ASSIGNMENT 10   ECE 103 (Spring 2009)

Due in tutorial on Monday, July 27.

1. The *balanced binary tree of height* n, T_n, is defined recursively as follows. T_0 is a single vertex. T_1 is obtained by adding two new vertices to T_0 and connecting each of them to the original vertex. In general, for $n \geq 2$, T_n is obtained from T_{n-1} by adding two new vertices for every leaf vertex and connecting each of them to the original leaf vertex.

   (a) Draw T_4.

   (b) How many leaves does T_n have?

   (c) How many vertices does T_n have?

   (d) How many edges does T_n have?

2. A graph with no cycles is called a *forest*. Prove that a forest with n vertices and m edges has $n - m$ components.

3. Prove or disprove the following statement: Given a graph G, if T and U are spanning trees of G, then T and U are isomorphic.

4. Construct a breadth-first search tree for the 3-cube. Denote each tree edge by an arrow from a vertex to its parent, and indicate each non-tree edge by a dashed line.