Name

Discussion Time (circle one)  
10am  11am  12pm  1pm  2pm  3pm

Discussion TA (circle one)  
BT  Daniel  Chris  Alex  Derek  Pei-Jo  Akbar  Justin L.  
Tal  Shiraj  Cameron  Eric  Kesha  Kameron  Michael S.  Michael P.

Instructions

• Do not start this quiz until you are told to do so.
• You have 15 minutes for this quiz.
• This is a closed book quiz. No notes or other aids are allowed.
• For partial credit, show all your work and clearly indicate your answers.

1. (5 points) Using the rules given below, show

\[ 1 \times 2 + 3 \Rightarrow 5 \]

In the rules, \( n \) is a metavariable that refers to an integer, while \( e \) is a metavariable that refers to an expression, whose form is according to the following grammar:

\[ e ::= n \mid e + e \mid e \times e \]

Here are the rules:

\[
\begin{align*}
\frac{\quad e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2 \quad n_3 \text{ is } n_1 + n_2}{n \Rightarrow n} \quad \\
\frac{e_1 + e_2 \Rightarrow n_3}{e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2}
\end{align*}
\]

\[
\begin{align*}
\frac{e_1 \Rightarrow n_1 \quad e_2 \Rightarrow n_2 \quad n_3 \text{ is } n_1 \times n_2}{e_1 \times e_2 \Rightarrow n_3}
\end{align*}
\]
2. (7 points) Using the rules given below, show:

\[
\text{let } y = 1 \text{ in let } x = 3 \text{ in } x \Rightarrow 3
\]

In the rules, \(x\) is a metavariable that refers to an identifier (variable), \(n\) is a metavariable that refers to an integer, while \(e\) is a metavariable that refers to an expression, and \(A\) is a metavariable that refers to an environment. Grammars for the latter two are as follows:

\[
e ::= x | n | \text{let } x = e \text{ in } e \\
A ::= \cdot | A, x:n
\]

In the above, \(\cdot\) represents an empty environment, while \(A, x:n\) is the environment that extends \(A\) with a mapping from \(x\) to \(n\) (overriding any other mapping that might already be in \(A\) for \(x\)).

\[
\begin{array}{c}
A; n \Rightarrow n \\
A; x \Rightarrow n \\
A; e_1 \Rightarrow v_1 \quad A, x : v_1; e_2 \Rightarrow v_2 \\
A; \text{let } x = e_1 \text{ in } e_2 \Rightarrow v_2
\end{array}
\]

3. (8 points) Give a Finite Automata that accepts a string on alphabet 0,1 if and only if it has an even number of 1's and exactly one zero.