# CMSC 330: Organization of Programming Languages

Theory of Regular Expressions

NFAs → DFAs

## Reminders

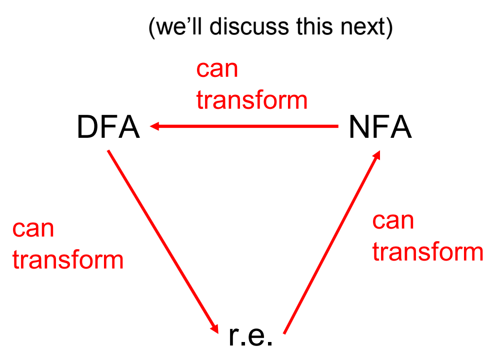
- Homework 1 due Sep. 20
- Project 1 due Sep. 24
- Exam 1 on Sep. 25
  - Study this weekend!
- Project 2 given out on Sep. 24.
  - Start soon!

## Review

- How are DFAs and NFAs different?
- When does an NFA accept a string?
- How do we convert from a regular expression to an NFA?
- What is the ε-closure of a state?



## Relating R.E.'s to DFAs and NFAs



## Reduction Complexity

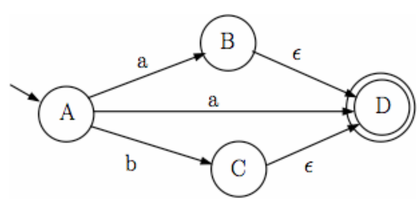
- Regular expression to NFA reduction:
  - O(n)

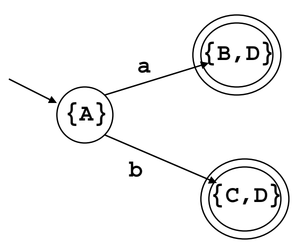
- NFA to DFA reduction
  - Intuition: Build DFA where each DFA state represents a set of NFA states
  - How many states could there be in the DFA?
  - Given NFA with n states, DFA may have $2^n$ states
  - Not so good, since DFAs are what we can implement easily



## NFA → DFA reduction

Example:



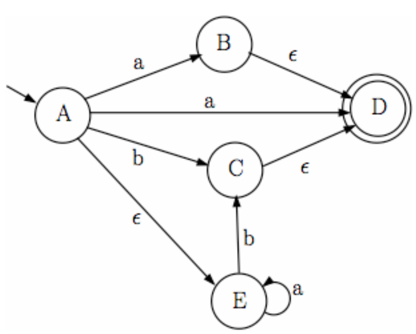
# NFA → DFA reduction Algorithm

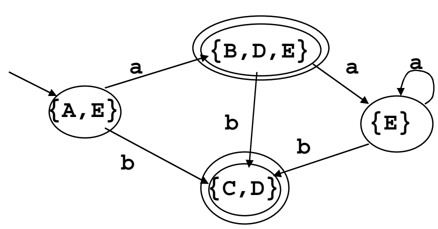
- Let $r_0$ be the $\varepsilon$-closure of $q_0$, add it to R
- While there is an unmarked state $r_i$ in R
  - Mark $r_i$
  - For each $a \in \Sigma$
    - Let $S = \{s \mid q \in r_i$ and for $\{q, a, B\} \in \delta, s \in B\}$
    - Let E = $\varepsilon$-closure(S)
    - If $E \notin R$
      - $R = E \cup R$
    - $\delta = \delta \cup \{ri, a, E\}$
- Let $r_f = \{r_i \mid \exists\, s \in r_i$ with $s \in q_f\}$

Notes: Let Q be the set of states for the NFA and R be the set of states for the DFA. All states are unmarked at creation.



