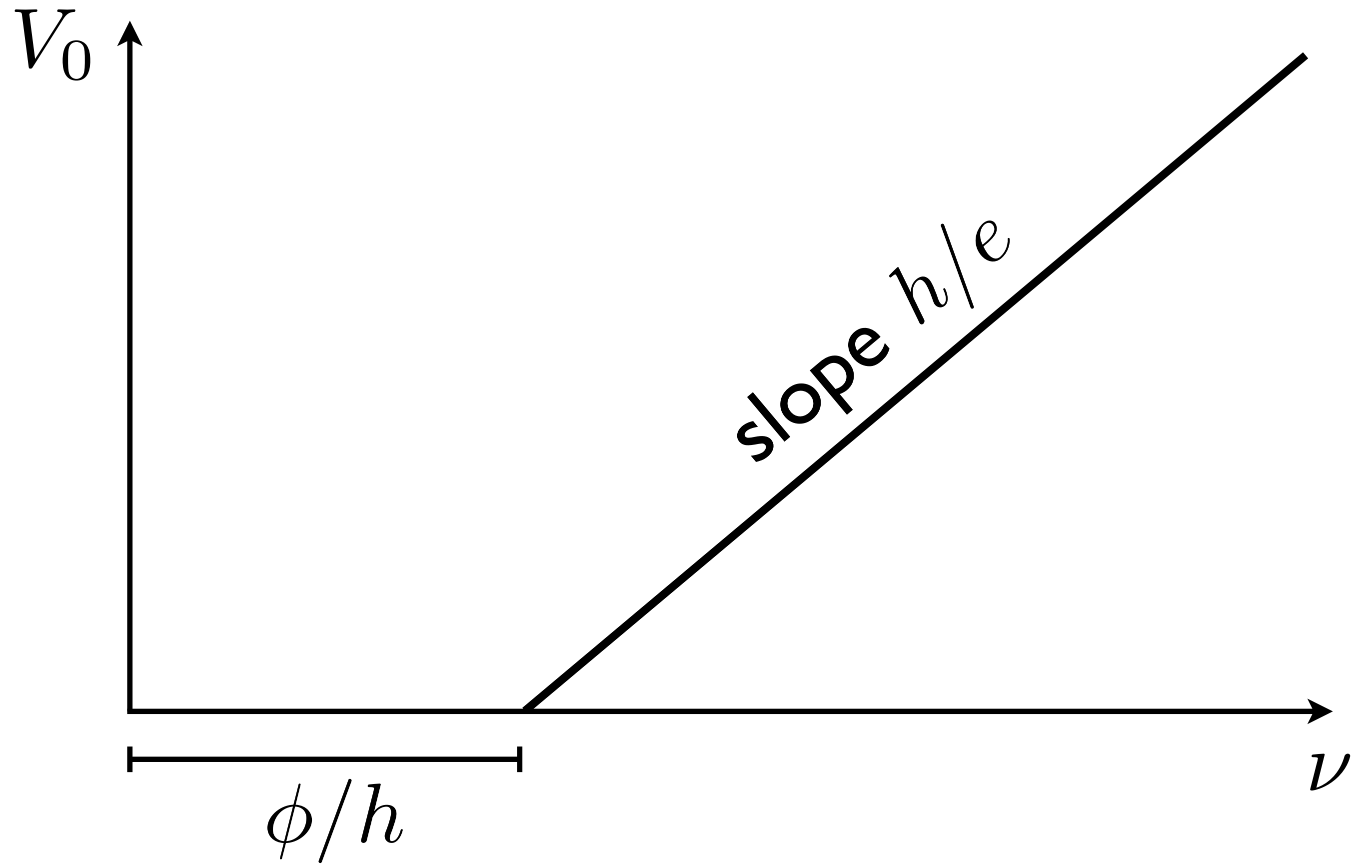

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Exercise: Photoelectric effect in platinum

For this problem, the following values may be useful:

$$h = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$$

$$c = 3 \times 10^{8}\ \mathrm{m/s}$$

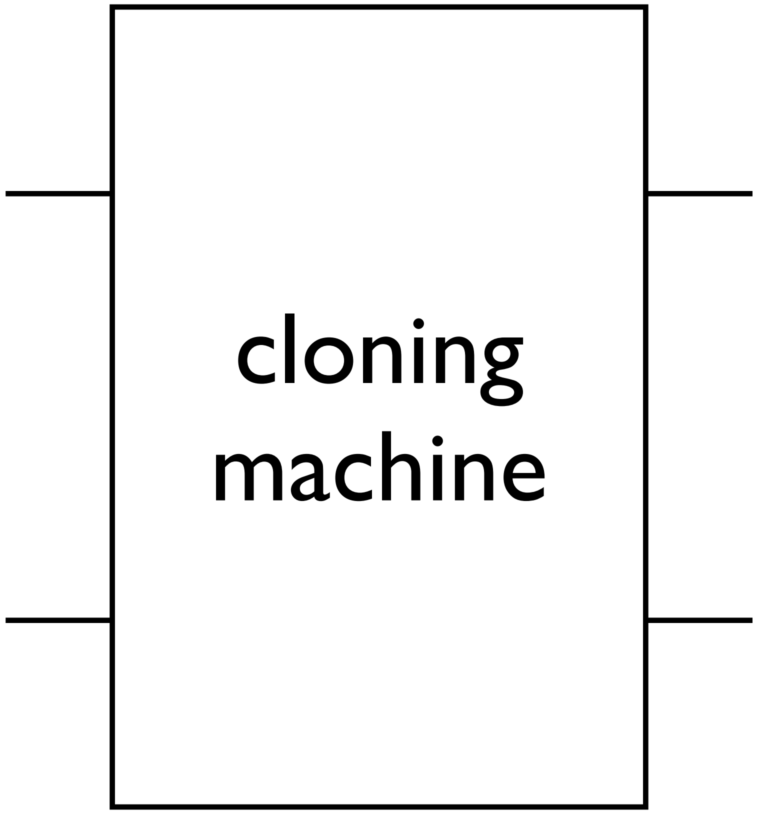
a. When a platinum electrode is illuminated with light of wavelength 150 nm, the stopping potential is 2 V. What is the work function of platinum in eV?

b. What is the maximum wavelength of light that will eject electrons from platinum?

Experiment
(end of the week)

No-cloning theorem

Classical cloning



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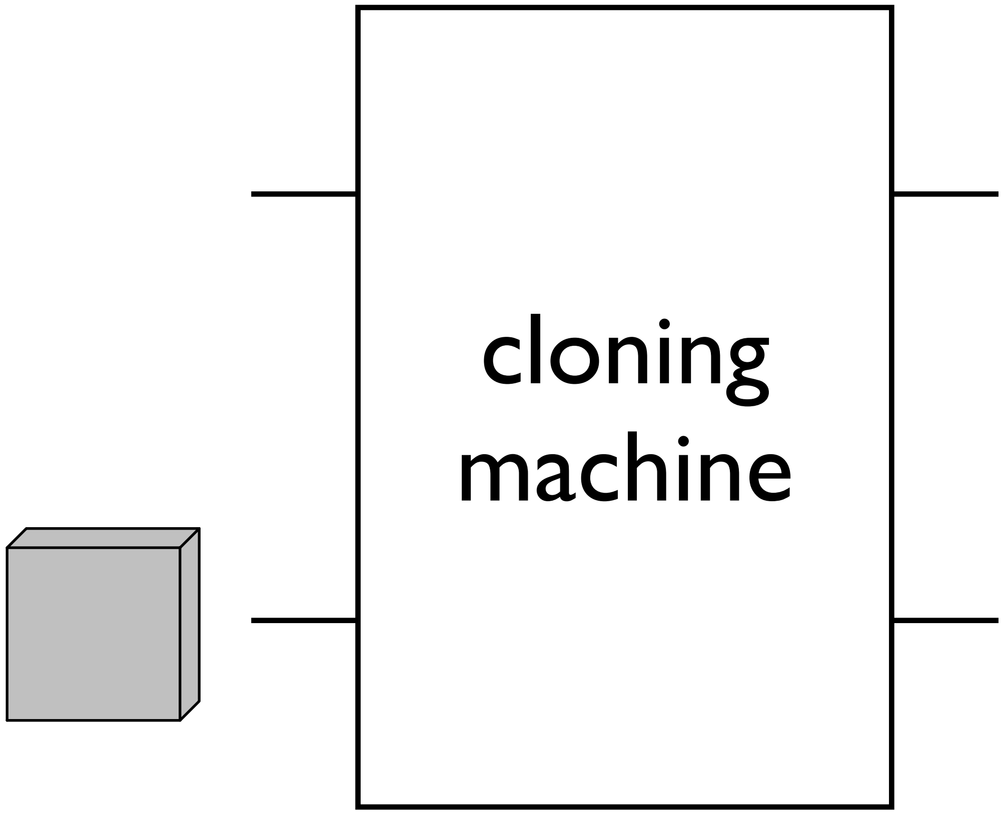

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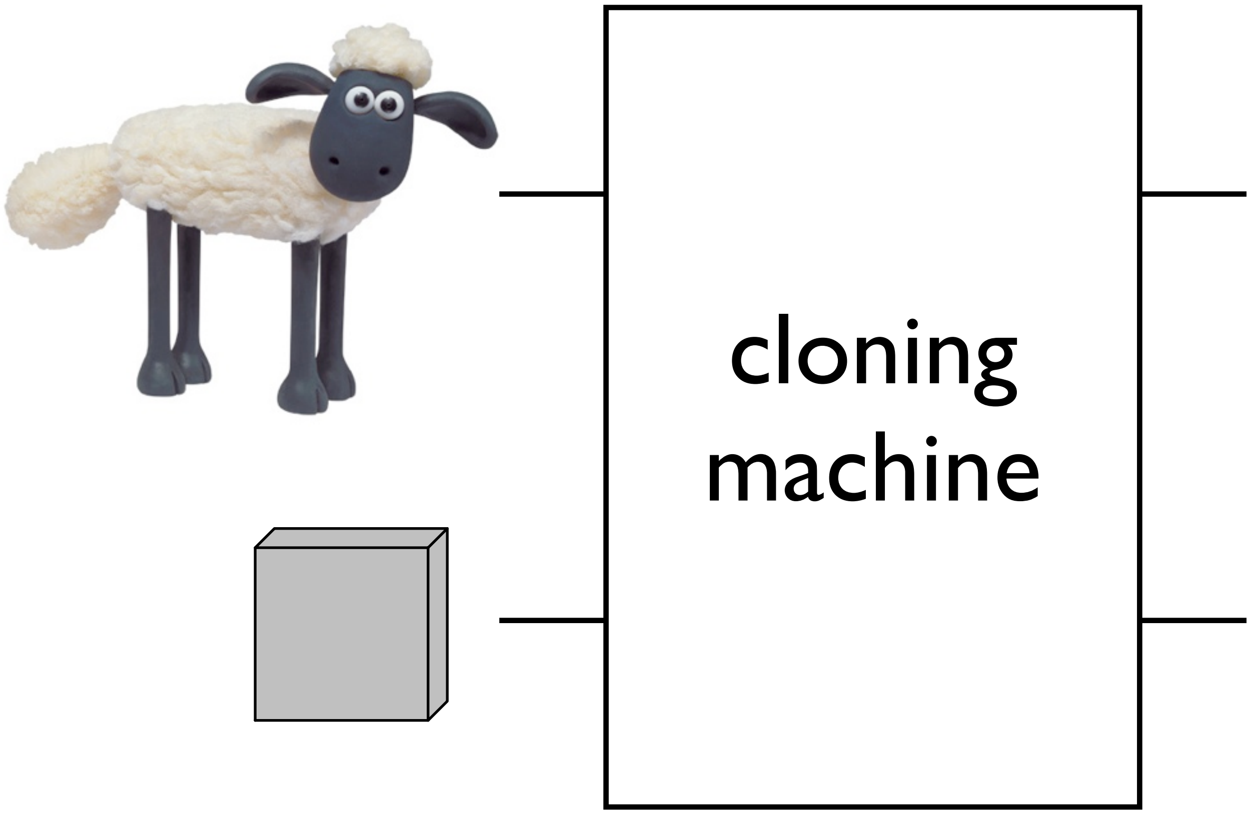

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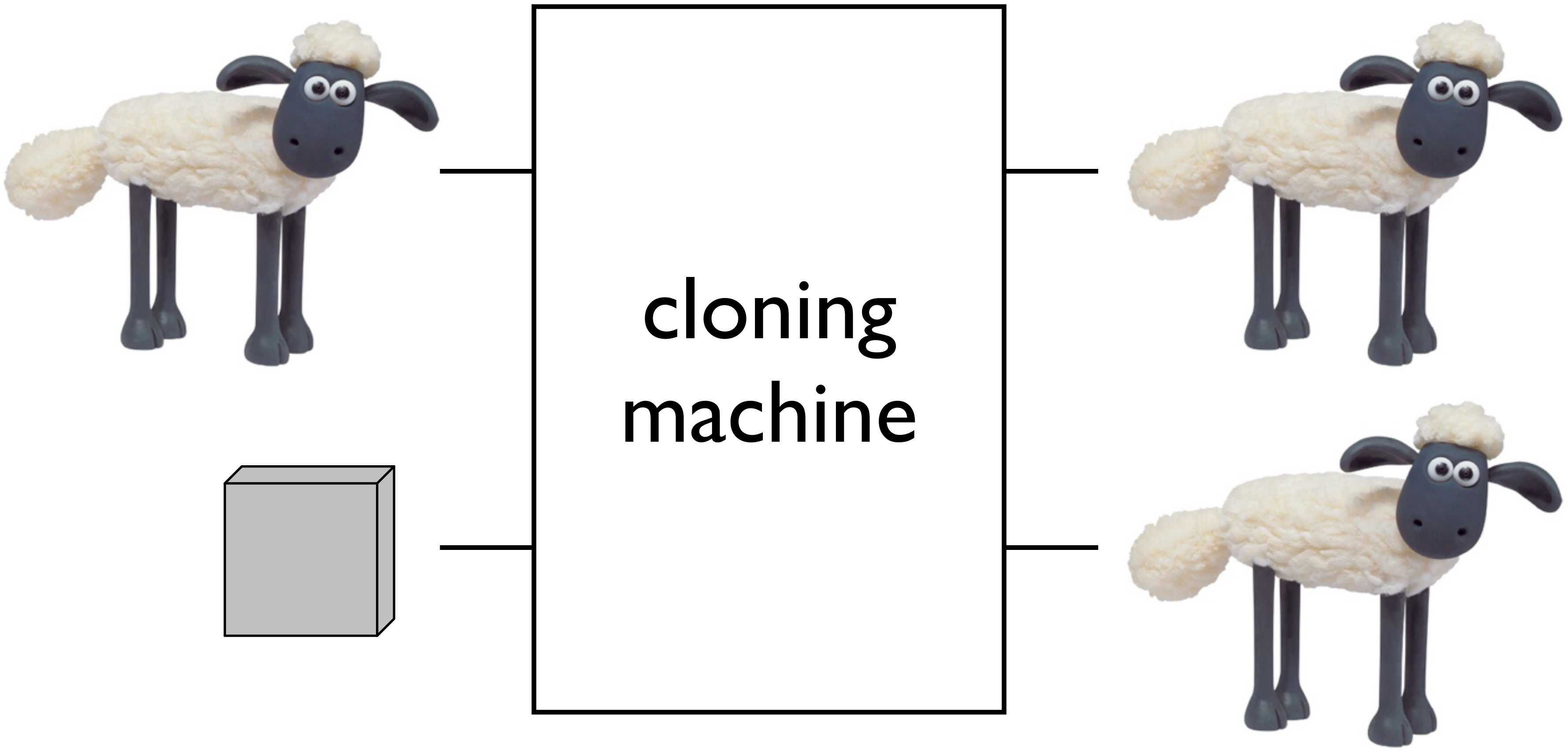

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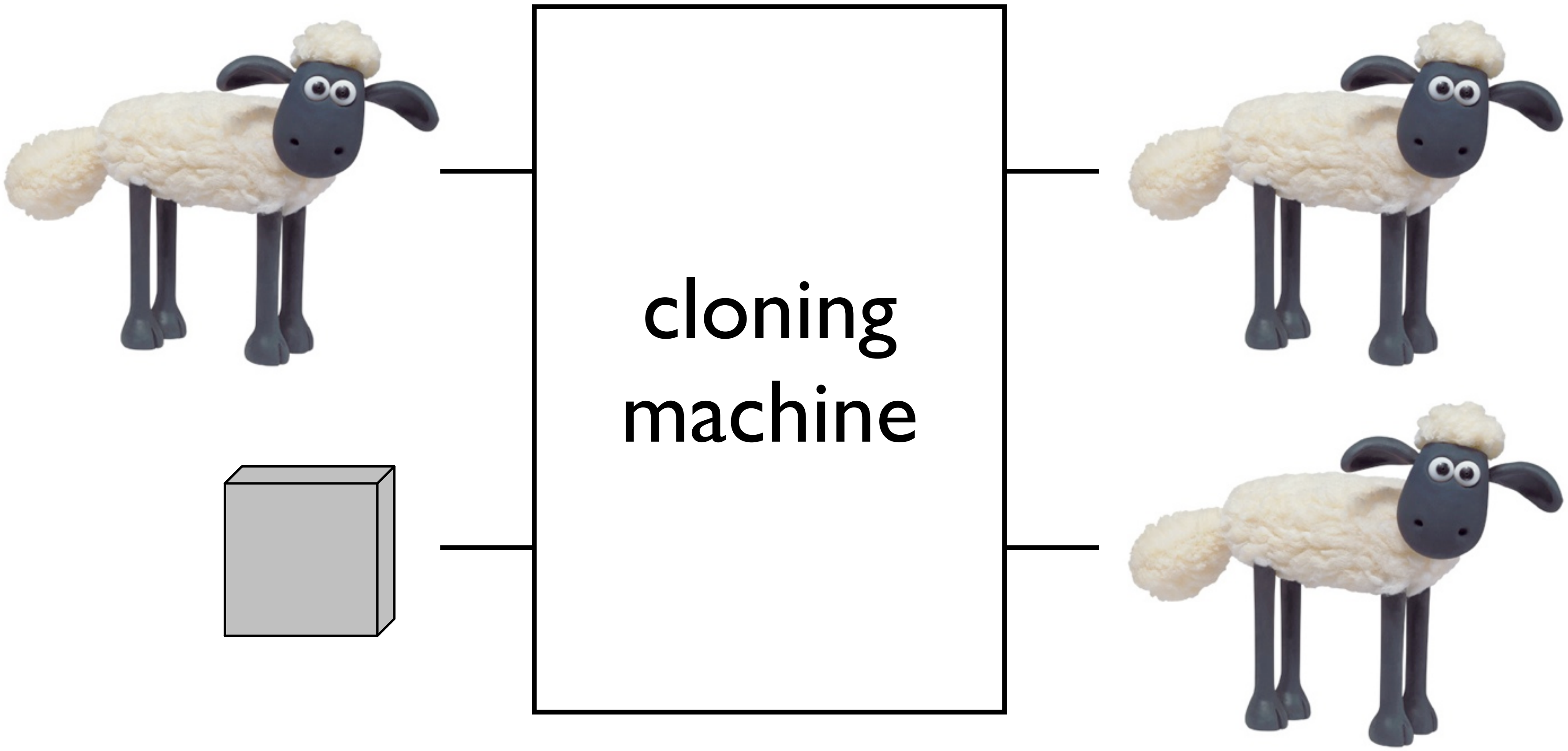
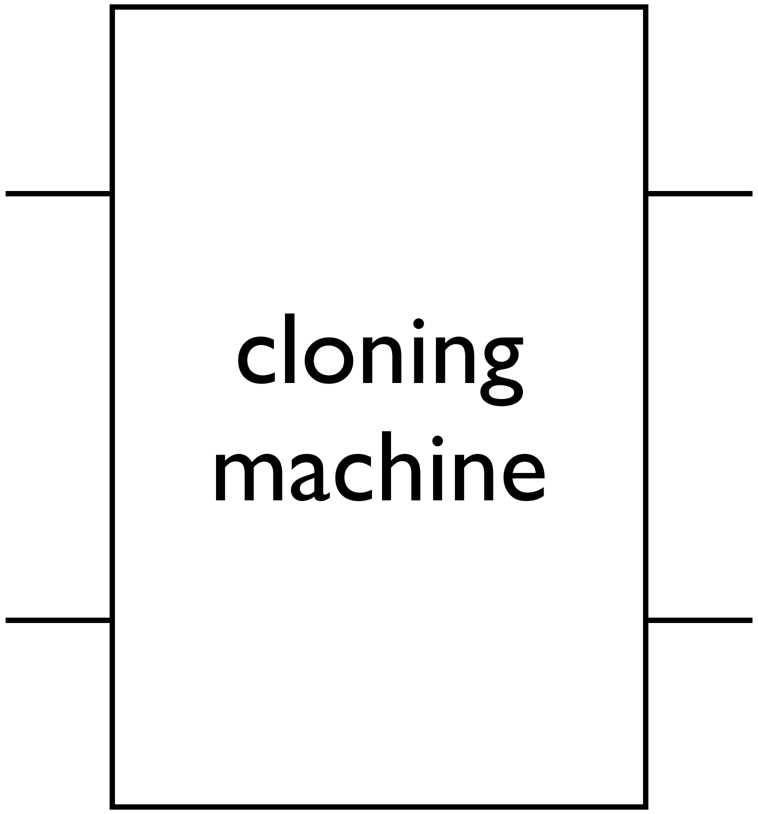


In principle, such a device is possible.

