CMSC 330, Spring 2018 Quiz 2

Name
Discussion Time (circle one) 10am 11am 12pm 1pm 2pm 3pm
Discussion TA (circle one) BT Daniel Chris Alex Derek Pei-Jo Akbar Justin L.
Tal Shriraj Cameron Eric Kesha Kameron Michael S. Michael P.

Instructions
- Do not start this quiz until you are told to do so.
- You have 15 minutes for this quiz.
- This is a closed book quiz. No notes or other aids are allowed.
- For partial credit, show all your work and clearly indicate your answers.

1. (5 points) Using the rules given below, show

$1 \cdot 2 + 3 \Rightarrow 5$

In the rules, $n$ is a metavariable that refers to an integer, while $e$ is a metavariable that refers to an expression, whose form is according to the following grammar:

$$e ::= n | e + e | e \cdot e$$

Here are the rules:

$\frac{n \Rightarrow n}{n_1 \Rightarrow e_1 + e_2 \Rightarrow n_3 \text{ is } n_1 + n_2}
\frac{e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2 \quad n_3 \text{ is } n_1 + n_2}{e_1 + e_2 \Rightarrow n_3}
\frac{n_1 \Rightarrow e_1 \quad n_2 \Rightarrow e_2 \quad n_3 \text{ is } n_1 \cdot n_2}{e_1 \cdot e_2 \Rightarrow n_3}
\frac{1 \Rightarrow 1 \quad 2 \Rightarrow 2 \quad 2 \text{ is } 1 \cdot 2}{1 \cdot 2 \Rightarrow 2}
\frac{3 \Rightarrow 3 \quad 5 \text{ is } 2 + 3}{(1 \cdot 2) + 3 \Rightarrow 5}$
2. (7 points) Using the rules given below, show:

\[
\text{let } y = 1 \text{ in let } x = 3 \text{ in } x \Rightarrow 3
\]

In the rules, \(x\) is a metavariable that refers to an identifier (variable), \(n\) is a metavariable that refers to an integer, while \(e\) is a metavariable that refers to an expression, and \(A\) is a metavariable that refers to an environment. Grammars for the latter two are as follows:

\[
e ::= x | n | \text{let } x = e \text{ in } e
\]

\[
A ::= \cdot | A, x : n
\]

In the above, \(\cdot\) represents an empty environment, while \(A, x : n\) is the environment that extends \(A\) with a mapping from \(x\) to \(n\) (overriding any other mapping that might already be in \(A\) for \(x\)).

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A; n \Rightarrow n)</td>
<td>(A(x) = n)</td>
</tr>
<tr>
<td>(A; x \Rightarrow n)</td>
<td>(A; e_1 \Rightarrow v_1) (A, x : v_1; e_2 \Rightarrow v_2)</td>
</tr>
<tr>
<td>(A; 1 \Rightarrow 1)</td>
<td>(A, y : 1, x : 3(x) = 3)</td>
</tr>
<tr>
<td>(A, y : 1; let x = 3 in x \Rightarrow 3)</td>
<td>(A, y : 1; let x = 3 in x \Rightarrow 3)</td>
</tr>
</tbody>
</table>

3. (8 points) Give a Finite Automata that accepts a string on alphabet 0,1 if and only if it has an even number of 1’s and exactly one zero.