# NFA → DFA example



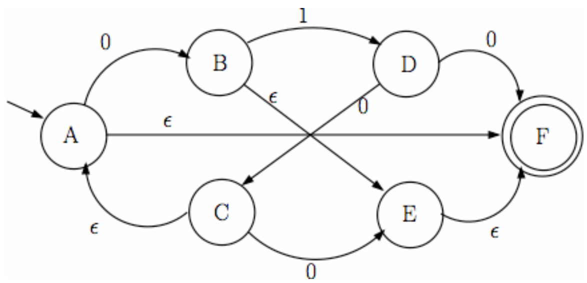




R = {{A,E}, {B,D,E}, {C,D}, {E}}

Language?
All strings that have exactly 1 b and end in b or the string a
Regular expression?
a*b|a

# Practice

Convert the NFA to a DFA:

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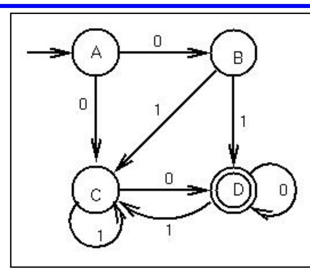
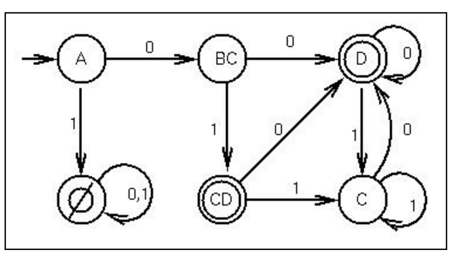
# Equivalence of DFAs and NFAs

- Any string from {A} to either {D} or {CD} represents a path from A to D in the original NFA.

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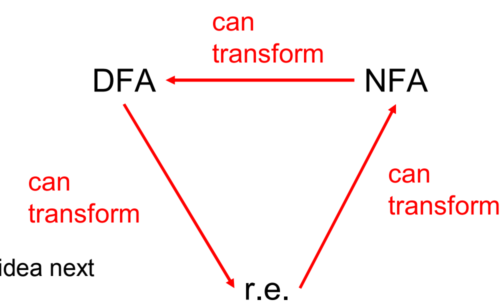
## Relating R.E.'s to DFAs and NFAs

- Regular expressions, NFAs, and DFAs accept the same languages!

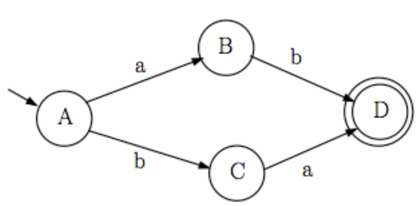


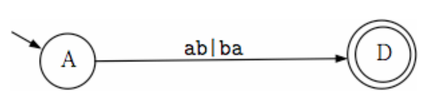

High-level idea next



## Converting from DFAs to REs

- General idea:
  - Remove states one by one, labeling transitions with regular expressions
  - When two states are left (start and final), the transition label is the regular expression for the DFA



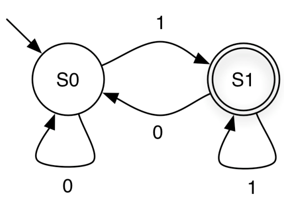
# Relating R.E's to DFAs and NFAs

- Why do we want to convert between these?
  - Can make it easier to express ideas
  - Can be easier to implement



# Implementing DFAs

It's easy to build a program which mimics a DFA




```
cur_state = 0;
while (1) {

  symbol = getchar();

  switch (cur_state) {

    case 0: switch (symbol) {
              case '0':  cur_state = 0; break;
              case '1':  cur_state = 1; break;
              case '\n': printf("rejected\n"); return 0;
              default:   printf("rejected\n"); return 0;
            }
            break;

    case 1: switch (symbol) {
              case '0':  cur_state = 0; break;
              case '1':  cur_state = 1; break;
              case '\n': printf("accepted\n"); return 1;
              default:   printf("rejected\n"); return 0;
            }
            break;

    default: printf("unknown state; I'm confused\n");
             break;
  }
}
```

## Implementing DFAs (Alternative)

Alternatively, use generic table-driven DFA

```
given components (Σ, Q, q0, F, δ) of a DFA:
let q = q0
while (there exists another symbol s of the input string)
   q := δ(q, s);
if q∈F then
   accept
else reject
```