Classical cloning (digital)



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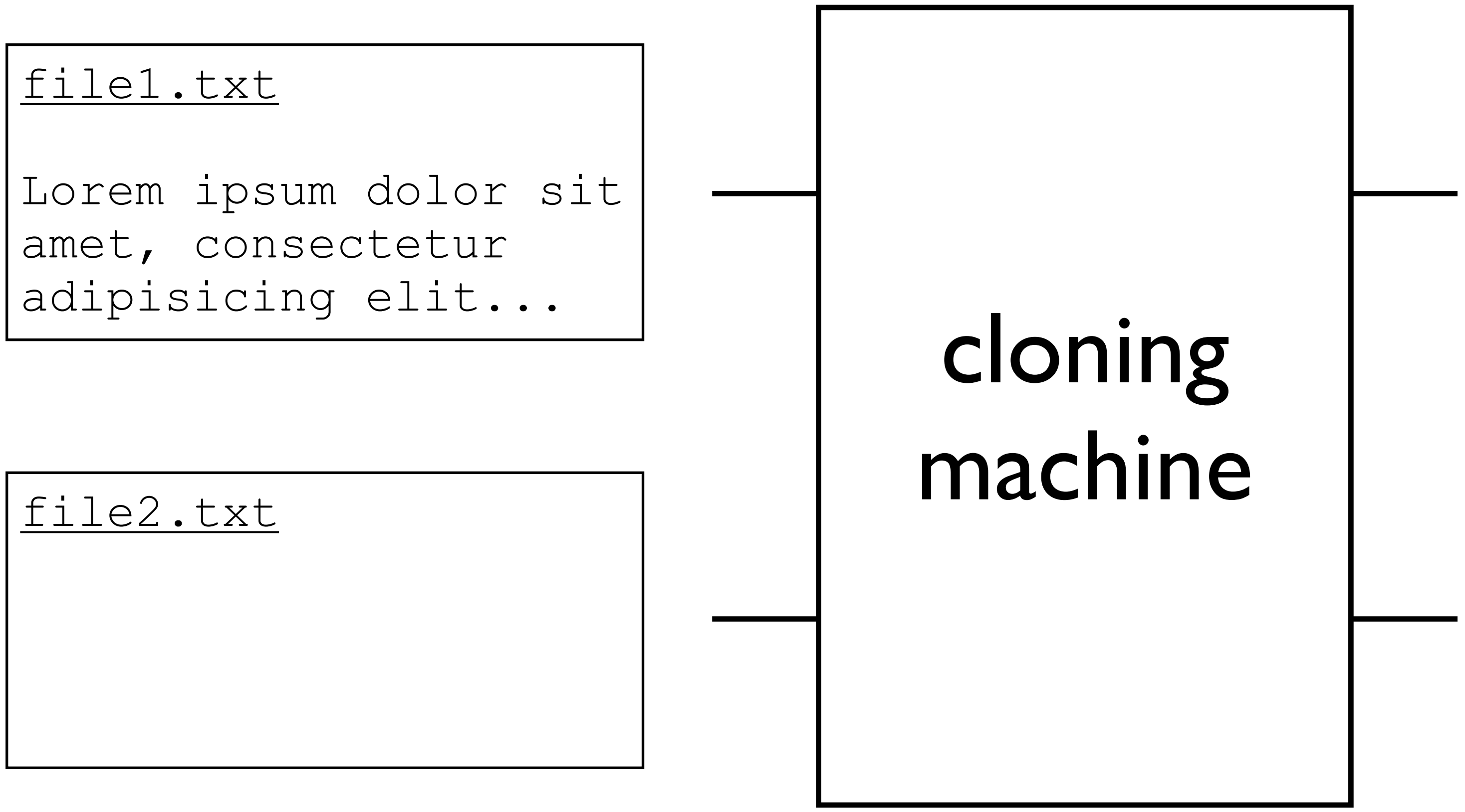

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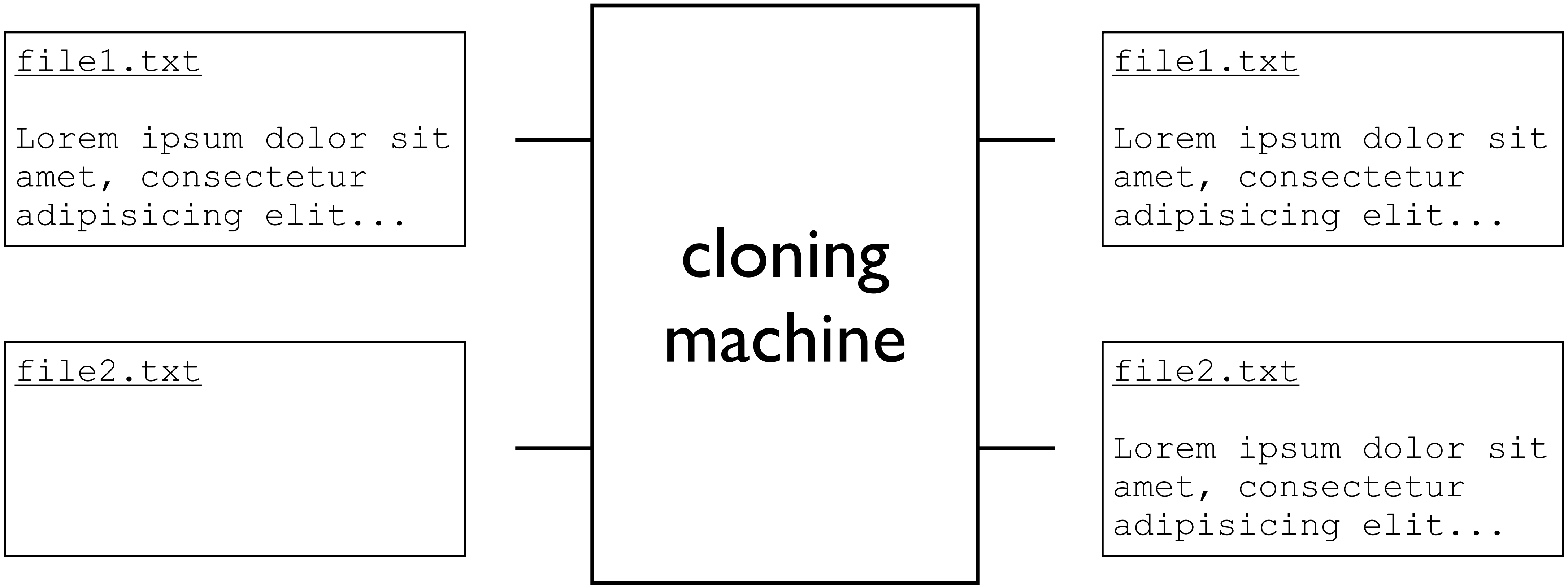
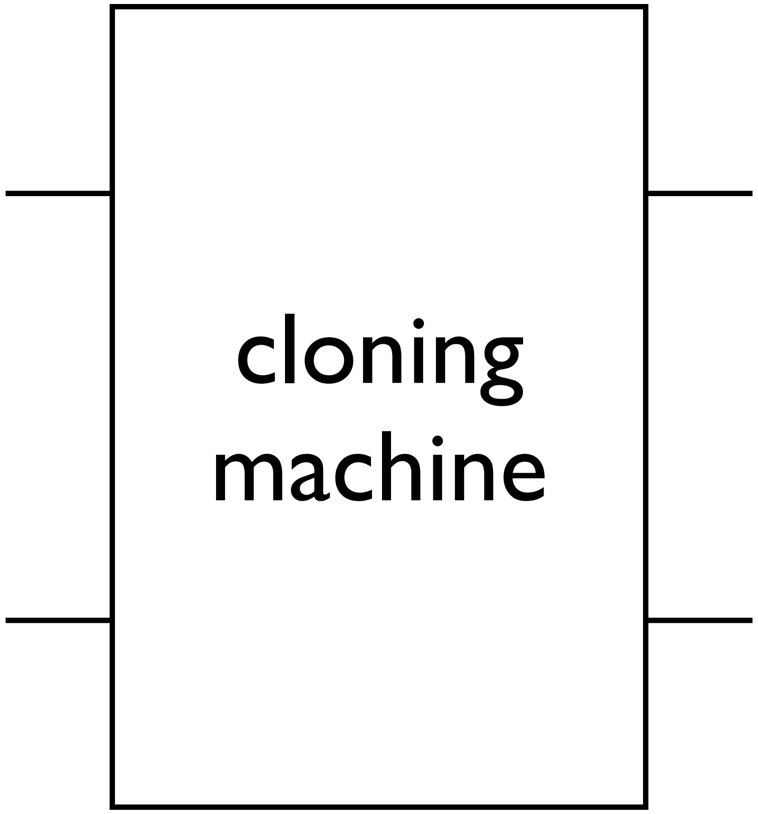


Quantum cloning?



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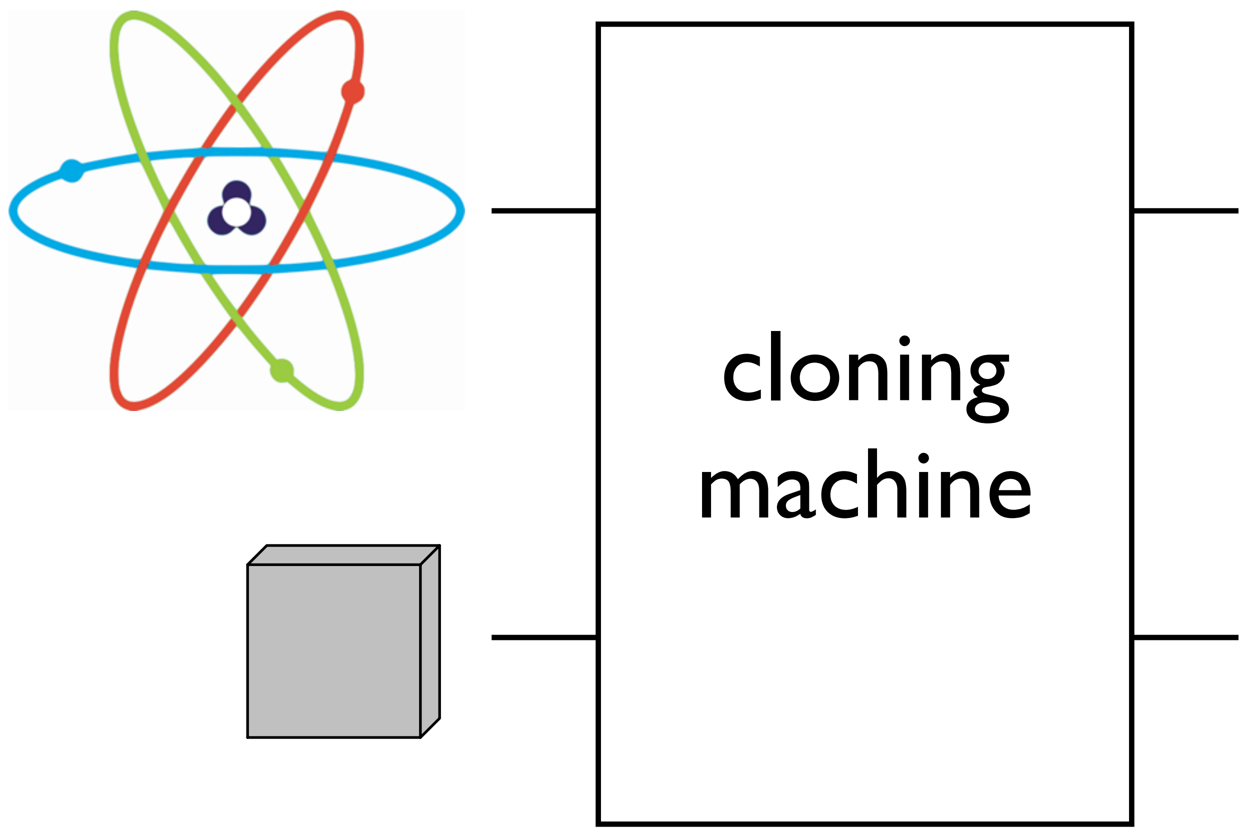

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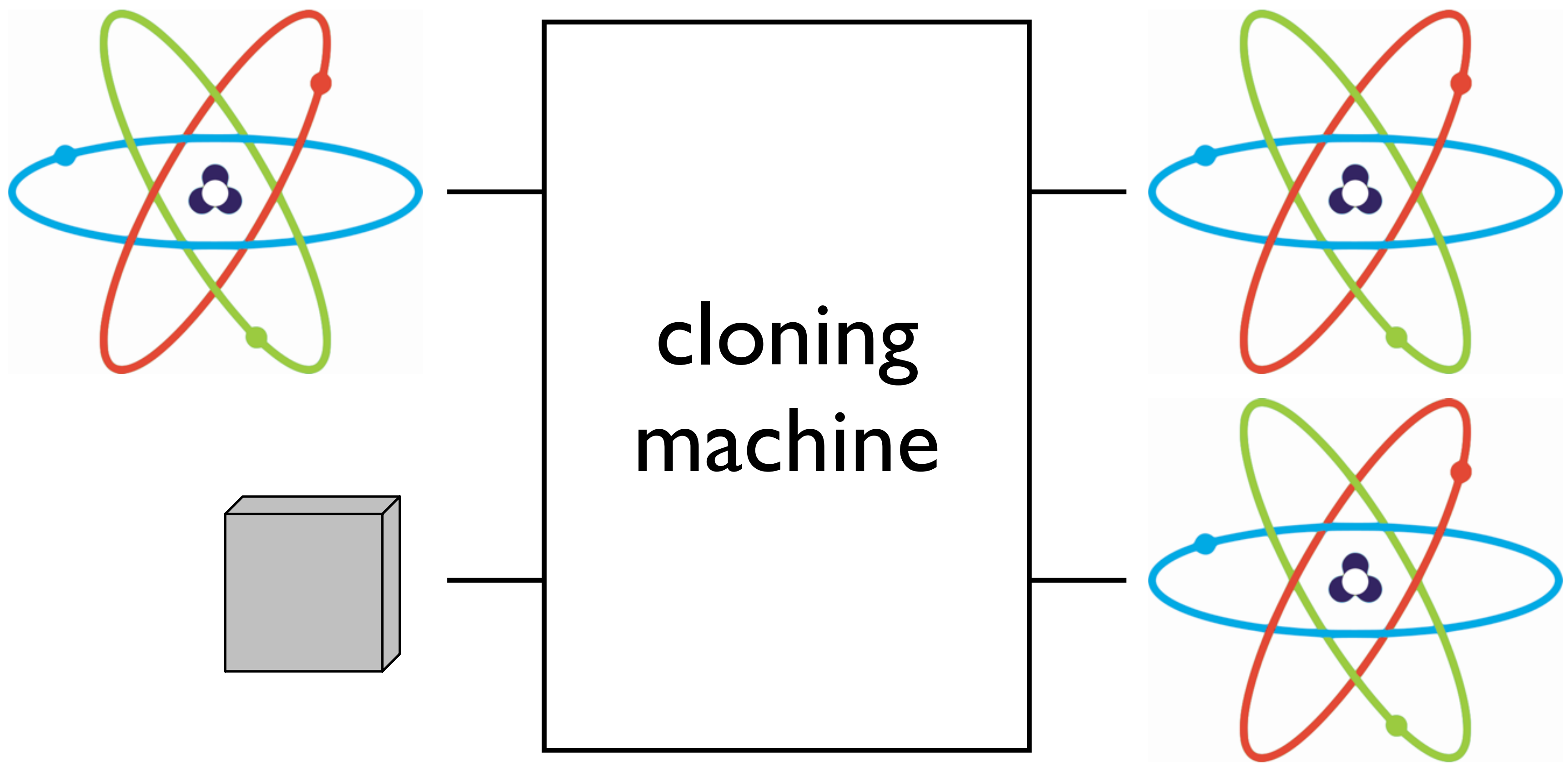

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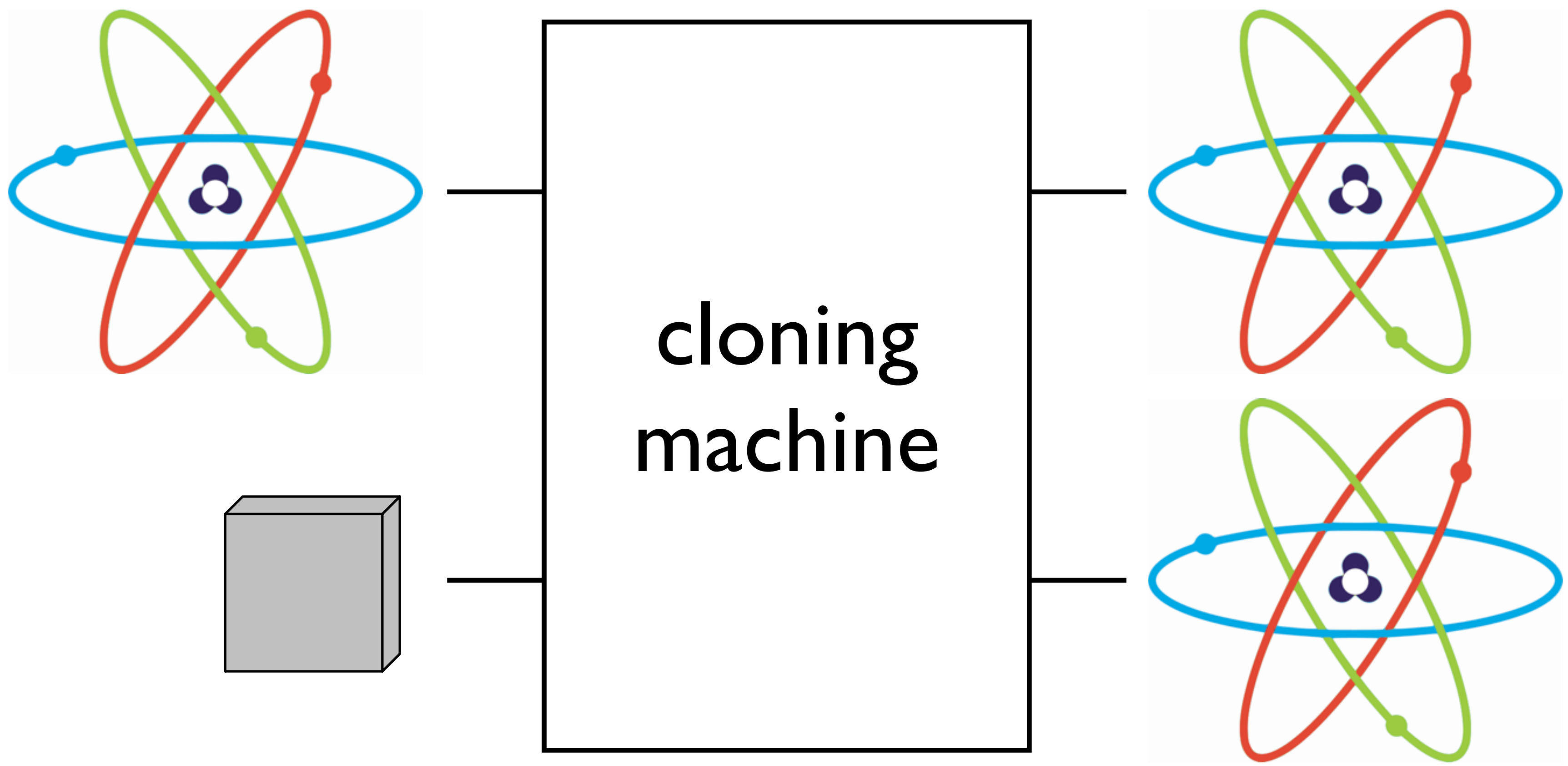

