Operational Semantics Practice

1. Using the rules given below, show: $1 + 1 \Rightarrow 2$

$$\frac{}{n \Rightarrow n} \qquad \frac{e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2 \quad n_3 \ is \ n_1 + n_2}{e_1 + e_2 \Rightarrow n_3}$$

$$\frac{1 \Rightarrow 1 \quad 1 \Rightarrow 1 \quad 2 \ is \ 1 + 1}{1 + 1 \Rightarrow 2}$$

2. Using the rules given below, show: $1 + (2 + 3) \Rightarrow 6$

$$\frac{}{n \Rightarrow n} \qquad \frac{e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2 \quad n_3 \ is \ n_1 + n_2}{e_1 + e_2 \Rightarrow n_3}$$

$$\frac{1 \Rightarrow 1 \qquad \dfrac{2 \Rightarrow 2 \quad 3 \Rightarrow 3 \quad 5 \ is \ 2 + 3}{2 + 3 \Rightarrow 5} \qquad 6 \ is \ 1 + 5}{1 + (2 + 3) \Rightarrow 6}$$

3. Using the rules given below, show: $A;\ let\ y = 1\ in\ y \Rightarrow 1$

$$\frac{}{A;\ n \Rightarrow n} \qquad \frac{A(x) = v}{A;\ x \Rightarrow v}$$

$$\frac{A;\ e_1 \Rightarrow v_1 \quad A, x : v_1;\ e_2 \Rightarrow v_2}{A;\ \textbf{let}\ x = e_1\ \textbf{in}\ e_2 \Rightarrow v_2} \qquad \frac{A;\ e_1 \Rightarrow n_1 \quad A;\ e_2 \Rightarrow n_2 \quad n_3 \ is \ n_1 + n_2}{A;\ e_1 + e_2 \Rightarrow n_3}$$

$$\frac{A;\ 1 \Rightarrow 1 \qquad \dfrac{A, y : 1(y) = 1}{A, y : 1;\ y \Rightarrow 1}}{A;\ let\ y = 1\ in\ y \Rightarrow 1}$$

4. Using the rules given below, show: $A;\ let\ y = 1\ in\ let\ x = 2\ in\ x \Rightarrow 2$

$$\frac{}{A;\ n \Rightarrow n} \qquad \frac{A(x) = v}{A;\ x \Rightarrow v}$$

$$\frac{A;\ e_1 \Rightarrow v_1 \quad A, x : v_1;\ e_2 \Rightarrow v_2}{A;\ \textbf{let}\ x = e_1\ \textbf{in}\ e_2 \Rightarrow v_2} \qquad \frac{A;\ e_1 \Rightarrow n_1 \quad A;\ e_2 \Rightarrow n_2 \quad n_3\ is\ n_1 + n_2}{A;\ e_1 + e_2 \Rightarrow n_3}$$

$$\dfrac{A;\ 1 \Rightarrow 1 \qquad \dfrac{A, y : 1;\ 2 \Rightarrow 2 \qquad \dfrac{A, y : 1, x : 2(x) = 2}{A, y : 1, x : 2;\ x \Rightarrow 2}}{A, y : 1;\ let\ x = 2\ in\ x \Rightarrow 2}}{A;\ let\ y = 1\ in\ let\ x = 2\ in\ x \Rightarrow 2}$$

5. Translate the following rules into English.

$$\text{identity:}\frac{}{n \Rightarrow n} \qquad \text{addition:}\frac{e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2 \quad n_3\ is\ n_1 + n_2}{e_1 + e_2 \Rightarrow n_3}$$

- identity: n evaluates to n
- addition: Assuming e_1 evaluates to n_1 and e_2 evaluates to n_2 and $n_1 + n_2\ is\ n_3$ then $e_1 + e_2$ evaluates to n_3.