- q is just an integer
- Represent δ using arrays or hash tables
- Represent F as a set



## Run Time of Algorithm

- Given a string s, how long does algorithm take to decide whether s is accepted?
  - Assume we can compute δ(q0, c) in constant time
  - Then the time per string s to determine acceptance is $O(|s|)$
  - Can't get much faster!
- But recall that constructing the DFA from the regular expression A may take $O(2^{|A|})$ time
  - But this is usually not the case in practice
- So there's the initial overhead, but then accepting strings is fast

## Regular Expressions in Practice

- Regular expressions are typically "compiled" into tables for the generic algorithm
  - Can think of this as a simple byte code interpreter
  - But really just a representation of $(\Sigma, Q_A, q_A, \{f_A\}, \delta_A)$, the components of the DFA produced from the r.e.
- Regular expression implementations often have extra constructs that are non-regular
  - I.e., can accept more than the regular languages
  - Can be useful in certain cases
  - Disadvantages: nonstandard, plus can have higher complexity



## Considering Ruby Again

- Interpreted
- Implicit declarations
- Dynamically typed
  - These three make it quick to write small programs
- Built-in regular expressions and easy string manipulation
  - This and the three above are the hallmark of scripting languages
- Object-oriented
  - Everything (!) is an object
- Code blocks
  - Easy higher-order programming!
  - Get ready for a lot more of this...

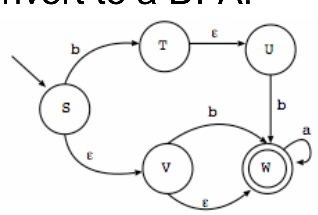
## Other Scripting Languages

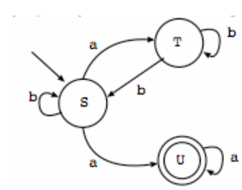
- Perl and Python are also popular scripting languages
  - Also are interpreted, use implicit declarations and dynamic typing, have easy string manipulation
  - Both include optional “compilation” for speed of loading/execution
- Will look fairly familiar to you after Ruby
  - Lots of the same core ideas
  - All three have their proponents and detractors
  - Use whichever one you like best



## Practice

Convert to a DFA:



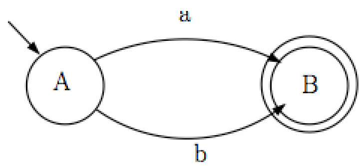

Convert to an NFA and then to a DFA:

- (0|1)*11|0*
- strings of alternating 0 and 1
- aba*|(ba|b)

## Complement of DFA

Given a DFA accepting language L, how can we create a DFA accepting its complement?

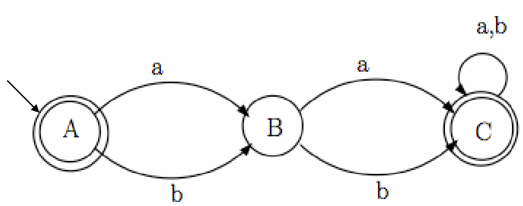
(the alphabet = {a,b})





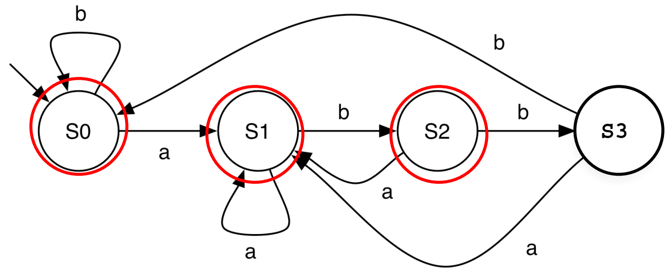

## Complement Steps

- Add implicit transitions to a dead state
- Change every accepting state to a non-accepting state and every non-accepting state to an accepting state
- Note: this *only* works with DFAs - Why?



## Practice

Make the DFA which accepts the complement of the language accepted by the DFA below.






## Practice

- Make the DFA which accepts all strings with a substring of 330
- Take the complement of this DFA