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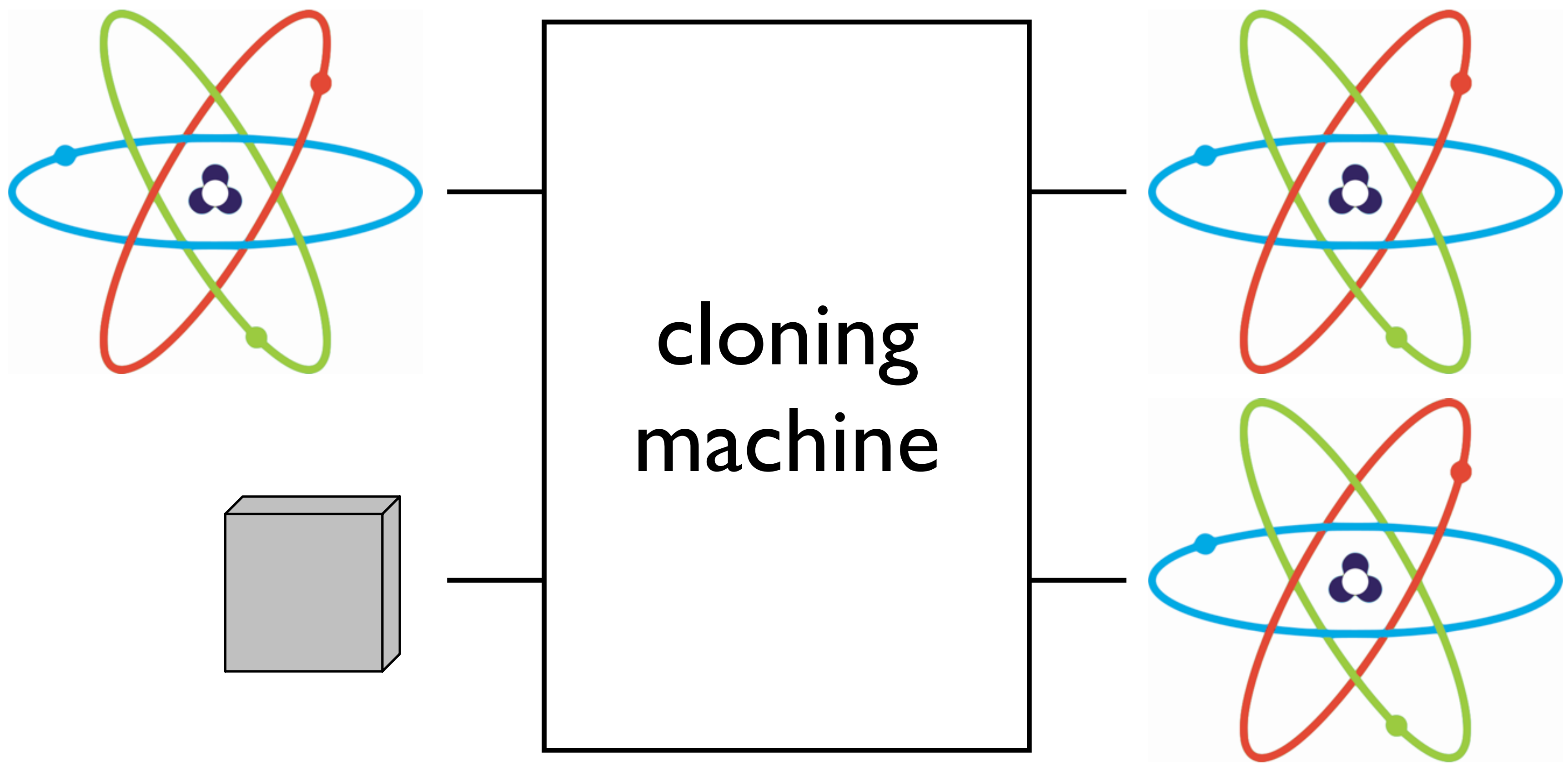
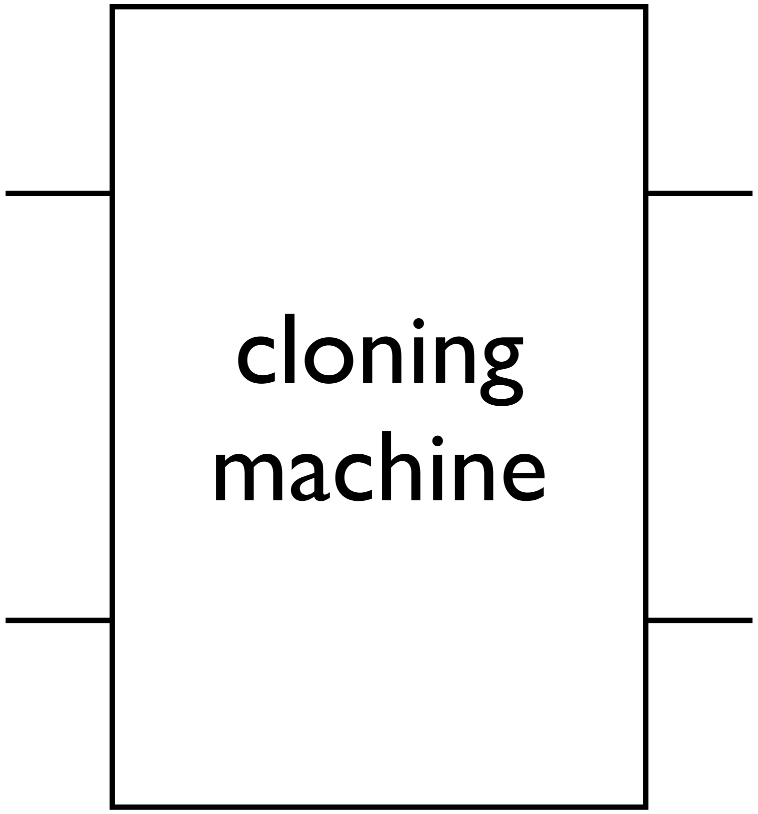


The uncertainty principle prevents us from learning an unknown quantum state.

Such a device is impossible!

Quantum cloning?



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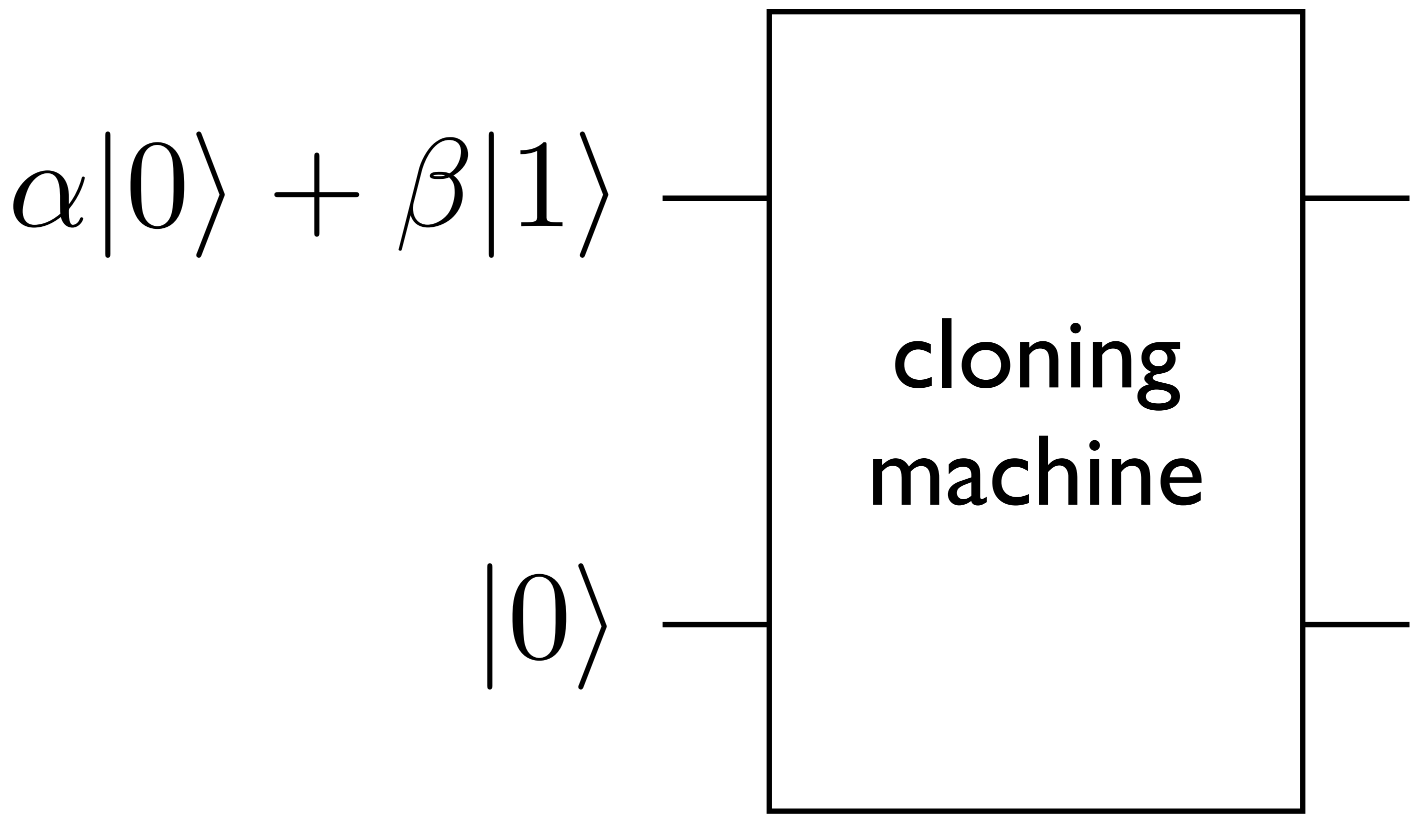

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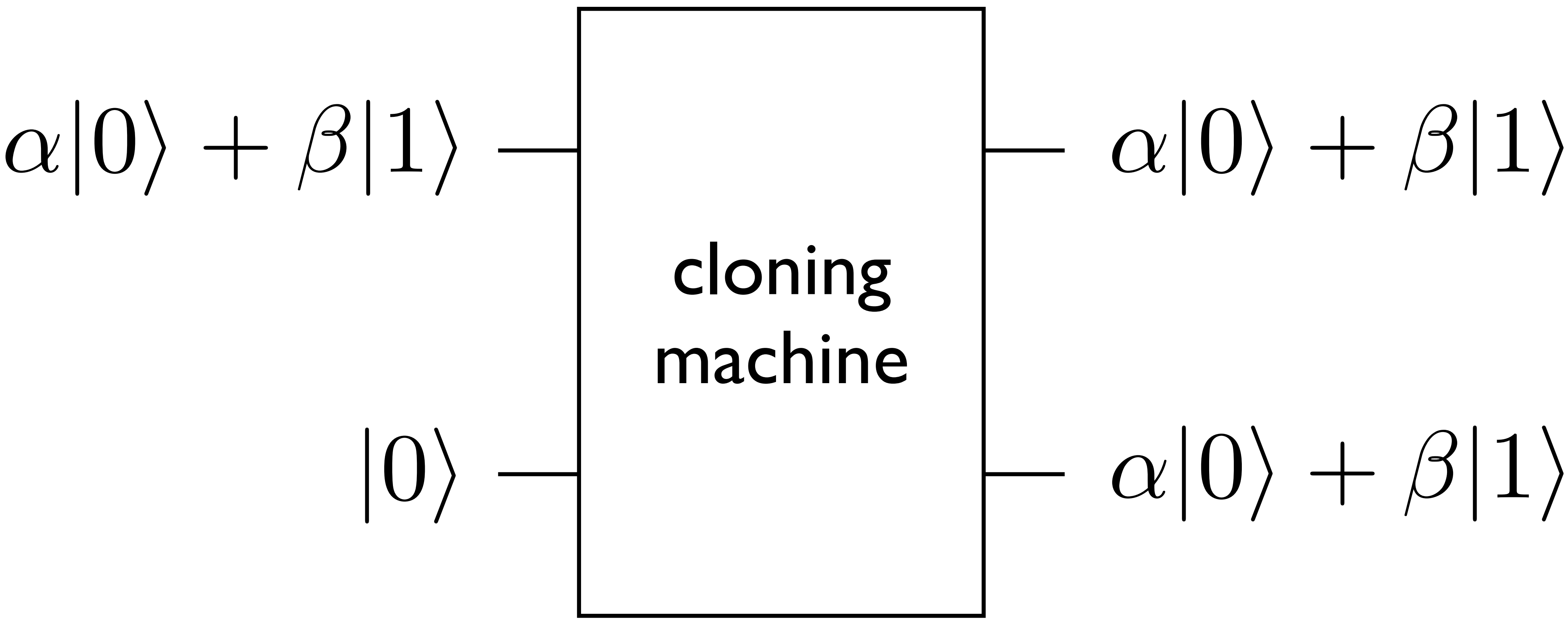

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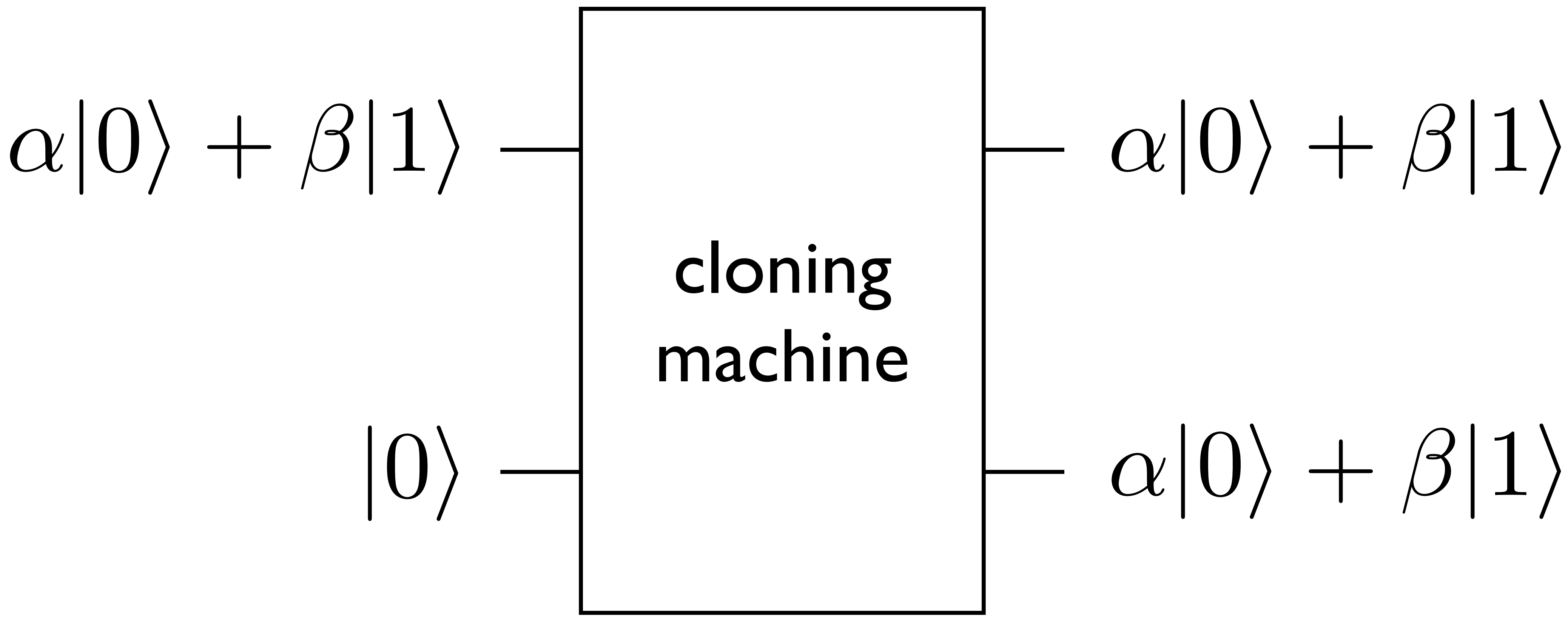

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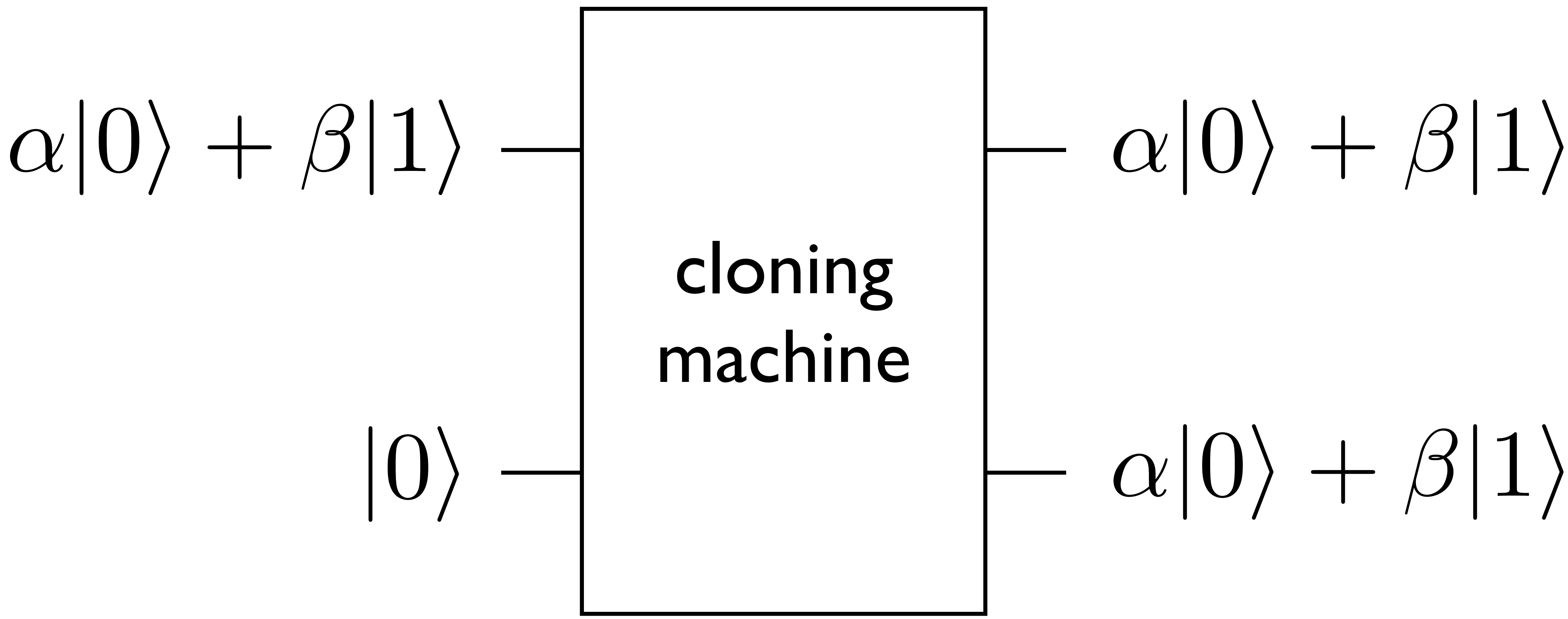
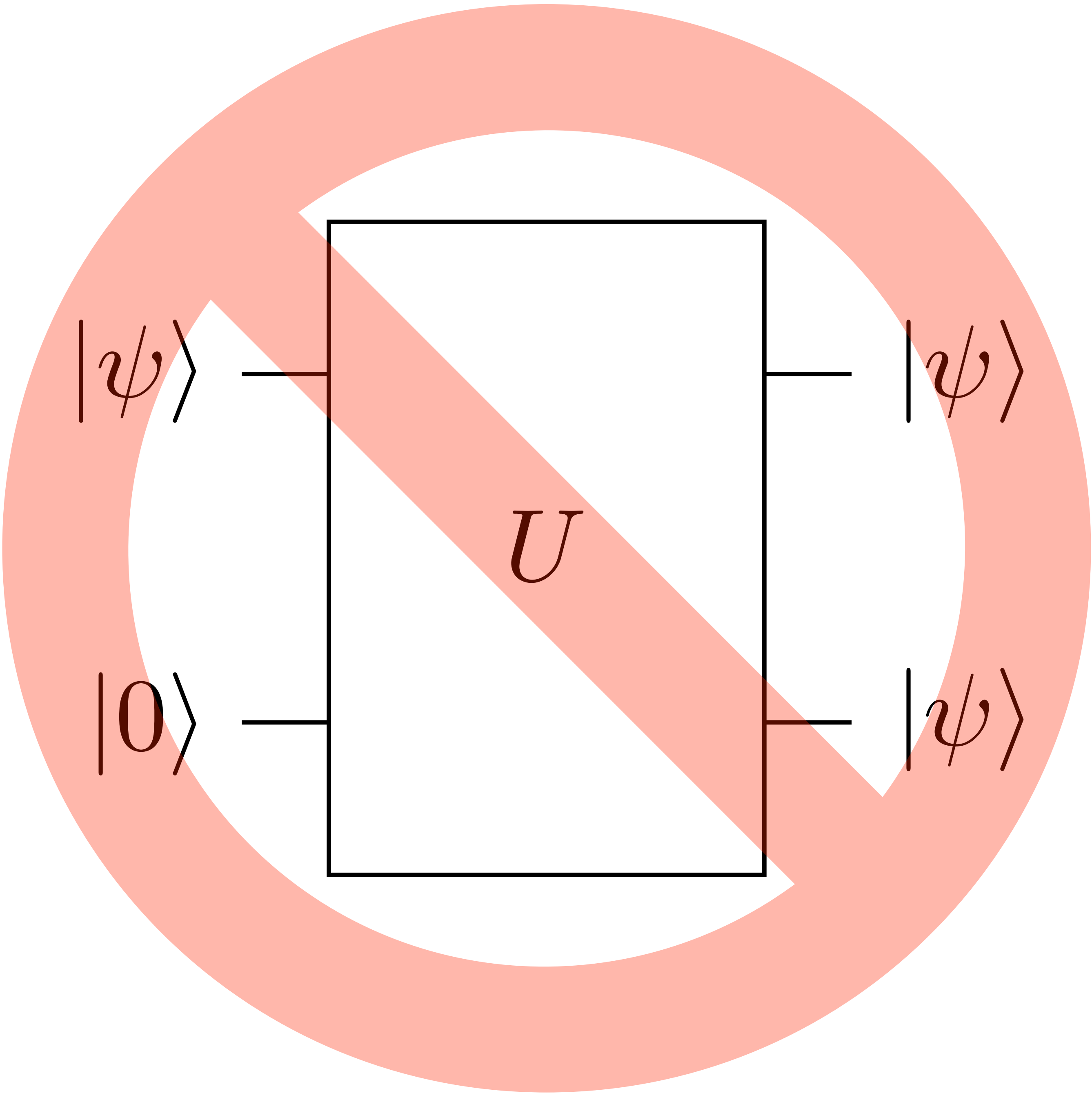


This is also impossible.

Even digital quantum information (qubits) cannot be cloned.

No-cloning theorem

Theorem [Wootters, Zurek, Dieks 1982]: There is no valid quantum process that takes as input an unknown quantum state $|\psi\rangle$ and an ancillary system in a known state, and outputs two copies of $|\psi\rangle$.



Proof of the no-cloning theorem

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Consider two orthogonal states $|\psi\rangle, |\phi\rangle$. By the definition of cloning,

$$U(|\psi\rangle \otimes |0\rangle) = |\psi\rangle \otimes |\psi\rangle$$

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By linearity,

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But again by the definition of cloning, we should have

$$U[(\alpha|\psi\rangle + \beta|\phi\rangle) \otimes |0\rangle]$$

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By linearity,

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So either $\alpha = 0$ or $\beta = 0$.

Cloning in a fixed basis

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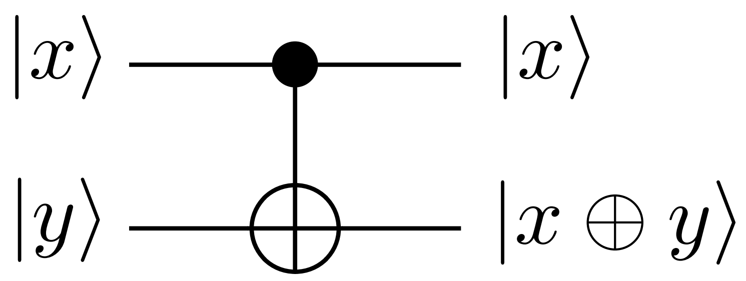
Example: Controlled-not gate



$$
\begin{aligned}
|00\rangle &\mapsto |00\rangle \\
|01\rangle &\mapsto |01\rangle \\
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\end{aligned}
$$

$$
\begin{pmatrix}
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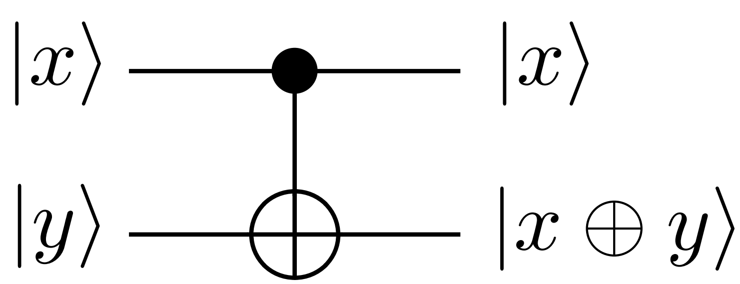

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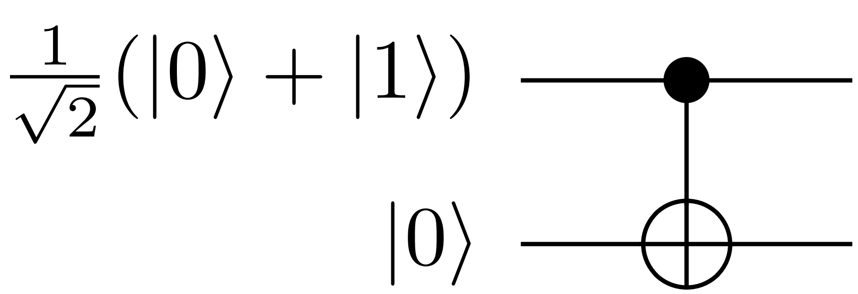
Inputting non-basis states produces an *entangled state*:



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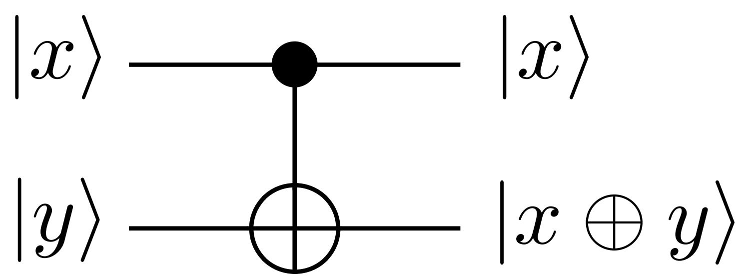

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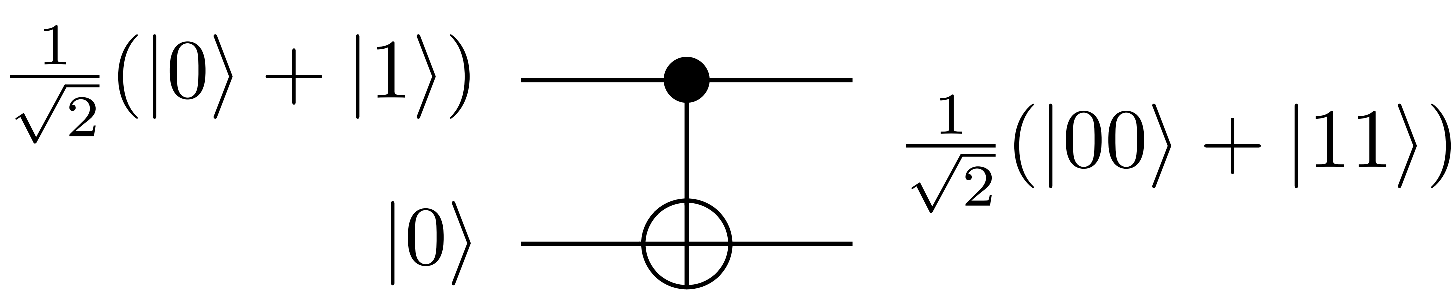


Exercise: Distinguishing non-orthogonal states

No device can clone two non-orthogonal states, and in particular, it is not possible to perfectly distinguish such states. But if we want to distinguish them, how well can we do?

Suppose Alice prepares the state $|0\rangle$ or $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, each with probability $\frac{1}{2}$.

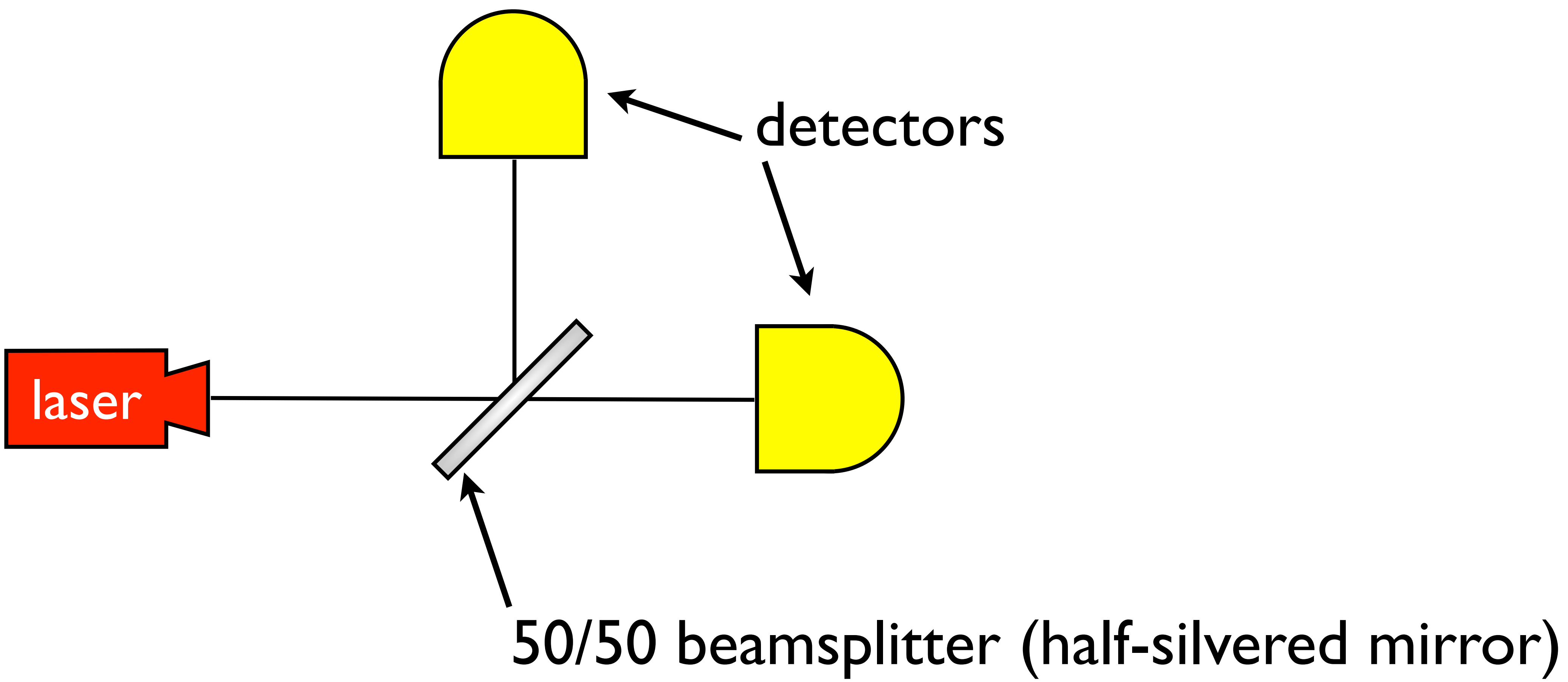
a. If you measure in the basis $\{|0\rangle, |1\rangle\}$, with what probability can you correctly guess which state Alice prepared?

b. What if you measure in the basis $\{|+\rangle, |-\rangle\}$, where $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$?

c. Can you think of another measurement that distinguishes the states with higher probability? (Hint: Consider the given states as polarizations of light. How would you orient a polarizer to get the most information about which polarization was prepared?)

Mach-Zehnder interferometer

A simple experiment



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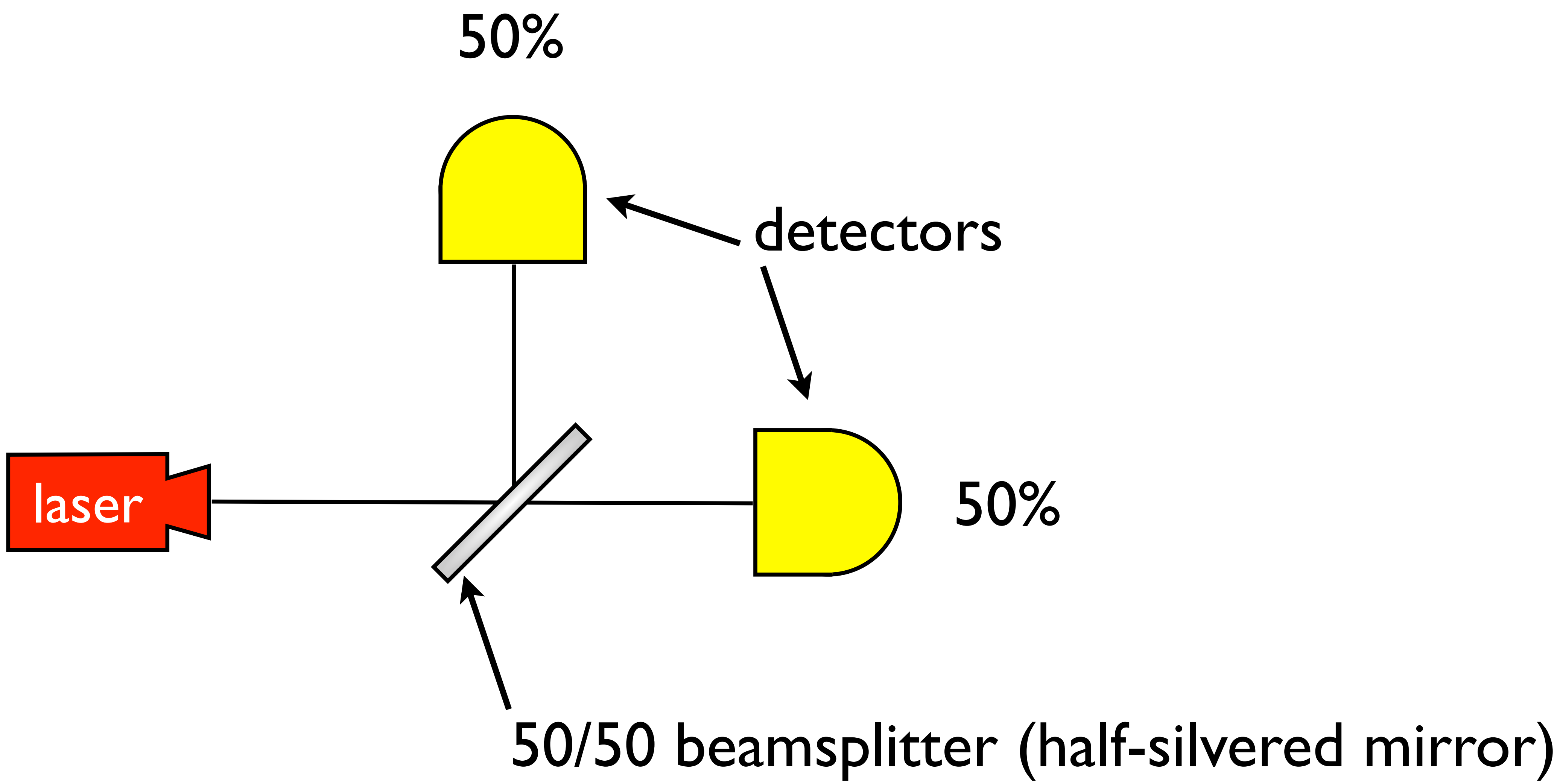
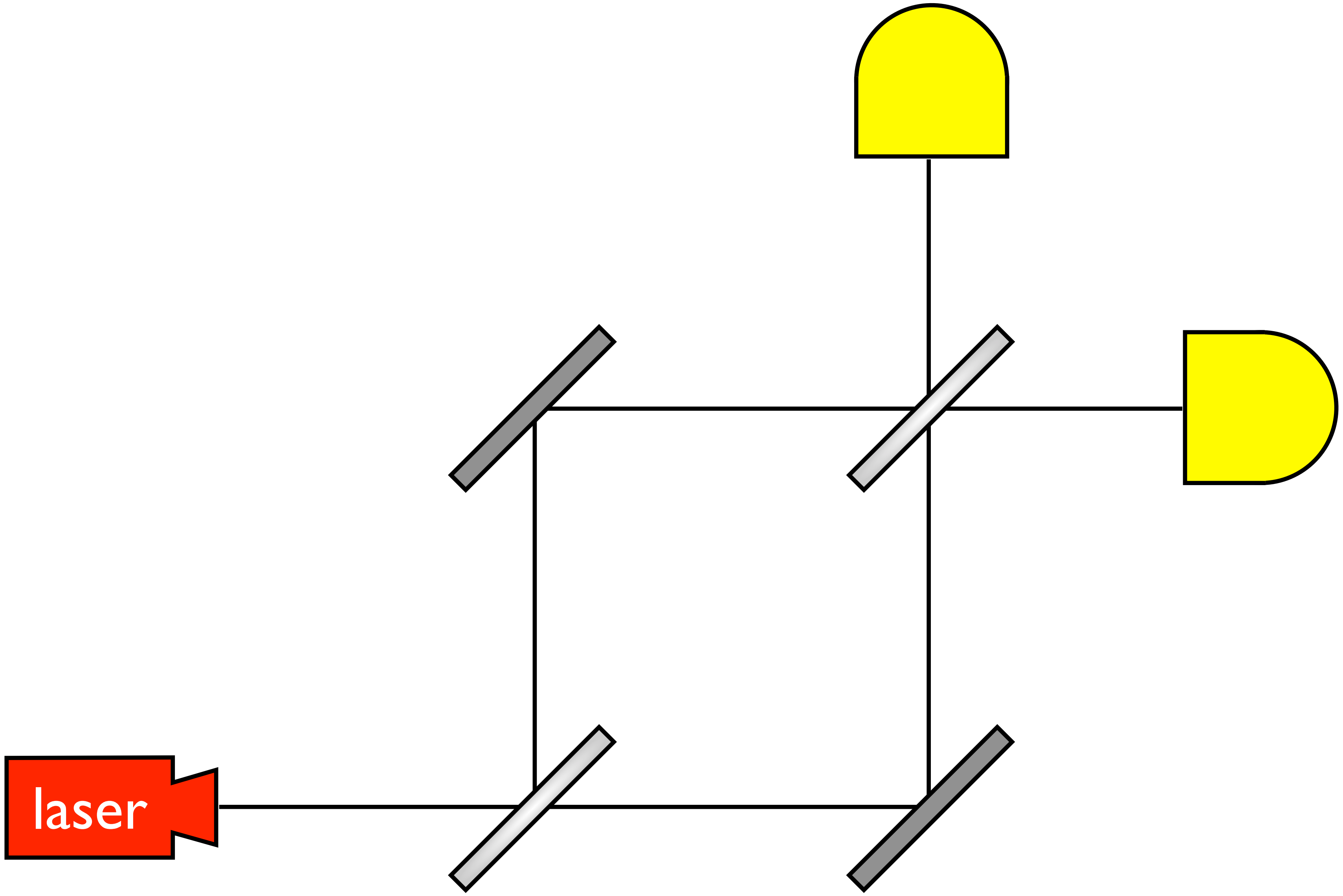


Interferometer



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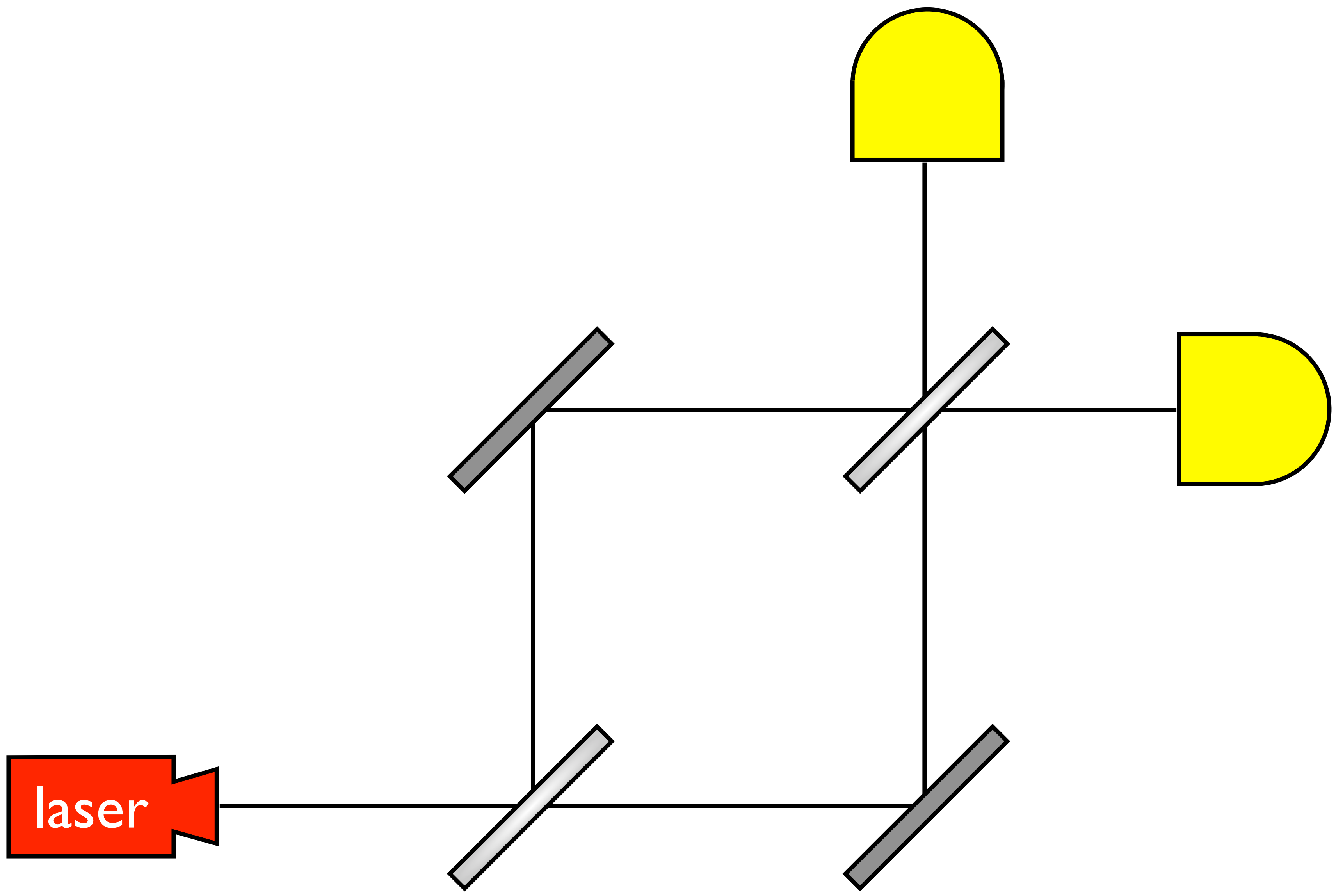

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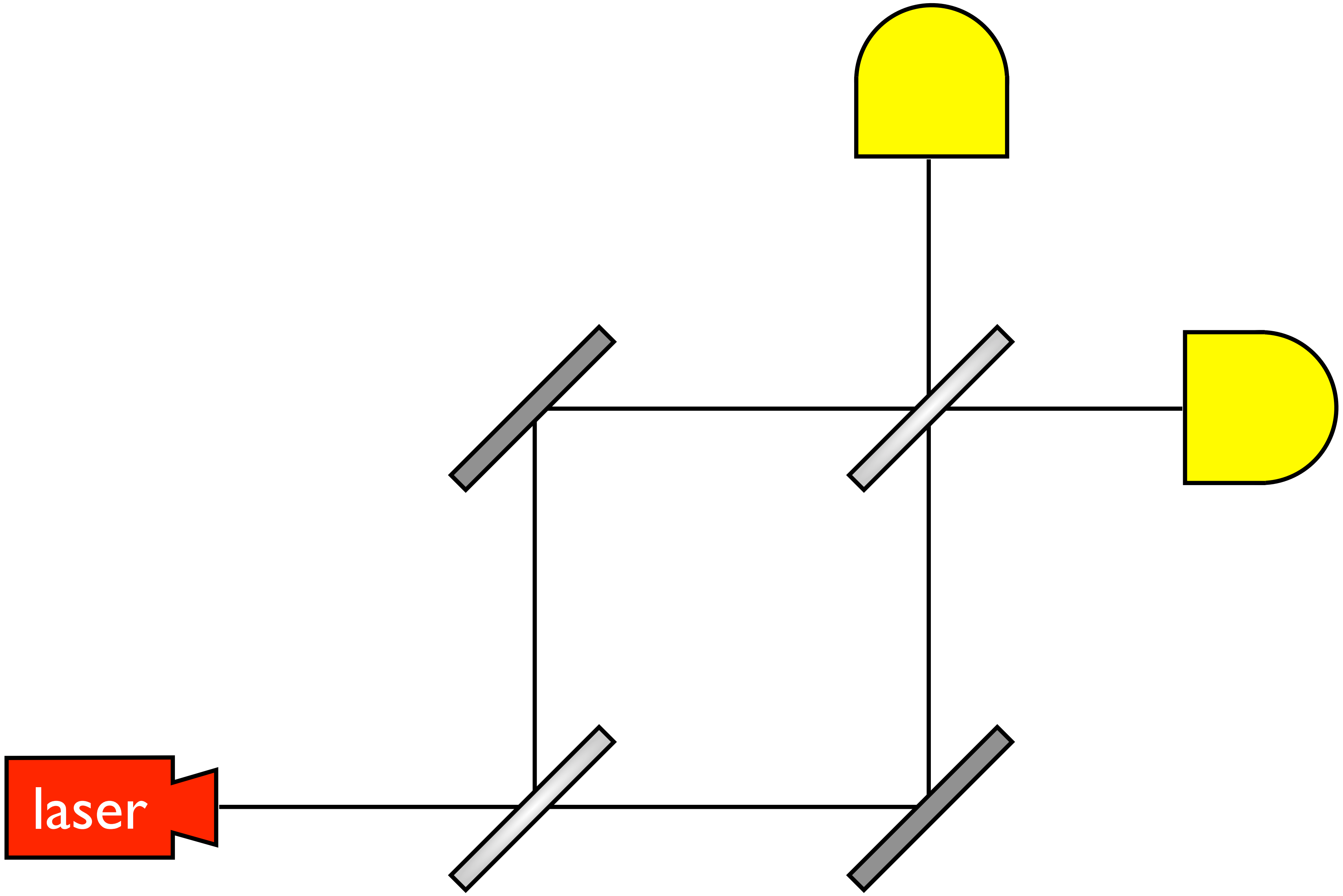

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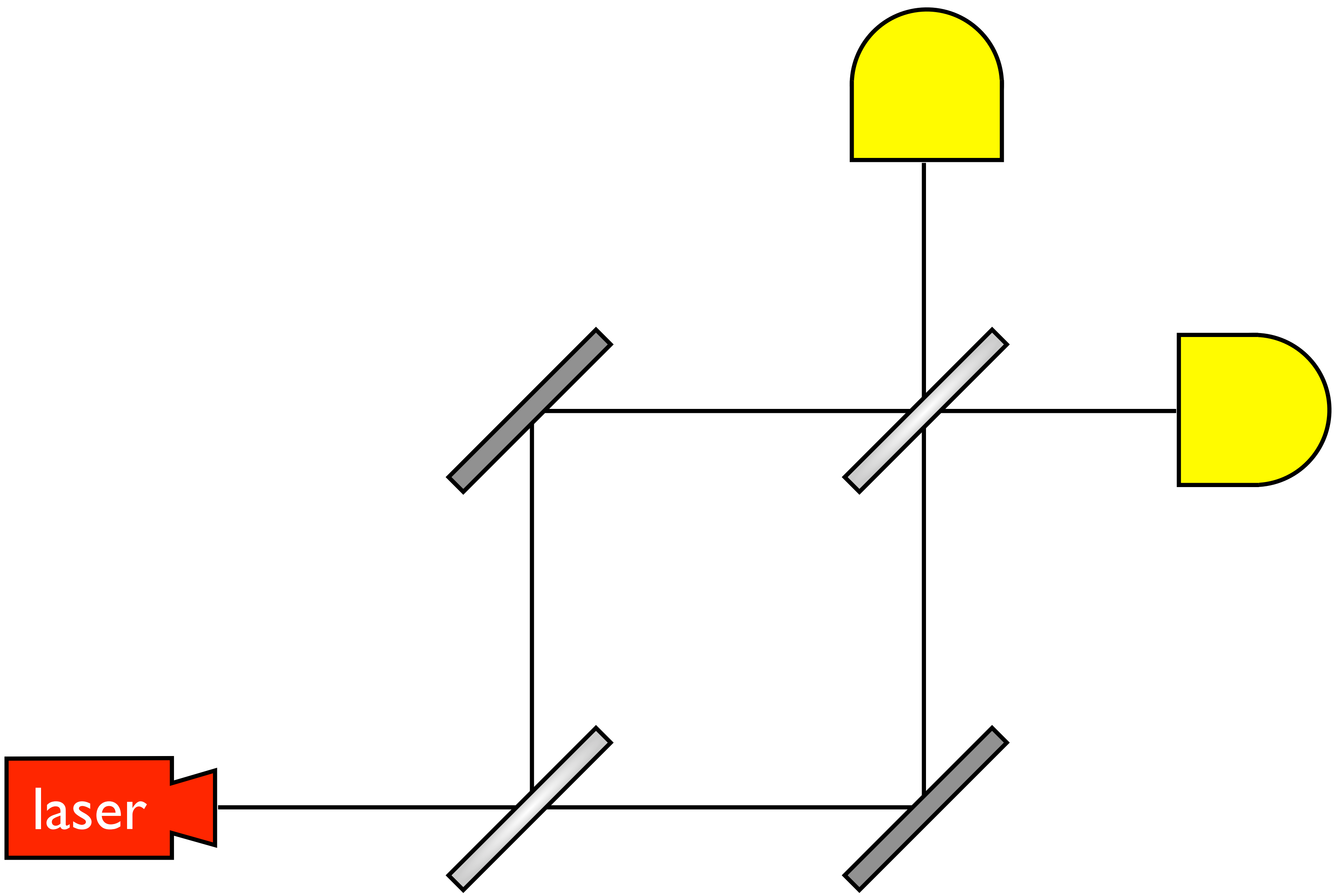

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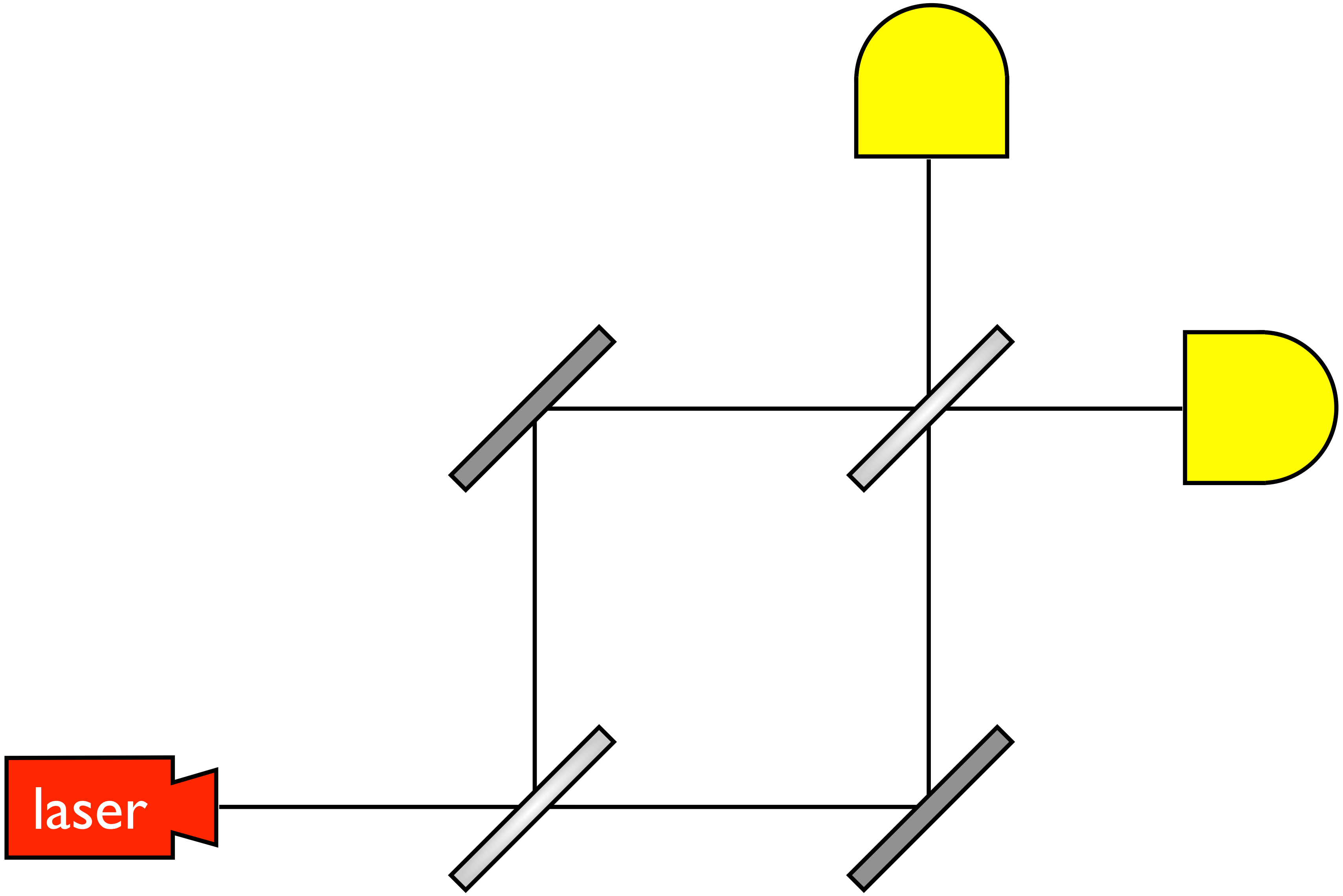

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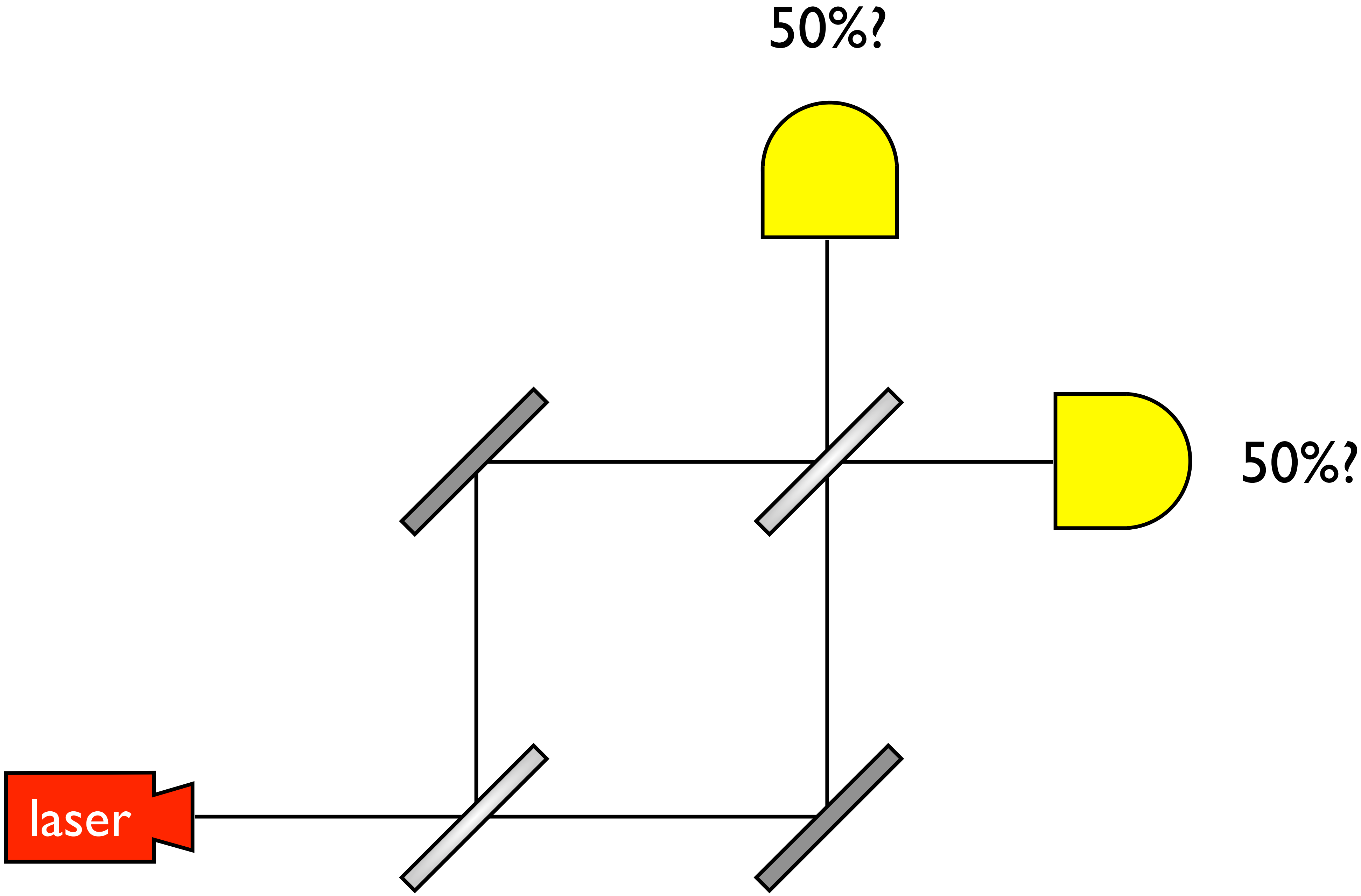

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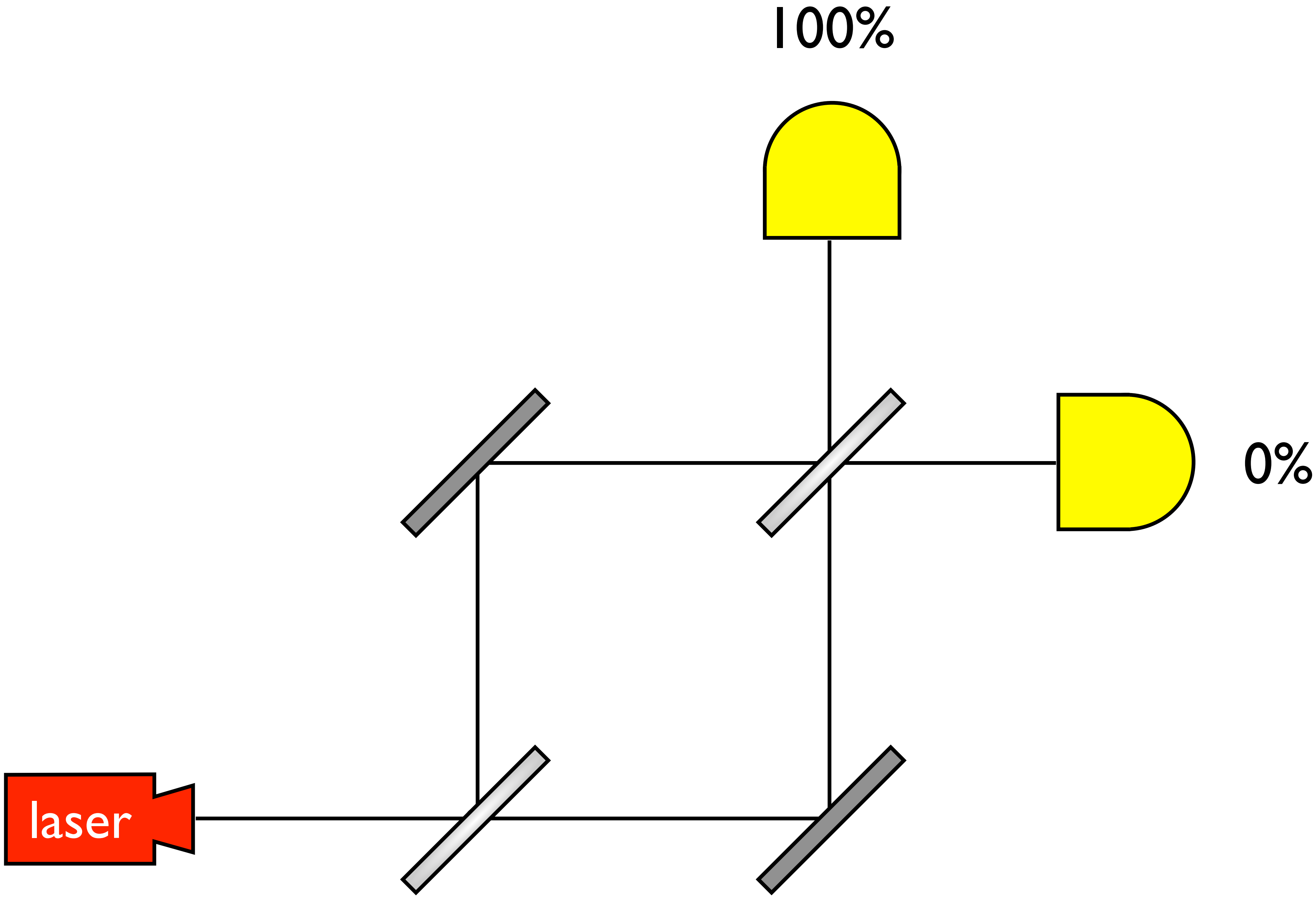
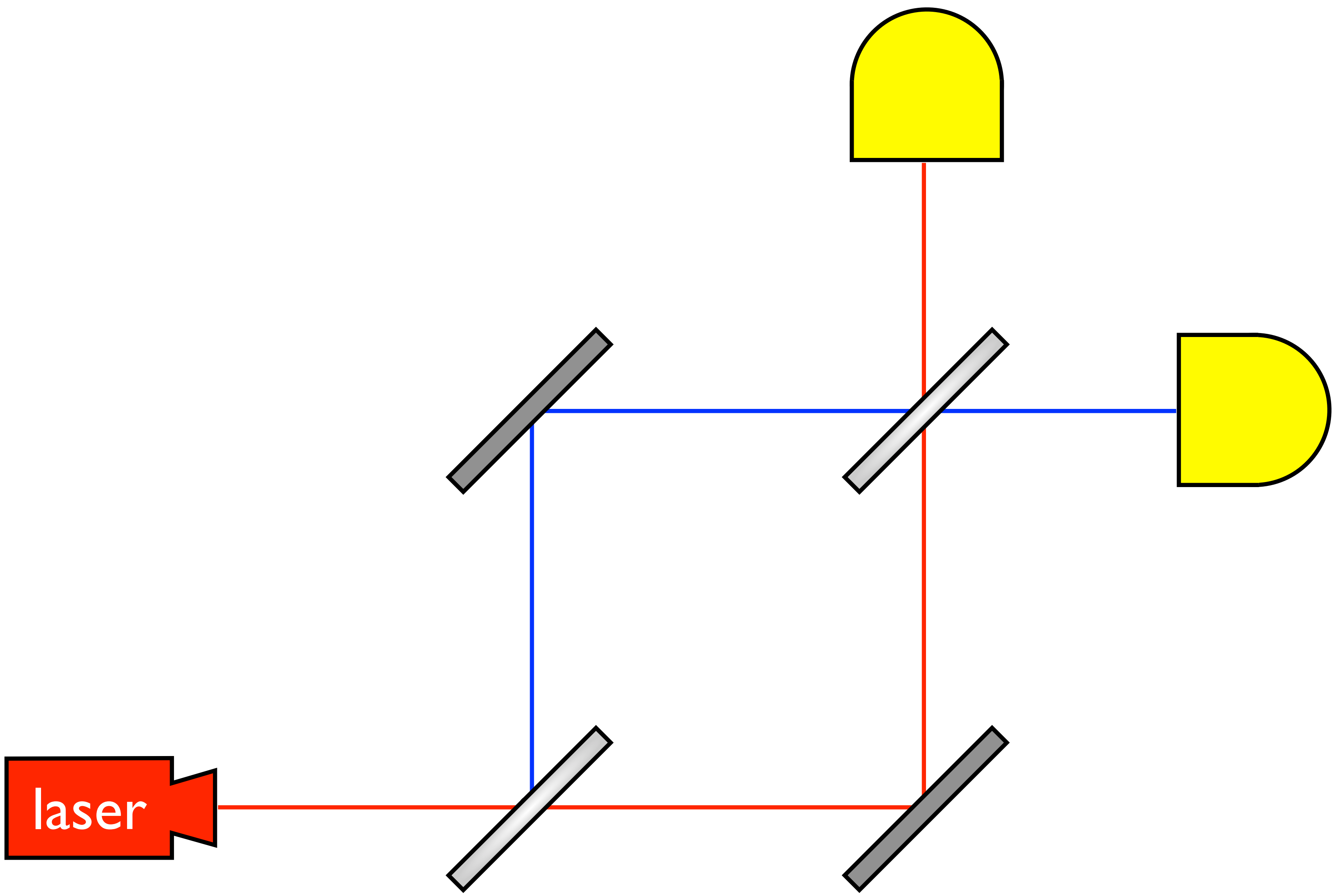


Mathematical model



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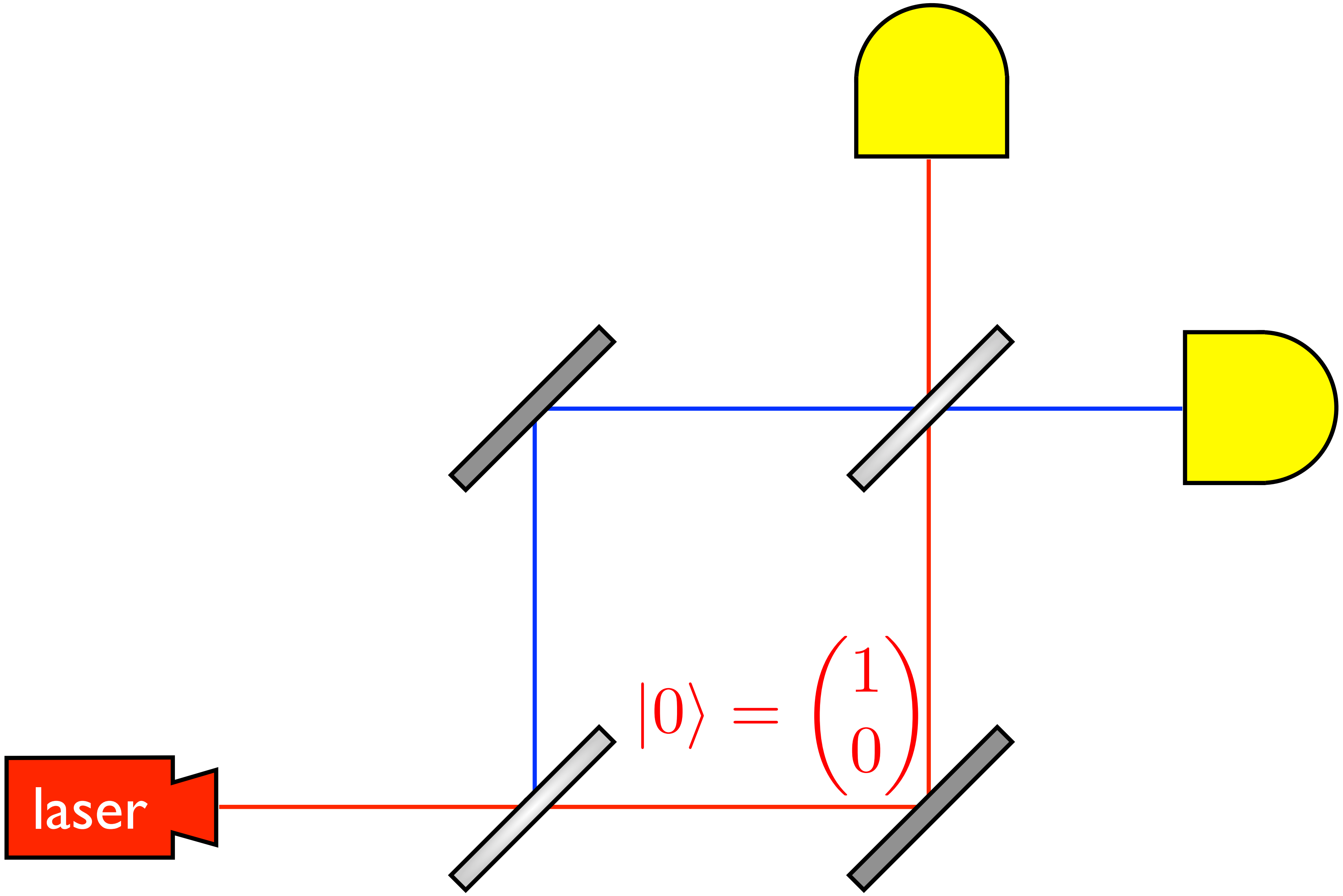

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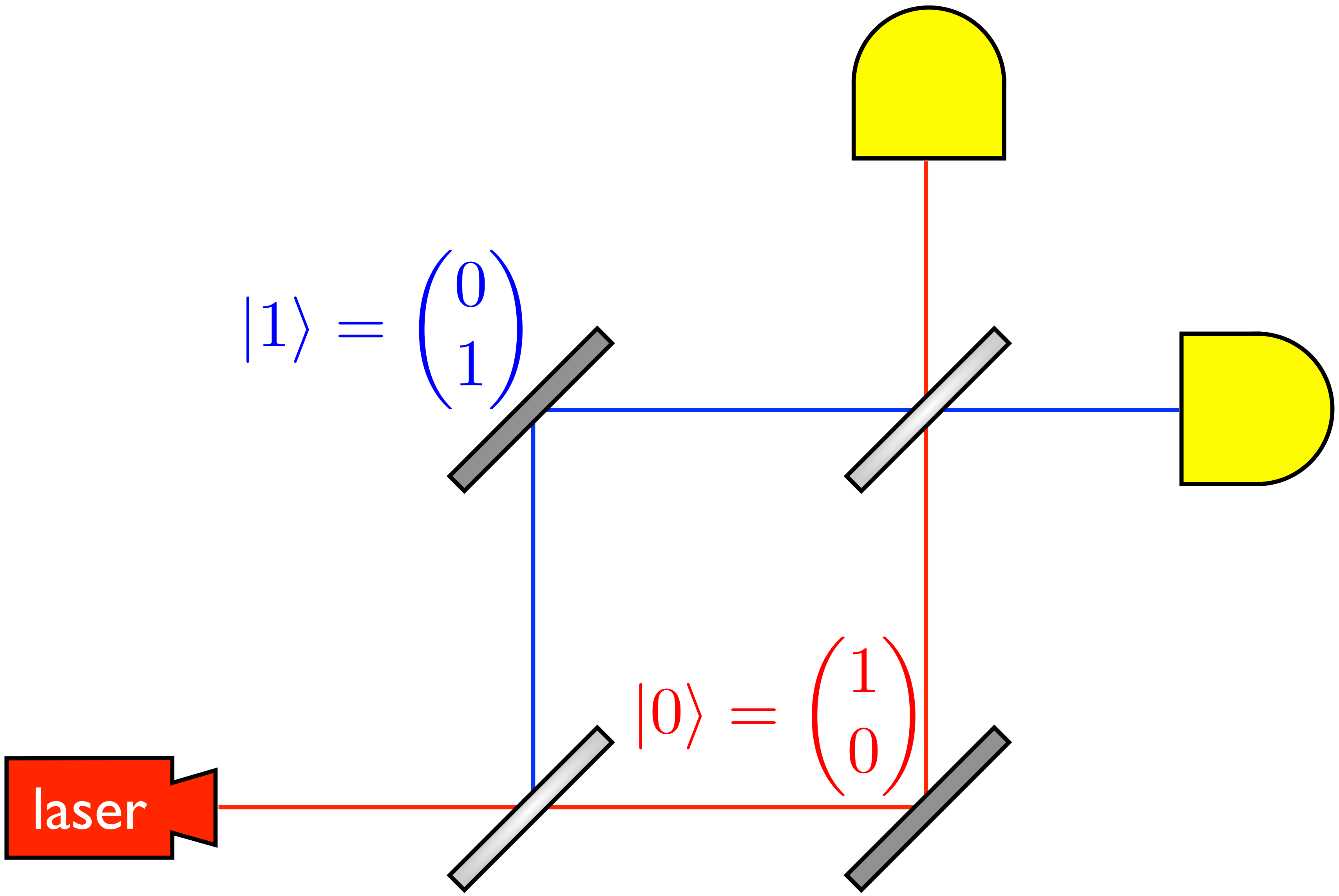

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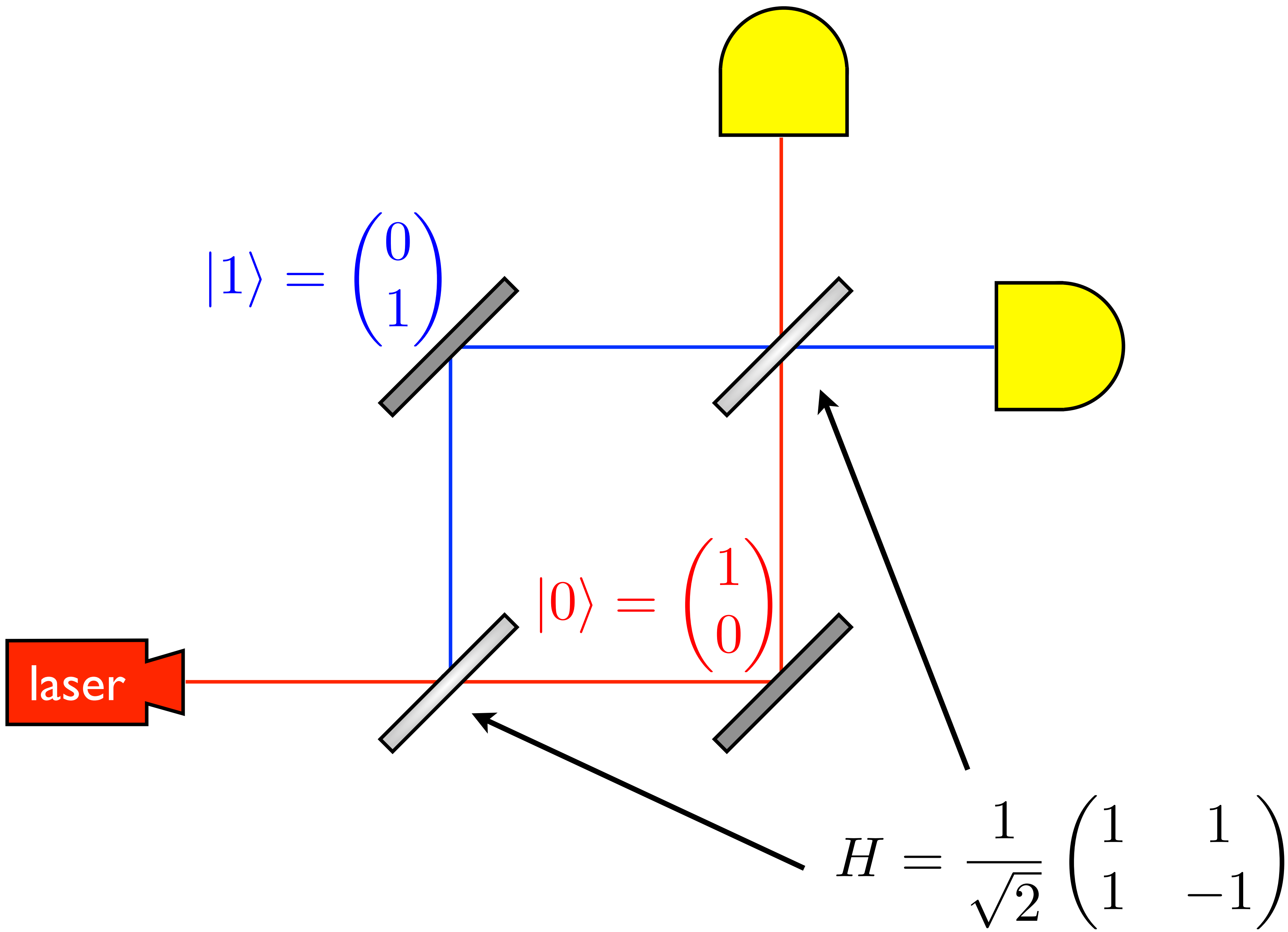

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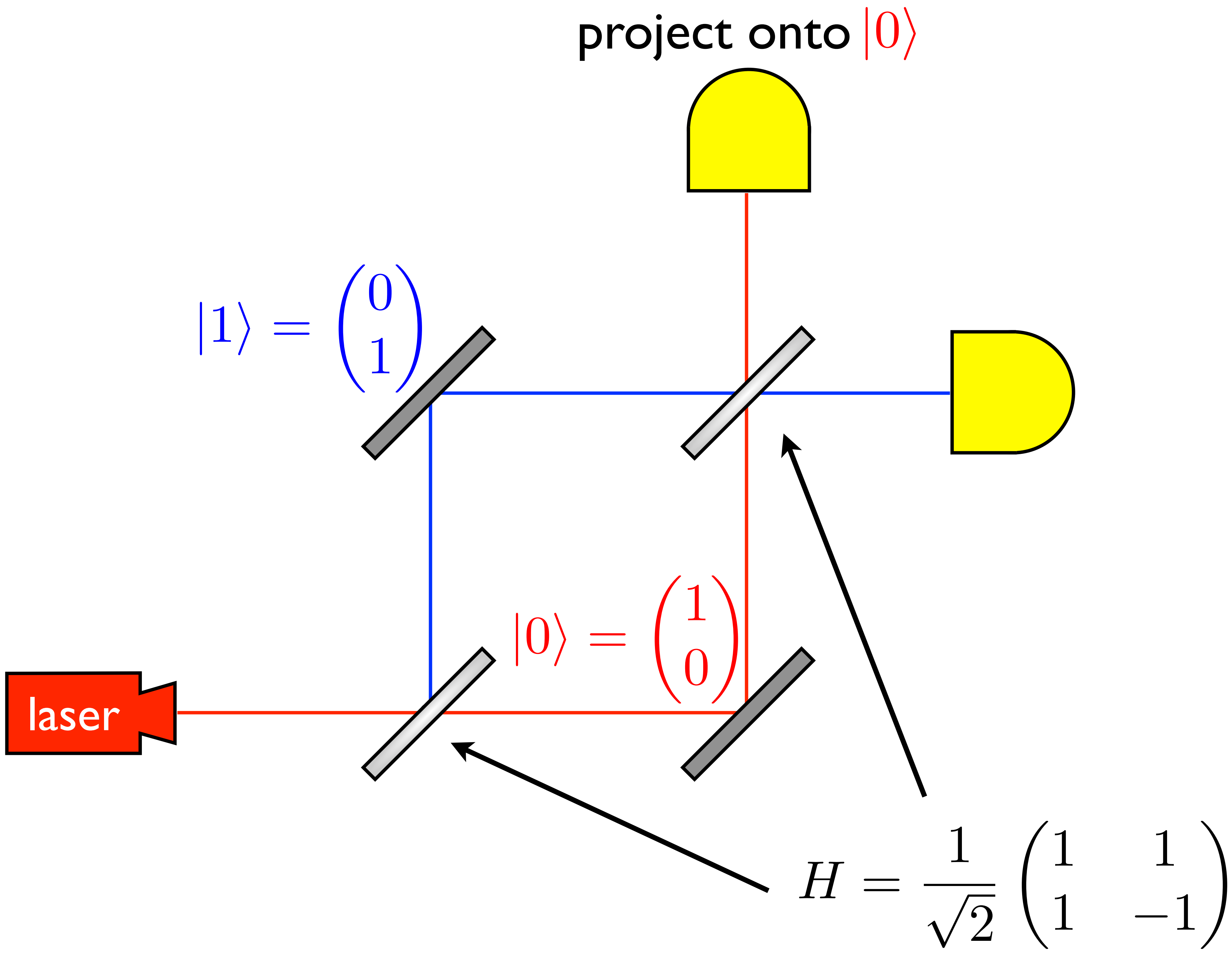

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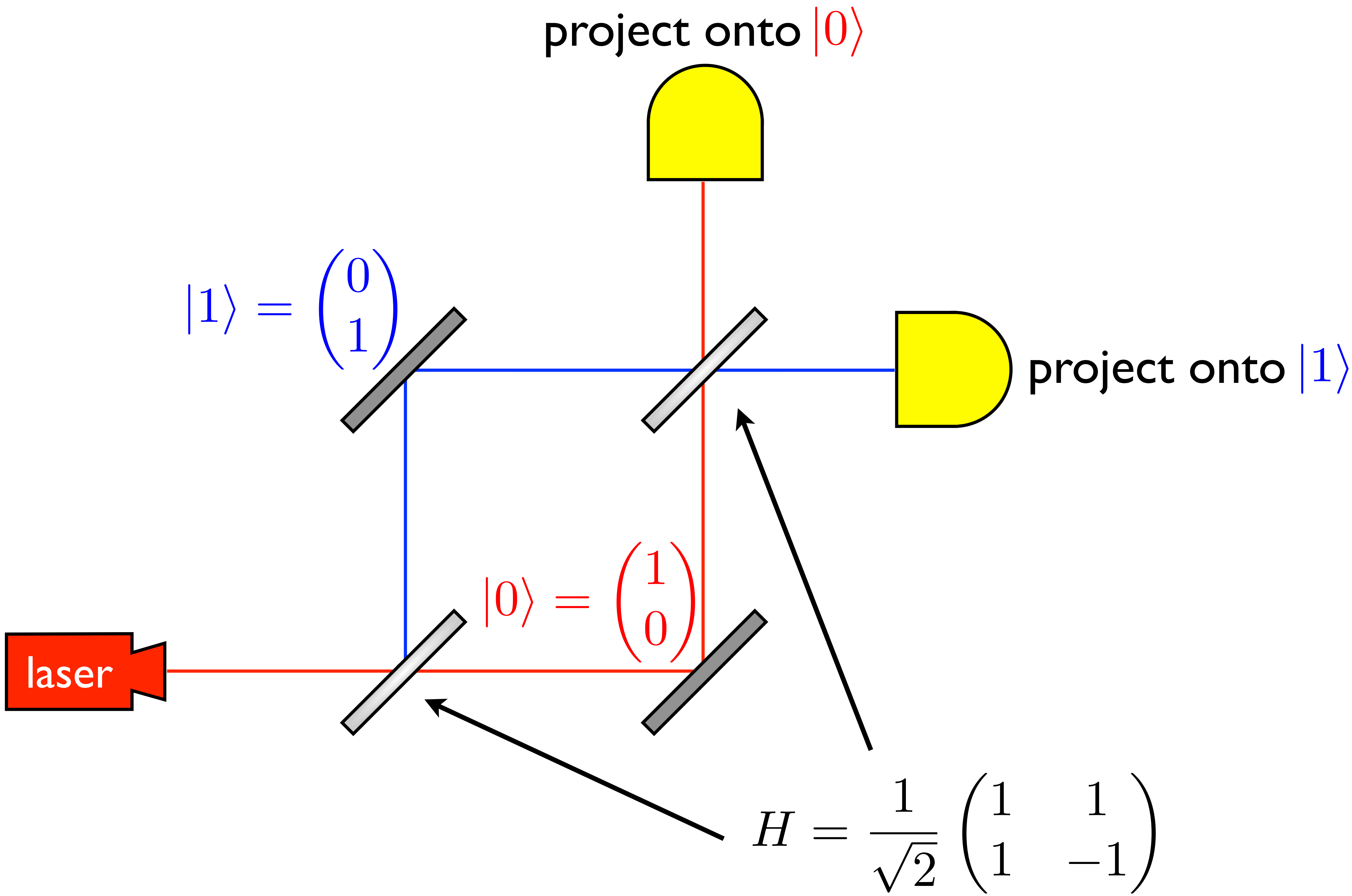
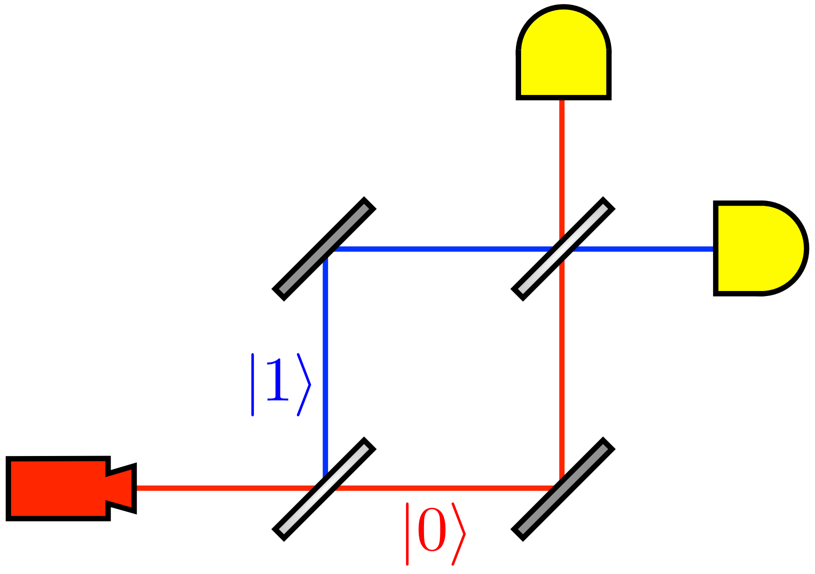


Calculation



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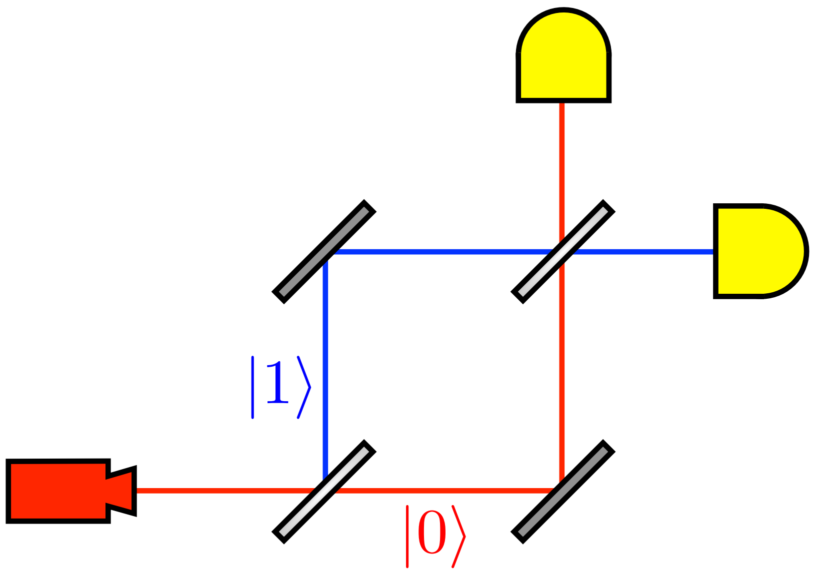

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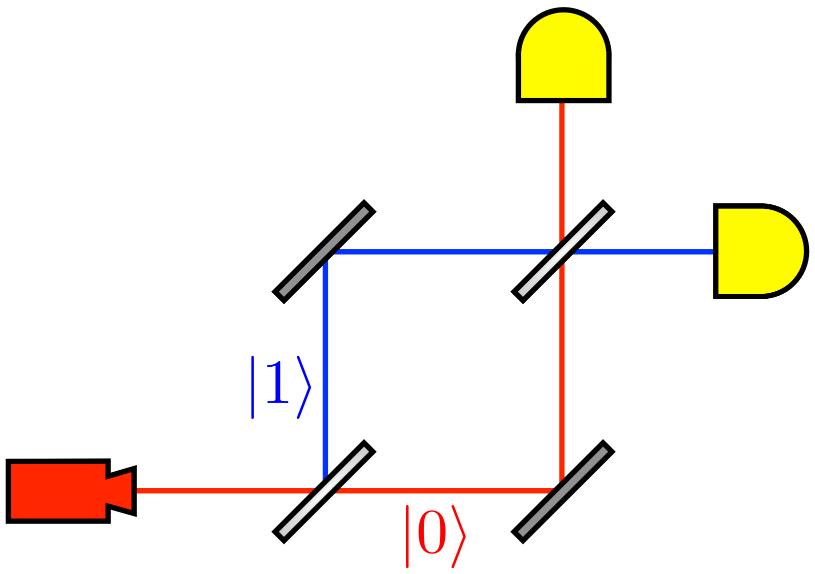

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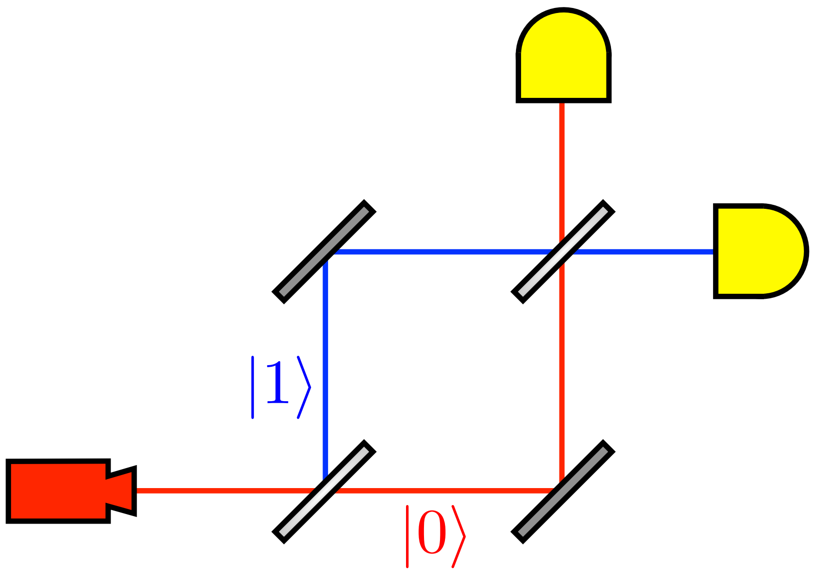

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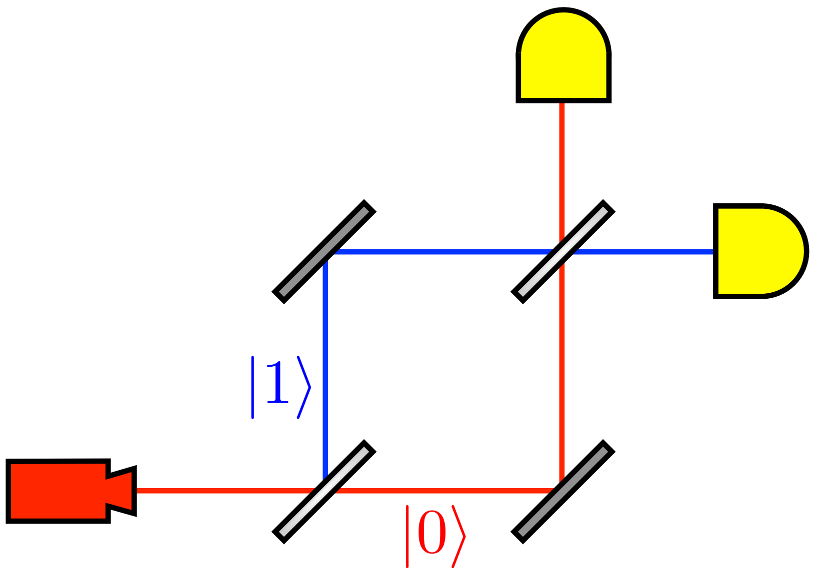

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$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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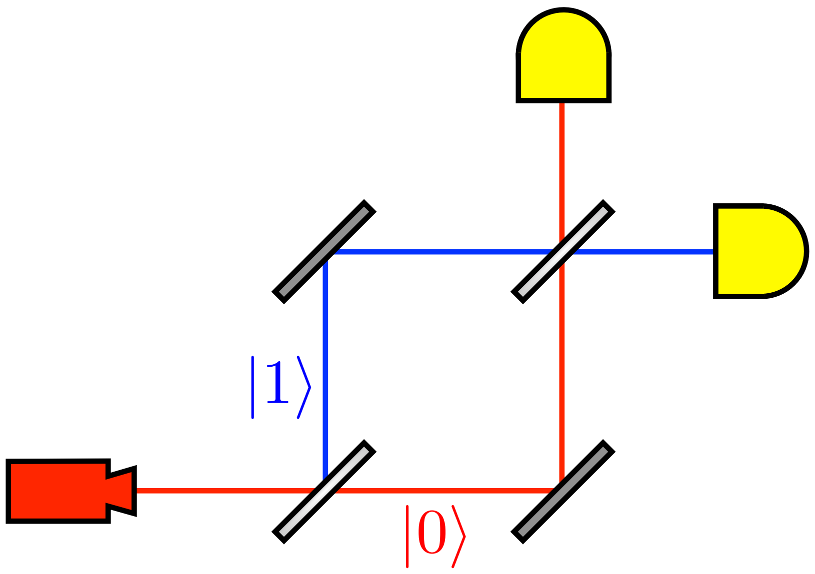

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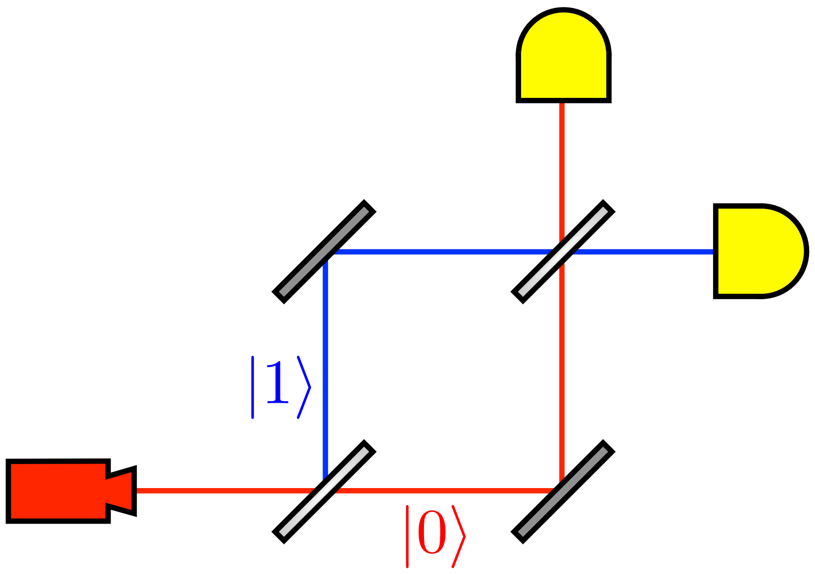

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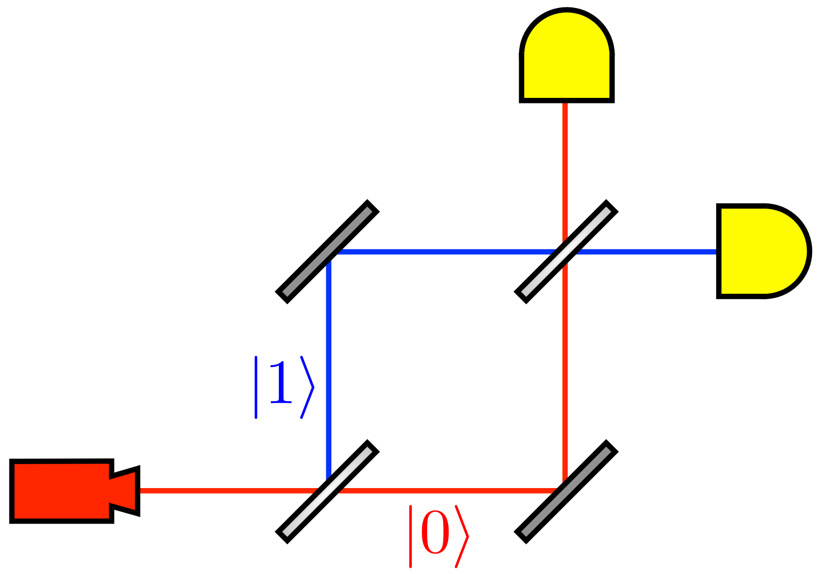

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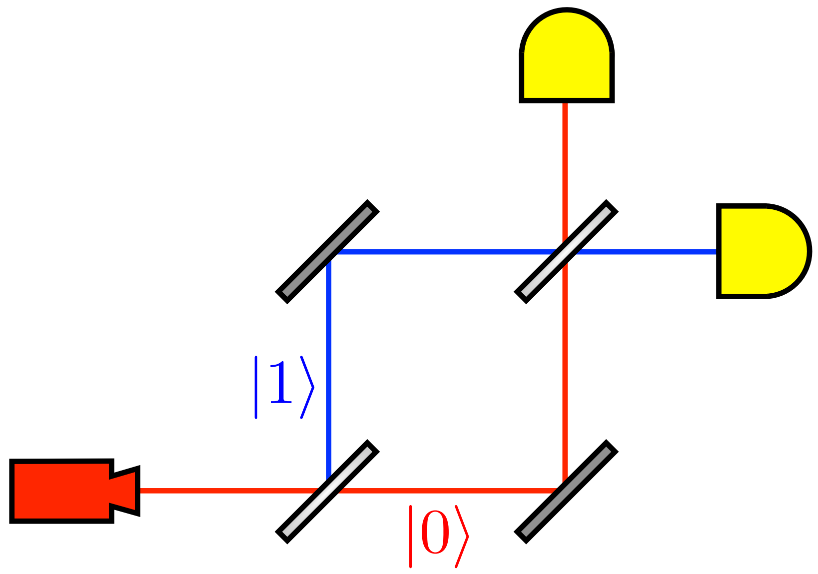

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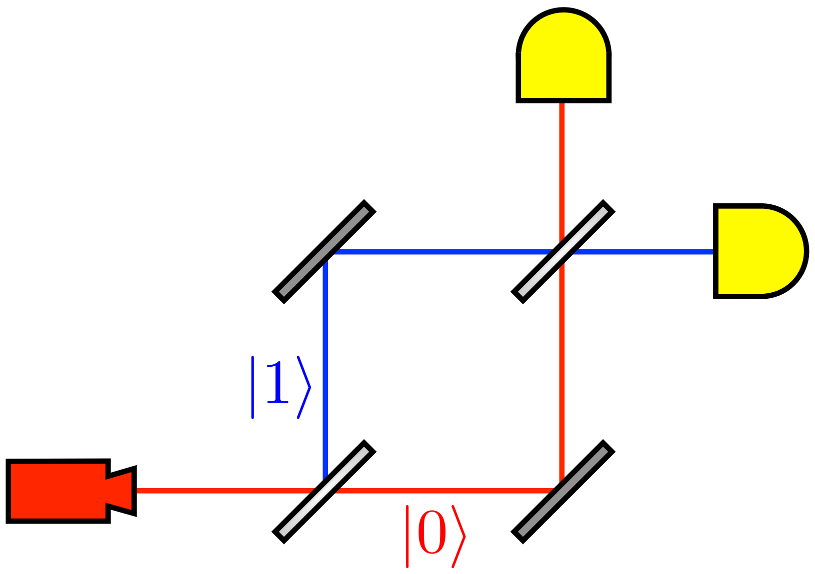

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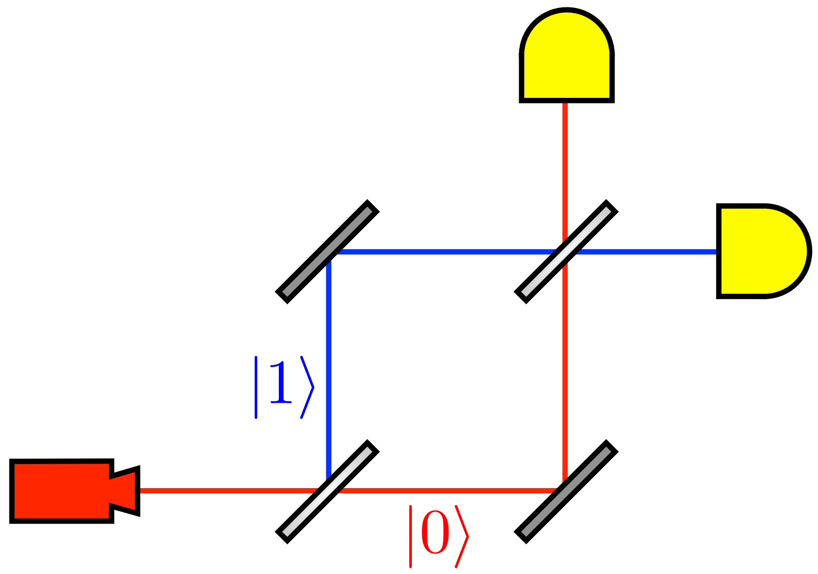

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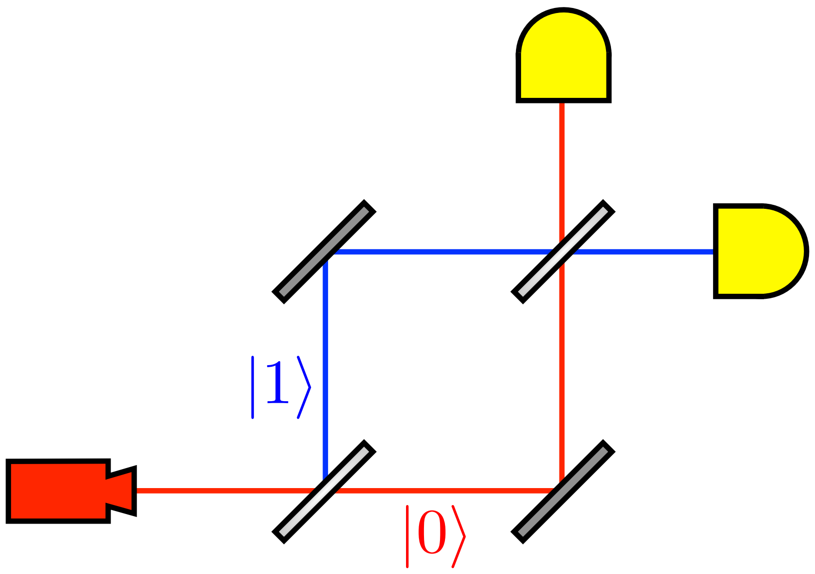
Calculation



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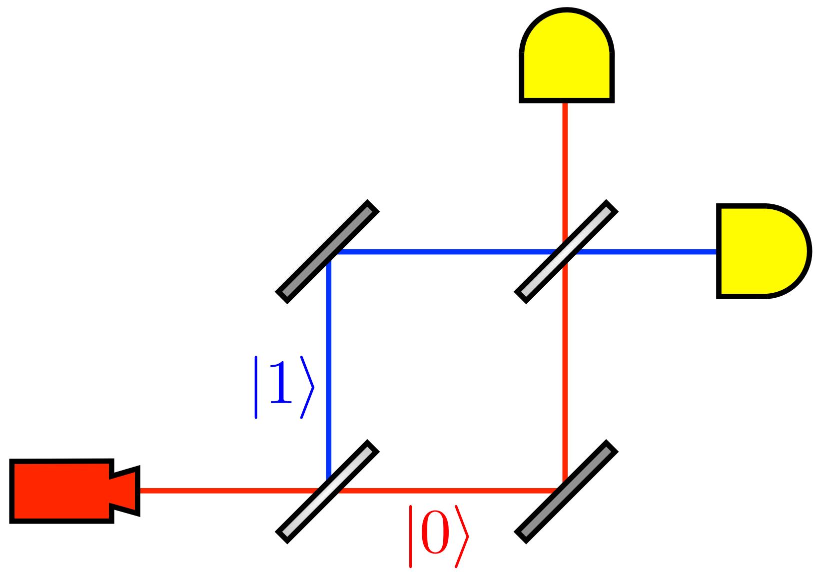

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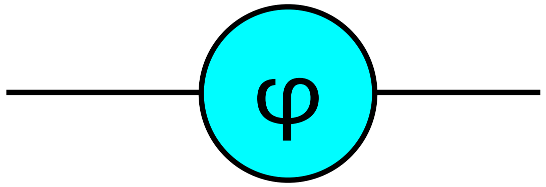

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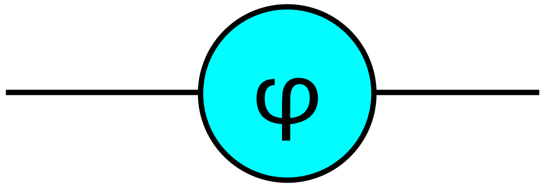

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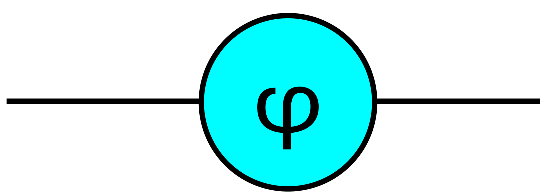

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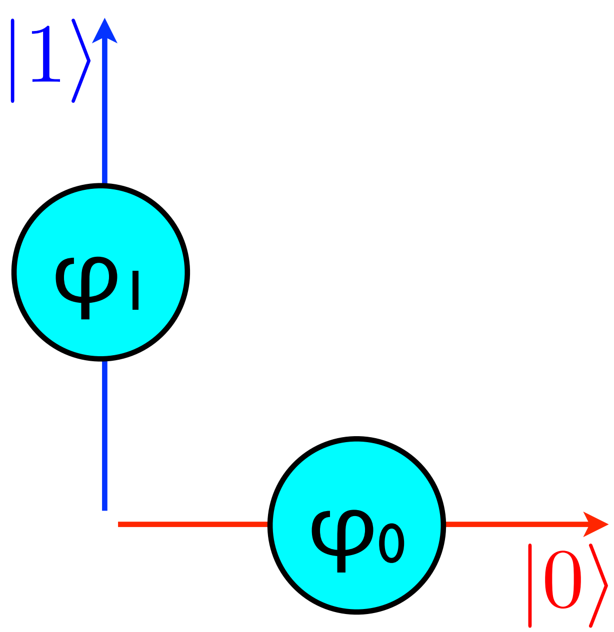

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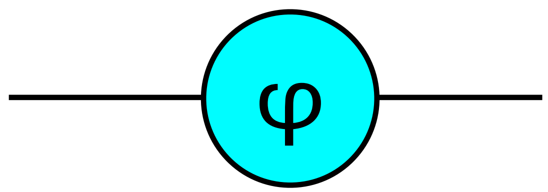
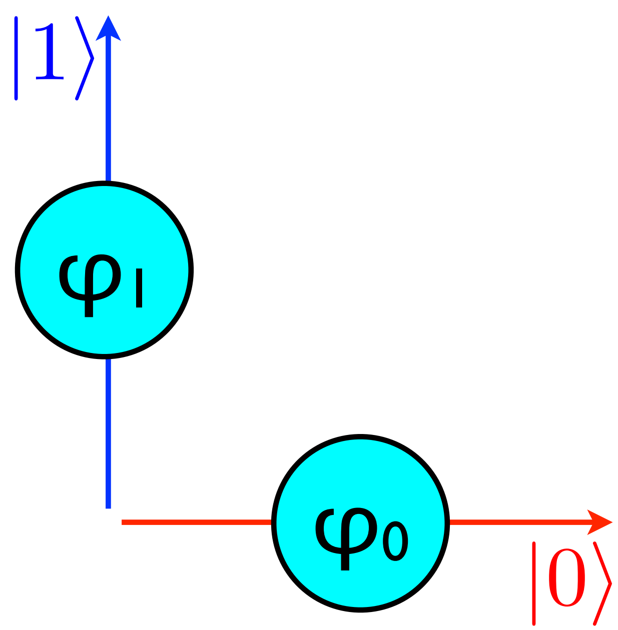


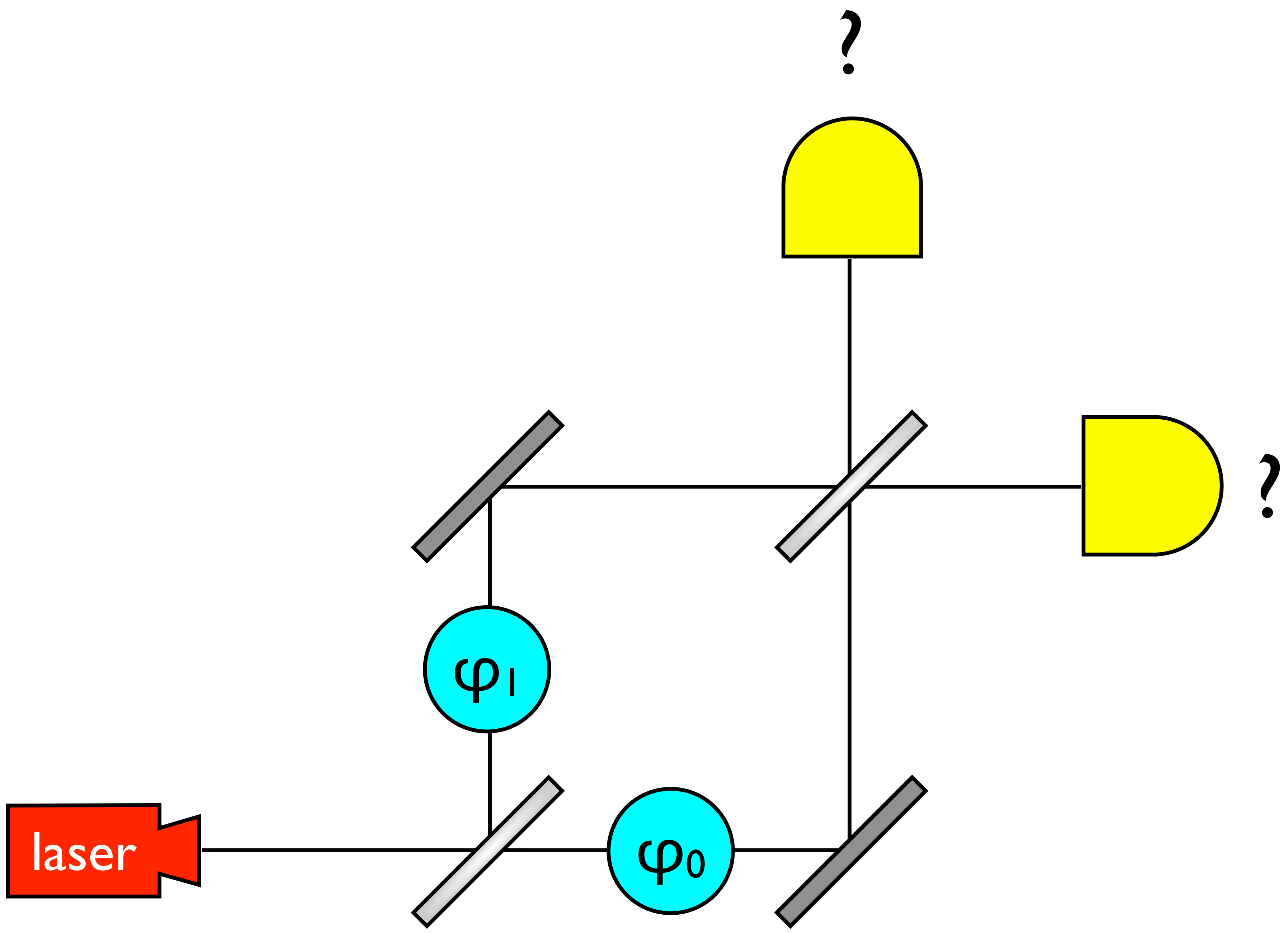
multiplies this portion of the state by $e^{i\varphi}$



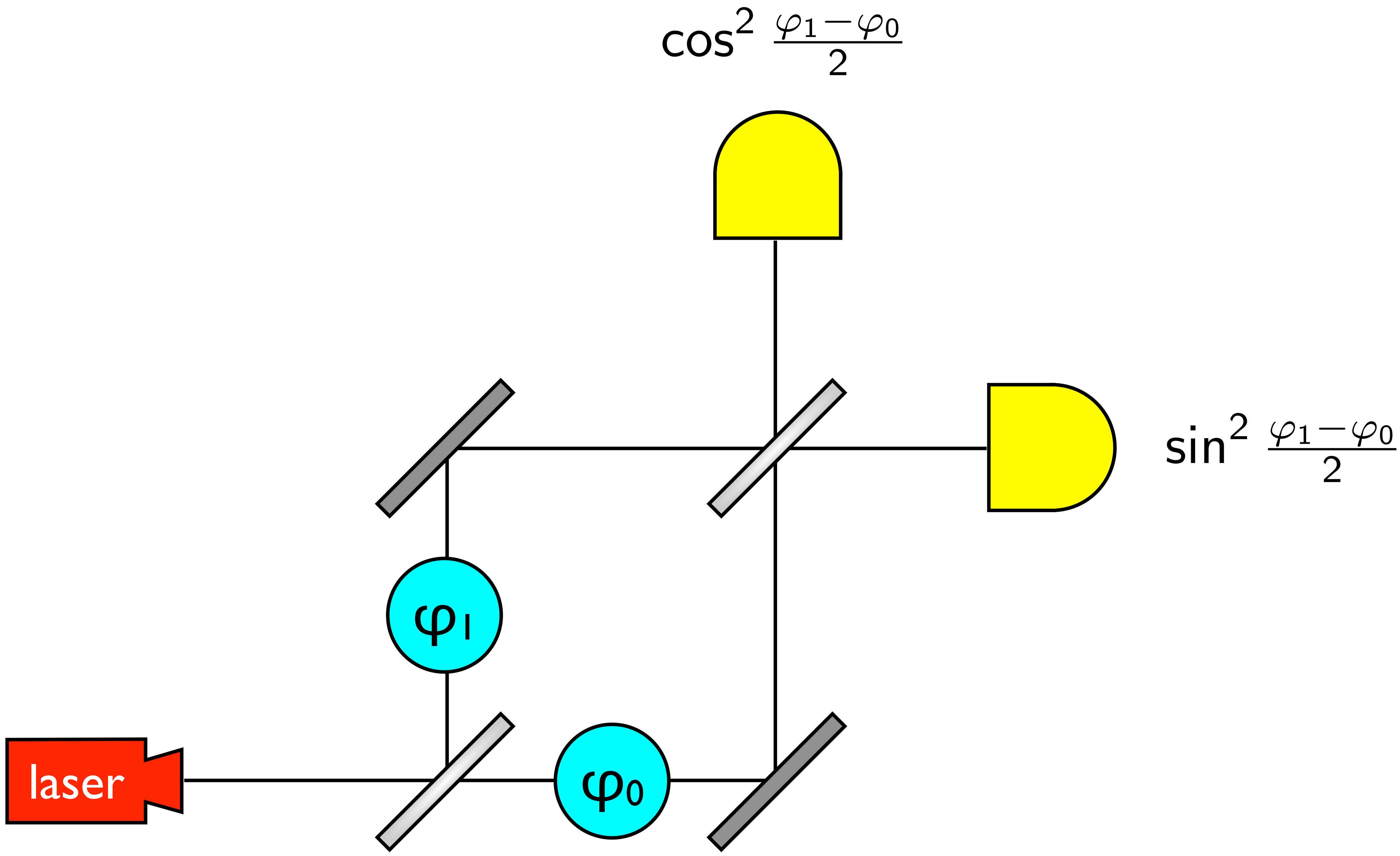
implements the unitary transformation

$$\begin{pmatrix} e^{i\varphi_0} & 0 \\ 0 & e^{i\varphi_1} \end{pmatrix}$$

Exercise: Interferometry with phase shifts



Exercise: Interferometry with phase shifts



Deutsch's problem

Given: A function $f\colon \{0,1\} \to \{0,1\}$

(As a black box: You can call the function f, but you can't read its source code.)

Task: Determine whether f is constant.

Deutsch's problem

Given: A function $f: \{0,1\} \rightarrow \{0,1\}$
(As a black box: You can call the function f, but you can't read its source code.)

Task: Determine whether f is constant.

Four possible functions:

x	$f_1(x)$
0	0
1	0

x	$f_2(x)$
0	1
1	1

x	$f_3(x)$
0	0
1	1

x	$f_4(x)$
0	1
1	0

Deutsch's problem

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Task: Determine whether f is constant.

Four possible functions:

constant

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x	$f_2(x)$
0	1
1	1

not constant

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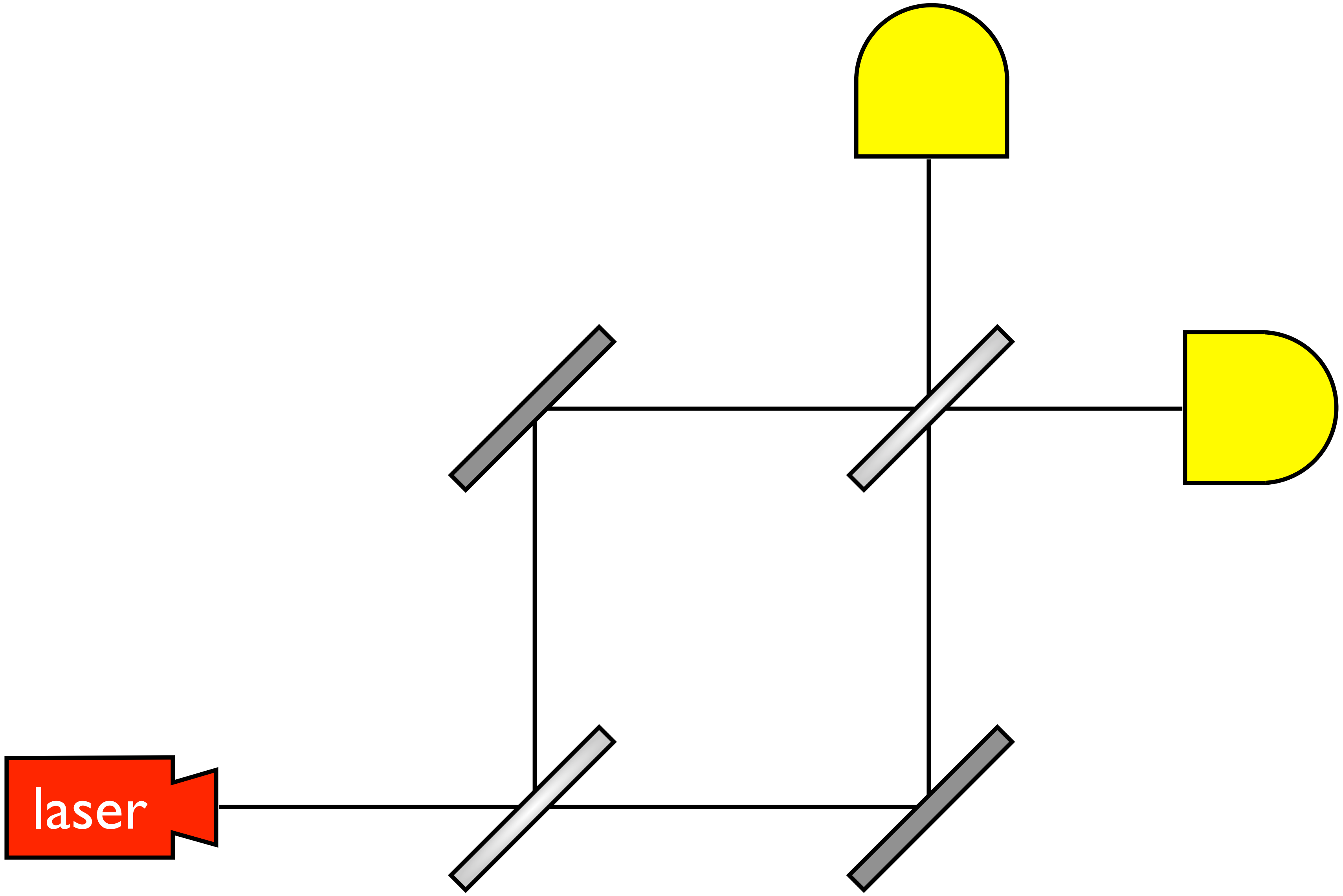
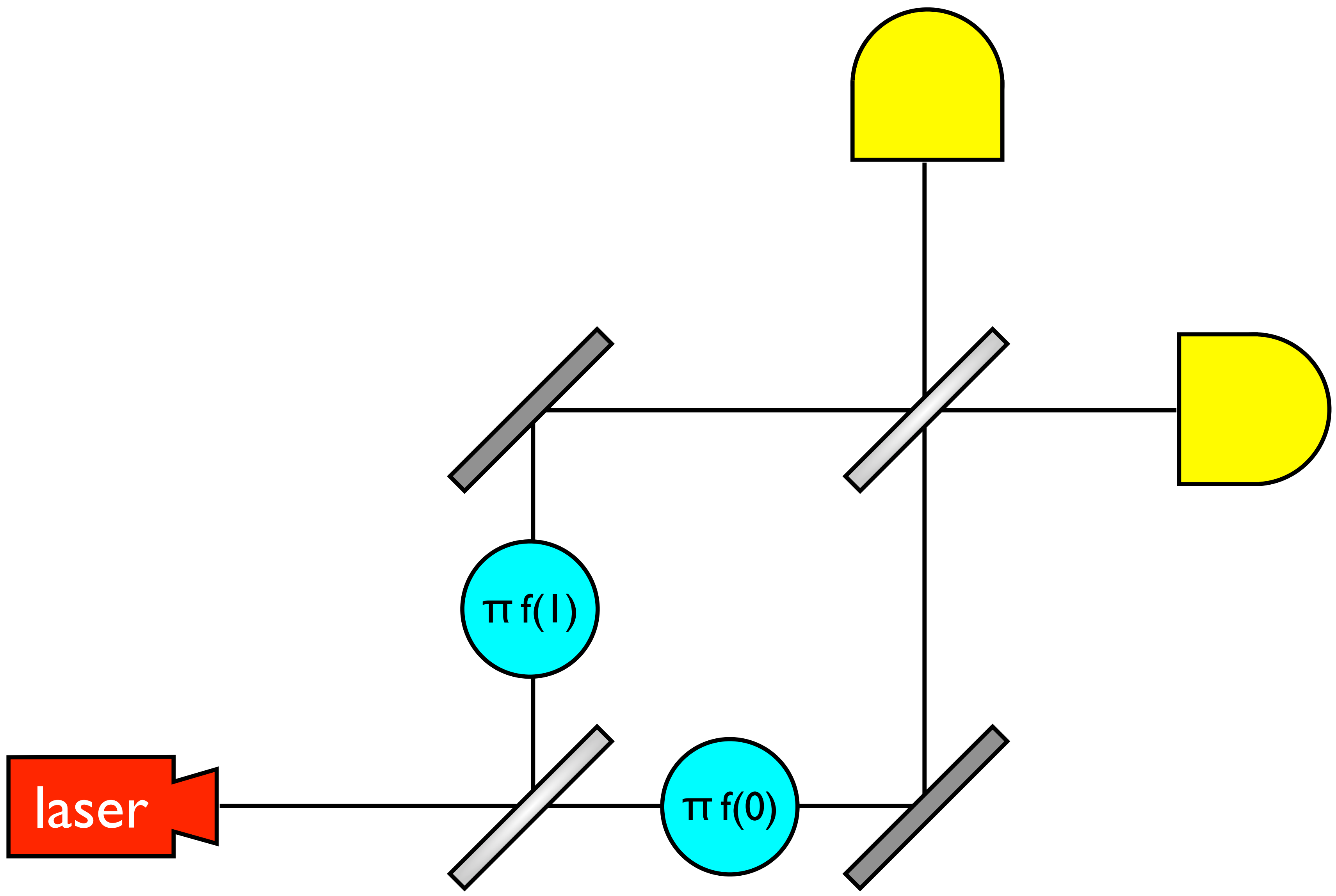
not constant

Classically, two function calls are required to solve this problem.

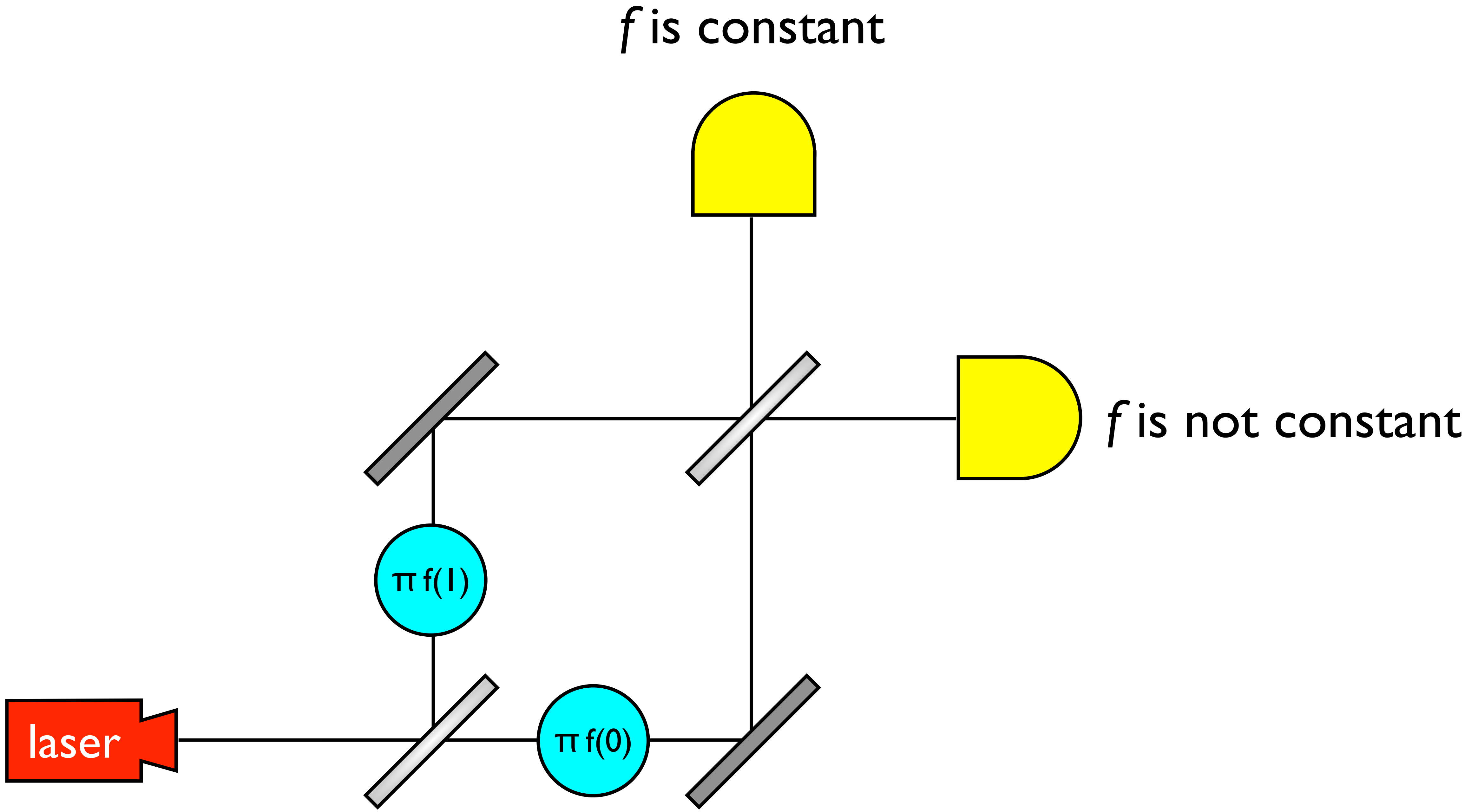
Deutsch's algorithm as interferometry



Deutsch's algorithm as interferometry



Deutsch's algorithm as interferometry



Exercise: More linear optics

What unitary transformation is implemented by the following optical setup?